Five Dimensional Bianchi Type VI_0 Dark Energy Cosmological Model in General Relativity

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In this paper, we have constructed a self-consistent system of Bianchi Type VI_0 cosmology in five dimensions with a binary mixture of perfect fluid and dark energy. The perfect fluid chosen here obeys the usual equation of state $p = \rho\gamma$ with $0 \leq \gamma \leq 1$. The dark energy chosen is considered to be either the quintessence or Chaplygin gas. Exact solutions to the corresponding Einstein's field equations are obtained as a quadrature. The equation of state parameter for dark energy ω is found to be consistent with the recent observations of SNe Ia data (SNe Ia data with CMBR anisotropy) and galaxy clustering statistics. Here, our models predict that the rate of expansion of universe would increase with passage of time. The physical and geometric aspects of the models are also discussed in detail.

1. Introduction

The study of higher-dimensional space-time is important because of the underlying idea that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. In the latest study of super-strings and super-gravity theories, Weinberg [1] studied the unification of the fundamental forces with gravity, which reveals that the space-time should be different from four dimensions. Since the concept of higher dimensions is not unphysical, the string theories are discussed in 10-dimensions or 26-dimensions of space-time. Because of this, studies in higher dimensions inspired many researchers to enter into such a field of study to explore the hidden knowledge of the universe. Chodos and Detweller [2], Lorentz-Petzold [3], Ibanez and Verdaguer [4], Gleiser and Diaz [5], Reddy and Venkateswara [6], Adhav et al. [7] have studied the multi-dimensional cosmological models in general relativity and in other alternative theories of gravitation. Further, Marciano [8] has suggested that the experimental observation of fundamental constants with varying time could produce the evidence of extra dimensions. Rao et al. [9], Sahoo and Mishra [10], Tripathy et al. [11,12] studied various aspects of cosmic strings in various theories of relativity.

Recently, observed astronomical phenomena have revolutionized the understanding of cosmology. Dark energy is one of the central problems in theoretical physics and cosmology, and there are many papers about dark energy, however still there are many attempts to understand the nature of dark energy. Dark energy is a special form of energy that permeates all of space and tends to increase the rate of expansion of the universe. Dark energy is the most accepted theory to explain recent observations that the universe appears to be expanding at an accelerating rate. Consequences of combined analysis of Ia Supernovae (SNe Ia) observations [13-18], galaxy cluster measurements [19] and cosmic microwave background (CMB) data [20] have shown that dark energy causing cosmic acceleration dominates the present universe. This acceleration is observed at a very small red-shift showing that it is a recent phenomenon in the late universe. Dark energy and the accelerated expansion of the universe have been the direct prediction of the distant supernovae Ia observations which are also supported, indirectly, by the observations of the CMB anistropies, gravitational lensing and the studies of galaxy clusters. It is generally believed that the distant SNe Ia observations predict an accelerating expansion of the universe powered by some hypothetical source with negative pressure generally termed as 'dark energy'. Supernovae at relatively high red shift are found fainter than that predicted for an earlier-thought slowing-expansion and indicate that expansion of universe is actually speeding up.

The paramount characteristic of the dark energy is a constant or slightly changing energy density as the universe expands, but we do not know the

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nature of dark energy very well [21-27]. Dark Energy (DE) has been conventionally characterized by the equation of state (EoS) parameter $\omega = p/\rho$, which is not necessarily constant. The simplest DE candidate is the vacuum energy ($\omega = -1$), which is mathematically equivalent to the cosmological constant Λ. The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence ($\omega > -1$), phantom energy ($\omega < -1$) and quintom (that can across from phantom region to quintessence region) as evolved and have time dependent EoS parameter. Some other limits obtained from observational results coming from SN Ia data [28] and SN Ia data collaborated with CMBR anisotropy and galaxy clustering statistics [29] are ($-1.67 < \omega < -0.62$) and ($-1.33 < \omega < -0.79$), respectively. However, it is not at all obligatory to use a constant value of ω. Due to the lack of observational evidence in making a distinction between constant and variable ω, usually the equation of state parameter is considered as a constant [30,31] with phase wise value -1,0, 1/3 and +1 for vacuum fluid, dust fluid, radiation and stiff fluid dominated universe, respectively. But in general, ω is a function of time or redshift [32,33]. For instance, quintessence models involving scalar fields give rise to time dependent EoS parameter ω [34-37]. Also some literature is available on models with varying fields, such as cosmological model with variable equation of state parameter in Kaluza-Klein metric and wormholes [38]. In recent years various form of time dependent ω have been used for variable Λ models [39,40]. Recently Ray [41], Akarsu and Kilinc [42] and Yadav et al. [43] have studied dark energy model with variable EoS parameter. Recently, it has been suggested that the change of behavior of the missing energy density might be regulated by the change in the equation of state of the background fluid instead of the form of the potential, thereby avoiding the above mentioned fine-tuning problems. This is achieved via the introduction, within the framework of FRW cosmology, of an exotic background fluid, the Chaplygin gas, described by the equation of state $p = -\frac{q}{\rho_c}$ where q being a positive constant.

Several authors have studied various aspects of Chaplygin gas and further used it to obtain different cosmological models. The generalized Chaplygin gas (GCG) [44] is a recent proposal in order to explain the observed acceleration of the universe. This exotic fluid has been considered as an alternative to quintessence and to the cosmological constant, which are other serious candidates to explain the accelerated expansion of the universe. Many observational constraints have been obtained for cosmological models based on the GCG [22]. This model gives the cosmological evolution from the initial dust-like matter to an asymptotic cosmological constant with an epoch that can be seen as a mixture of a cosmological constant and a fluid obeying an equation of state $p = \omega\rho$. In particular, the Chaplygin gas behaves as pressureless fluid for small values of scale factor and as a cosmological constant for large values of scale factor which tends to accelerate the expansion. Mishra and Sahoo [45] investigated kink space-time in scale invariant theory with wet dark fluid, which is a candidate for dark energy.

In the present paper, we have considered a Bianchi type VI_0 space-time when the universe is filled with perfect fluid and dark energy. We also study the dark energy, which is considered to be either the Quintessence or the Chaplygin gas. The physical and kinematical properties of the models are also discussed.

2. Metric and Field Equations

The Einstein field equations are in the form

$$R_i^j - \frac{1}{2}\delta_i^j R = \kappa T_i^j \tag{1}$$

Where, R_i^j is the Ricci tensor, R is Ricci Scalar and κ is the Einstein gravitational constant. We study the gravitational field given by five dimensional Bianchi Type VI_0 cosmological model as choose it in the from

$$\begin{aligned} ds^2 &= dt^2 - A^2dx^2 - B^2e^{-2m^2x}dy^2 \\ &\quad -C^2e^{2m^2x}dz^2 - D^2e^{-2mx}d\psi^2 \end{aligned} \tag{2}$$

Where the metric functions A ,B ,C, D are functions of 't' only and m is a constant.

The energy momentum-tensor of the source is given by

$$T_i^j = (\rho_+ p)u_i u^j - p\delta_i^j \tag{3}$$

Where, u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1 \tag{4}$$

Here ρ is the total energy density of perfect fluid and dark energy while p is the corresponding pressure. p and ρ are related by an equation of state, $p = \gamma\rho$.

In a co-moving system of co-ordinates form (Eqn. (3)), we get

$$T_1^1 = \rho, T_2^2 = T_3^3 = T_4^4 = -p \quad (5)$$

The Einstein field equations for the metric (Eqn. (2)) can be obtained as

$$\frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{A'D'}{AD} + \frac{B'C'}{BC} + \frac{B'D'}{BD} + \frac{C'D'}{CD} - \frac{3m^4}{A^2} = \kappa\rho \quad (6)$$

$$\frac{B''}{B} + \frac{C''}{C} + \frac{D''}{D} + \frac{B'C'}{BC} + \frac{B'D'}{BD} + \frac{C'D'}{CD} = -\kappa p \quad (7)$$

$$\frac{A''}{A} + \frac{C''}{C} + \frac{D''}{D} + \frac{A'C'}{AC} + \frac{A'D'}{AD} + \frac{C'D'}{CD} - \frac{2m^4}{A^2} = -\kappa p \quad (8)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{D''}{D} + \frac{A'B'}{AB} + \frac{A'D'}{AD} + \frac{B'D'}{BD} - \frac{2m^4}{A^2} = -\kappa p \quad (9)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{C''}{C} + \frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} - \frac{2m^4}{A^2} = -\kappa p \quad (10)$$

$$\frac{A'}{A} + \frac{C'}{C} = \frac{B'}{B} + \frac{D'}{D} \quad (11)$$

Here afterwards the dash over field variable represents ordinary differentiation with respect to t.

From Eqn. (11), we obtained

$$AC = BD \quad (12)$$

Let V be the function of time t defined by

$$V = ABCD \quad (13)$$

From Eqns. (12) and (13), we obtained

$$V = A^2C^2 \quad (14)$$

Now adding Eqns. (7), (8), (9), (10) and four times of Eqn. (6), we found

$$\left(\frac{A''}{A} + \frac{B''}{B} + \frac{C''}{C} + \frac{D''}{D}\right)$$

$$+2\left(\frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{A'D'}{AD} + \frac{B'C'}{BC} + \frac{B'D'}{BD} + \frac{C'D'}{CD}\right)$$

$$-\frac{6m^4}{A^2} = \frac{4\kappa}{3}(\rho - p) \quad (15)$$

From Eqns. (13) and (15), we obtained

$$\frac{V''}{V} - \frac{6m^4}{A^2} = \frac{4\kappa}{3}(\rho - p) \quad (16)$$

The conservational law for the energy momentum tenser gives

$$\rho' = -\frac{V''}{V}(\rho + p) \quad (17)$$

3. Solutions

Here we discuss two cases:

Case I: When $A = \sqrt{V}$, then Eqn. (16) reduces to

$$\frac{V''}{V} - \frac{6m^4}{V} = \frac{4\kappa}{3}(\rho - p) \quad (18)$$

Now, from Eqns. (17) and (18), we have

$$V' = \pm\sqrt{2\left[\frac{2}{3}\kappa\rho V^2 + 3m^4 V + C_1\right]} \quad (19)$$

Where, C_1 is the integration constant.

Rewriting Eqn. (17) in the form

$$\frac{\rho'}{(\rho + p)} = -\frac{V'}{V} \quad (20)$$

We know that the pressure and the energy density obeys an equation of state of type $p = f(\rho)$. So we conclude that ρ and p are function of V. Hence the right-hand side of Eqn. (16) is a function V. Now Eqn. (16) can be written as

$$V'' = \frac{4\kappa}{3}(\rho - p)V + 6m^4 \cong F(V) \quad (21)$$

From the mechanical point of view, Eqn. (21) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then

$$V' = \pm\sqrt{2(\epsilon - U(V))} \quad (22)$$

Here ϵ also can be viewed as energy and U is the potential energy of the force F.

Comparing Eqns. (19) and (22), we get

$$\epsilon = C_1 \text{ and } U = -\left(\frac{2\kappa}{3}V^2\rho + 3m^4 V\right) \quad (23)$$

Finally, the solution to Eqn. (19) in quadrature form can be written as

$$\int \frac{dV}{\sqrt{2\left(\frac{2\kappa}{3}V^2\rho + 3m^4V + C_1\right)}} = t + t_0 \quad (24)$$

Where the integration constant t_0 taken as zero, since it only gives a shift in time.

Case II: When $C = \sqrt{V}$, then Eqn. (16) reduces to

$$\frac{V^{''}}{V} - 6m^4 = \frac{4\kappa}{3}(\rho - p) \quad (25)$$

Now, from Eqns. (17) and (25), we have

$$V^{'} = \pm\sqrt{2\left[\frac{2}{3}\kappa\rho V^2 + \frac{3}{2}m^4V^2 + C_2\right]} \quad (26)$$

Where, C_2 is the constant of integration.

As before, the pressure and the energy density obeys an equation of state of type $p = f(\rho)$. So we conclude that ρ and p are function of V. Hence the right-hand side of Eqn. (16) is a function V. Now Eqn. (16) can be written as

$$V^{''} = \frac{4\kappa}{3}(\rho - p)V + 6m^4 \cong F(V) \quad (27)$$

From the mechanical point of view Eqn. (27) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then

$$V^{'} = \pm\sqrt{2(\epsilon - U(V))} \quad (28)$$

Here also ϵ also can be viewed as energy and $U(V)$ is the potential energy of the force F.

Comparing Eqns. (26) and (28), we get

$$\epsilon = C_2 \text{ and } U(V) = -\left(\frac{2\kappa}{3}V^2\rho + \frac{3}{2}m^4V^2\right) \quad (29)$$

Finally, we obtained the solution to Eqn. (26) in quadrature form as

$$\int \frac{dV}{\sqrt{2\left(\frac{2\kappa}{3}V^2\rho + \frac{3}{2}m^4V^2 + C_2\right)}} = t + t_0 \quad (30)$$

Where the integration constant t_0 taken as zero, since it only gives a shift in time.

4. Universe with Perfect Fluid and Dark Energy

We consider the evolution of the Bianchi type VI_0 universe filled with perfect fluid and dark energy.

$$\rho = \rho_{PF} + \rho_{DE}; p = p_{PF} + p_{DE} \quad (31)$$

The energy momentum tensor can be decomposed as

$$\begin{aligned} T_i^j &= (\rho_{PF} + \rho_{DE} + p_{PF} + p_{DE})U_iU^j \\ &\quad -(p_{DE} + p_{PF})\delta_i^j \end{aligned} \quad (32)$$

Here ρ_{DE} and p_{DE} respectively denotes the dark energy density and pressure whereas ρ_{PF} and p_{PF} denotes the energy density and pressure of the perfect fluid, respectively. The perfect fluid obeys the equation of state

$$p_{PF} = \gamma\rho_{PF} \quad (33)$$

Where, $0 \leq \gamma \leq 1$ is a constant.

Depending on numerical valuesof γ, it describes the following types of universe

$$\gamma = 0 \text{ (Dust Universe)} \quad (34)$$

$$\gamma = \frac{1}{3} \text{ (Radiation Universe)} \quad (35)$$

$$\gamma = (0, 1) \text{ (Hard Universe)} \quad (36)$$

$$\gamma = 1 \text{ (Zeldovich Universe)} \quad (37)$$

In a co-moving frame, the conservation law of energy momentum tensor leads to the balance equation for the energy density.

$$\rho_{DE}^{'} + \rho_{PF}^{'} = -\frac{V'}{V}(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}) \quad (38)$$

The dark energy is supposed to interact with itself only and it is minimally coupled to the gravitational field. As a result, the evolution for the energy density decouples from that of the perfect fluid and from Eqn. (38), we obtained the two balance equations as

$$\rho_{DE}^{'} + \frac{V'}{V}(\rho_{DE} + p_{DE}) = 0 \quad (39)$$

$$\rho_{PF}^{'} + \frac{V'}{V}(\rho_{PF} + p_{PF}) = 0 \quad (40)$$

From Eqns. (32) and (39), we get

$$\rho_{PF}^{'} = \frac{\rho_0}{V^{1+\gamma}}, p_{PF} = \frac{\rho_0\gamma}{V^{1+\gamma}} \quad (41)$$

Where, ρ_0 is an integration constant.

5. Solutions with Perfect Fluid and Dark Energy

Here we consider the cases where dark energy is given by quintessence and represented by Chaplygin gas.

5.1. Dark energy by quintessence

Here the dark energy is given by a quintessence which obeys the equation of state

$$p_q = \omega_q \rho_q \tag{42}$$

where the constant $\omega_q = [-1, 0]$. From Eqns. (39) and (46), we obtained

$$\rho_q = \frac{\rho_{0_q}}{V^{1+\omega_q}}; p_q = \frac{\omega_q \rho_{0_q}}{V^{1+\omega_q}} \tag{43}$$

Where, ρ_{0_q} is an integration constant.

Case I: When $A = \sqrt{V}$, then the evolution of V can be written as

$$V^{''} = \frac{4\kappa}{3}\left[\frac{(1-\gamma)\rho_0}{V^\gamma} + \frac{(1-\omega_q)\rho_{0_q}}{V^{\omega_q}}\right] + 6m^4 \tag{44}$$

Eqn. (48) can be written in quadrature form as

$$\int \frac{dV}{\sqrt{2\left[\frac{2}{3}\left(\kappa\rho_0 V^{1-\gamma} + \kappa\rho_{0_q} V^{(1-\omega_q)}\right) + 3m^4 V + C_1\right]}} = t + t_0 \tag{45}$$

Here t_0 is a constant of integration and can be taken as zero.

Case II: When $C = \sqrt{V}$, then the evolution of V can be written as

$$V^{''} = \frac{4\kappa}{3}\left[\frac{(1-\gamma)\rho_0}{V^\gamma} + \frac{(1-\omega_q)\rho_{0_q}}{V^{\omega_q}}\right] + 6m^4 V \tag{46}$$

Eqn. (50) can be written in quadrature form as

$$\int \frac{dV}{\sqrt{2\left[\frac{2}{3}\left(\kappa\rho_0 V^{1-\gamma} + \kappa\rho_{0_q} V^{(1-\omega_q)}\right) + 3m^4 V^2 + C_2\right]}} = t + t_0 \tag{47}$$

Here t_0 is a constant of integration and can be taken as zero. In the limit of high matter densities ($\gamma = 1$), the general solution of the gravitational equations for Bianchi type VI_0 cannot be expressed in an exact analytic form.

5.2. Dark energy by Chaplygin gas

In this case, we consider the dark energy is represented by Chaplygin gas governed by the equation of state given as

$$p_c = -\frac{q}{\rho_c} \tag{48}$$

Where, q being a positive constant.

From Eqns. (39) and (52), we get

$$\rho_c = \sqrt{(\frac{\rho_{0_c}}{V^2} + q)}; p_c = -\frac{q}{\sqrt{(\frac{\rho_{0_c}}{V^2} + q)}} \tag{49}$$

Where, ρ_{0_c} is a constant of integration.

Case I: When $A = \sqrt{V}$, then the evolution equation of (Eqn. (15)) for V can be written as

$$V^{''} = \frac{4\kappa}{3}\left[\frac{(1-\gamma)\rho_0}{V^\gamma} + \sqrt{\rho_{0_c} + qV^2} + \frac{qV^2}{\sqrt{\rho_{0_c} + qV^2}}\right] + 6m^4 \tag{50}$$

The corresponding solution in quadrature form is written as

$$\int \frac{dV}{\sqrt{2\left[\frac{2}{3}\kappa\left(\rho_0 V^{1-\gamma} + \sqrt{\rho_{o_c} V^2 + qV^4}\right) + 3m^4 V + C_1\right]}} = t + t_0 \tag{51}$$

Here t_0 is a constant of integration and can be taken as zero.

Case II: When $C = \sqrt{V}$, then the evolution equation of (15) for V can be written as

$$V^{''} = \frac{4\kappa}{3}\left[\frac{(1-\gamma)\rho_0}{V^\gamma} + \sqrt{\rho_{0_c} + qV^2} + \frac{qV^2}{\sqrt{\rho_{0_c} + qV^2}}\right] + 6m^4 V \tag{52}$$

The corresponding equation in quadrature form as

$$\int \frac{dV}{\sqrt{2\left[\frac{2}{3}\kappa\left(\rho_0 V^{1-\gamma} + \sqrt{\rho_{o_c} V^2 + qV^4}\right) + \frac{3}{2}m^4 V^2 + C_2\right]}} = t \tag{53}$$

where the second integration is taken to be zero.

6. A Particular Case

When $\gamma = 0$

Case I: When $A = \sqrt{V}$. For $C_1 = 0$, Eqn. (42) reduces to

$$\int \frac{dV}{\sqrt{2\left(\frac{2}{3}\kappa V\rho_0 + 3m^4 V\right)}} = t \tag{54}$$

which gives

$$V = \left(\frac{1}{3}\kappa\rho_0 + \frac{3}{2}m^4\right)t^2 \tag{55}$$

with the increase in t, the volume increases and $V \to \infty$ as $t \to \infty$.

From Eqns. (41) and (59), we get

$$p = 0 \tag{56}$$

and

$$\rho = \frac{\rho_0}{\left(\frac{1}{3}\kappa\rho_0 + \frac{3}{2}m^4\right)t^2} \tag{57}$$

It should be noted that at the early stage of evolution of universe when the volume scale V was close to zero, the energy density of the universe was infinitely large. On the other hand, with the expansion of the universe i.e., with the increase of V, the energy density ρ decreases and an becomes infinitely large as $V \to 0$. By using Eqns. (59) and (60), we obtain

$$\omega' = \frac{p}{\rho} = 0 \tag{58}$$

From Eqns. (60), (61) and (62), it is observed that for this particular value of γ, our model represents dust universe. This is natural for ordinary non-relativistic matter.

Case II: When $C = \sqrt{V}$. For $C_2 = 0$, Eqn. (44) reduces to

$$\int \frac{dV}{\sqrt{2\left(\frac{2}{3}\kappa V\rho_0 + \frac{3}{2}m^4 V^2\right)}} = t \tag{59}$$

which gives

$$V = \frac{1}{2}\left[e^{t\sqrt{3m^4}} + \left(\frac{2\kappa\rho_0}{9m^4}\right)^2 e^{-t\sqrt{3m^4}} - \left(\frac{4\kappa\rho_0}{9m^4}\right)\right] \tag{60}$$

with an increase in t, the volume increases and $V \to \infty$ as $t \to \infty$.

From Eqns. (41) and (64), we get

$$p = 0 \tag{61}$$

$$\rho = \frac{\rho_0}{\frac{1}{2}\left[e^{t\sqrt{3m^4}} + \left(\frac{2\kappa\rho_0}{9m^4}\right)^2 e^{-t\sqrt{3m^4}} - \left(\frac{4\kappa\rho_0}{9m^4}\right)\right]} \tag{62}$$

It is noted here that when the volume scale V is close to zero, at an early stage of evolution of universe, the energy density of the universe is infinitely large. Moreover, with the expansion of the universe, that is, with the increase of V, the energy density ρ decreases and becomes infinitely large when $V \to 0$.

Using Eqns. (65) and (66), we again obtain

$$\omega' = \frac{p}{\rho} = 0 \tag{63}$$

Again from Eqns. (65), (66) and (67), it is observed that our model represents the dust universe.

7. Conclusions

The Bianchi type VI_0 universe has been considered for a mixture of a perfect fluid and dark energy in five dimensions. The solution has been obtained in quadrature form. Here, it is interesting to note that the inclusion of dark energy into the system gives rise to an accelerated expansion of the model. As a result, volume V approaches to infinite quicker than it does when the universe is filled with perfect fluid alone. The equation of state parameter for dark energy ω is found to be consistent with recent observations. The energy density ρ tends to zero as time increases indefinitely and that it leads to singularities at $t = 0$. The significance of the work satisfies the recent cosmological observations that our universe experiences accelerated expansion.

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