Gain Analysis for a 2-Pump Fibre Optical Parametric Amplifier

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In this study, we present the analysis of gain flatness in 2-pump Fibre Optical Parametric Amplifiers (FOPA) based on four-wave mixing (FWM) using numerical simulations. The influence of highly nonlinear fibre (HNLF) and higher order dispersion parameters is closely looked into. Results show that the nonlinear coefficient, γ affect the gain flatness of a FOPA. The third order and fourth order dispersion parameters play a vital role in predicting the gain profile of the FOPA. Polarization Mode Dispersion (PMD) is found to induce fluctuations that alter the FOPA gain magnitude and bandwidth. It is also found that amplifier gain increases with respective increase in fiber length. These results help in improving the transmission capacity of long haul system and dense wavelength division multiplexing (DWDM).

1. Introduction

The development of Fiber Optical Parametric Amplifiers (FOPA) has led to a dramatic increase in the transport capacity of fiber communication systems. FOPA is versatile and can be used as an optical amplifier (stand-alone high-intensity light source) as well as in signal processing (as gain media) such as wavelength conversion, optical multiplexing, sampling, limiting, switching, and dispersion monitoring [1]. The original intent in fiber amplifier development was to provide a simpler alternative to the electronic repeater, chiefly by allowing the signal to remain in optical form throughout a link or network [2]. However, they have attracted attention because of their versatility. They are practically used in most long-haul optical fiber links and advanced large-scale networks. Several designs, both in single and dual pump schemes, have been investigated both theoretically and experimentally. These developments have been made with emphasis on high and wide gain [3,4] as well as fluctuations such as zero-dispersion wavelengths [5], PMD [6] and polarization dependence of the gain [7].

The most important advantage of dual pump FOPA is that they can provide relatively flat gain over a much wider bandwidth than what is possible with single-pump FOPA [6]. More work needs to be done to improve the gain and its flatness and thus increase the operation window of FOPAs. FOPAs make use of the nonlinearities in the fiber.

Nonlinear effects in optical fibers such as Stimulated Raman Scattering (SRS), Stimulated Brillouin Scattering (SBS) and optical Kerr effect have many useful applications in tele-communication and optical signal processing. The optical Kerr effect, in which the refractive index changes with optical power, leads to various secondary effects, such as Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM), four wave mixing (FWM), and modulation instability. Influence of FWM has been reported in our previous work [8]. Applications using the Kerr effects include optical parametric amplification, frequency conversion, optical phase conjugation, pulse compression and regeneration and optical soliton propagation. In this work, we investigate the evolution of the gain spectra of two-pump FOPAs over their complete range, with emphasis on the gain flatness. We first present the relevant theory, and then present theoretical results for the FOPA on a HNLF.

2. Theory

Four-wave mixing in FOPAs makes use of the cubic non linearity in glass and requires a strong pump(s) of frequency ω_p and power P_p. When a signal (ω_s) and two pump light waves (ω_{p1}, ω_{p2}) are input into the nonlinear glass fiber, an additional new light wave known as an "idler" (ω_i) is generated

$$\omega_i = \omega_{p1} + \omega_{p2} - \omega_s \quad (1)$$

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That is, energy is conserved [9]. The following equations describe the propagation characteristics of the signal, the idler and the pump lights

$$\frac{dP_p}{dz} = \alpha P_p - 4\gamma\left[\left(P_s P_i P_p^2\right)^{\frac{1}{2}} \sin\theta\right] \tag{2}$$

$$\frac{dP_s}{dz} = \alpha P_s + 2\gamma\left[\left(P_s P_i P_p^2\right)^{\frac{1}{2}} \sin\theta\right] \tag{3}$$

$$\frac{dP_i}{dz} = \alpha P_i + 2\gamma\left[\left(P_s P_i P_p^2\right)^{\frac{1}{2}} \sin\theta\right] \tag{4}$$

Where, P $(_{j=p,s,i})$-Power (pump, signal and idler), α-attenuation constant, γ-nonlinear coefficient, z-longitudinal coordinate along the transmission media and θ is the phase angle.

The propagation characteristics of the relative phase is given by

$$\frac{d\theta}{dz} = \Delta\beta + \gamma\left(2P_p - P_s - P_i\right) + \gamma\left[\left(\frac{P_p^2 P_i}{P_s}\right)^{\frac{1}{2}} + \left(\frac{P_p^2 P_s}{P_i}\right)^{\frac{1}{2}} - 4\left(P_s P_i\right)^{\frac{1}{2}}\right]\cos\theta \tag{5}$$

$\Delta\beta$ is the linear phase shift while the remaining part is nonlinear phase shift due to the nonlinearity of the fiber.

This process requires phase matching, and its efficiency depends on the wave number mismatch, k, defined by

$$k = \Delta k + \gamma\left(P_1 + P_2\right) \tag{6}$$

Where, Δk is the linear phase mismatch while the remaining term is the nonlinear phase mismatch.

To simulate the frequency dependence of the optical amplification characteristics based on FWM we need to expand Δk (from Eqn. 6) in Taylor series up to fourth order around the zero dispersion wavelengths ω_0.

$$\Delta\beta = \left\{\begin{matrix} \beta_3\left(\omega_p - \omega_0\right) + \\ \frac{\beta_4}{2}\left[\begin{matrix}\left(\omega_p - \omega_0\right)^2 + \\ \frac{1}{6}\left(\omega_p - \omega_s\right)^2\end{matrix}\right]\end{matrix}\right\}\left(\omega_p - \omega_s\right)^2 \tag{7}$$

When, $\omega_p = \omega_0$, Δk depend on β_4 only.

The parametric gain can be obtained by taking into account the signal and idler waves. This is given by

$$g = \sqrt{4\gamma^2 P_1 P_2 - \left(\frac{k + \delta k}{2}\right)^2} \tag{8}$$

Where, *k* is the standard phase mismatch and δ*k* is the instantaneous phase mismatch due to pump phase modulation.

$$\delta k = \frac{\beta_3}{2}\left(\Delta\omega_s^2 - \Delta\omega_p^2\right)\left(\varphi_{1,\tau} + \varphi_{2,\tau}\right) \tag{9}$$

Where, $\varphi_{i,\tau}$ is the first order time derivative of the phase.

Eqns. (8) and (9) show that the parametric gain depends both on the fiber dispersion slope and on the pump phase modulation frequency [5,10]. The net signal gain, G is given by

$$G = 1 + \left(\frac{2\gamma\sqrt{P_1 P_2}}{g}\sinh\left(gL\right)\right)^2 \tag{10}$$

Where, g is the parametric gain and L is the fibre length [6].

With the inclusion of the pump depletion, the characteristic of the parametric amplifier is modified [11]. This is because of the analytic solution in terms of elliptic functions that can be obtained under specific boundary conditions on the four coupled equations in the paraxial approximation. Considering Eqns. (2) - (5), an amplification or attenuation of the signal, or idler depends on the relative phase θ defined by

$$\theta = \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 \tag{11}$$

Where, ϕ_j is the phase of the amplitude for four waves propagating waves the fiber. The relative phase alters the instantaneous phase mismatch through polarization thus varying the parametric gain.

PMD in fibers is caused by the birefringence and polarization mode coupling. Birefringence is as a result of asymmetries in the fiber core. It causes temporal separation between primary orthogonal modes of the fiber core. It rotates the State of Polarization (SOP) of optical fields with different frequencies at different rates. When SOP of the pumps vary with time then the FOPA gain exhibit Polarization Dependent Loses (PDL) and

Polarization Dependent Gains (PDG) thus affecting the gain [12,13].

Therefore, in FOPA the gain is exponential with input power P_0 and position z (as shown in Eqn. 10) when the phase matching condition is fulfilled, i.e., k = 0. The phase matching is mainly determined by the nonlinear phase shift, γP_0, whereas the spectral gain profile in the ZDW region is determined by the dispersion and polarization in the fibers. The magnitude and shape of the gain can thereby be optimized by tuning the length and dispersion in the fibers segment [14,15].

3. Simulation Setup

In the setup (Fig. 1), two laser sources of wavelengths λ_1 and λ_2 from a continuous WDM source were used as the pumps and a third laser of wavelength λ_0 was the probe. The probe was set at 1550.0 nm while pumps λ_1 and λ_2 were varied depending on channel spacing. The pumps were multiplexed using a multiplexer and launched into a 400-m-long highly nonlinear fiber (HNLF). The signals were provided by a continuous wave WDM source. The probe state of polarization (SOP) was adjusted by a polarization controller PC 2 while PC 1 and PC 3 were used to maximize the pump power into the fiber.

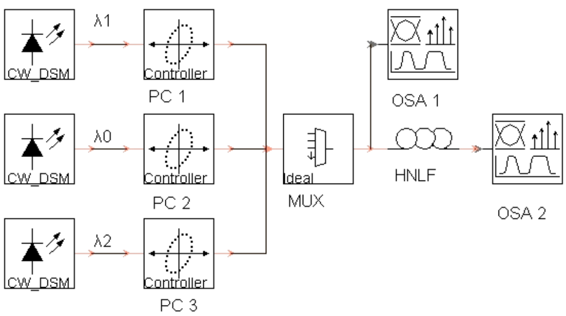

Fig.1: The simulation setup.

The spectrum of the three multiplexed sources was recorded by OSA 1 before propagation and by OSA 2 after propagation through the fiber. The lengths of the fiber used were 400 m and 600 m. The non-linear coefficient was varied and the gain obtained taken.

4. Results and Discussion

Figs. 2 (a) and (b) represent the effect of increasing fiber length at constant γ=10 s/W while Figs. 2 (c) and (d) at constant γ=20 s/W. Figs. 2 (a) and (c) represent the FWM products as γ is varied from 10 s/W to 20 s/W at constant 400 m fiber length, while Figs. 2 (b) and (d) at constant L=600 m.

FWM products increase in magnitude and intensity as γ is increased when phase matching condition is fulfilled. Moreover, increase in fibre length enhances FWM as interaction length is increased.

Fig. 3 shows that nonlinear coefficient affects the flatness of the gain. FOPA gain also increases with fiber length (agrees with Eqn. (10)). A flat gain over the S-, C- and L-band (1500-1603 nm) was realized on a HNLF of dispersion slope, s=7.5x10^{-5} ps/nm^2/m and fourth order dispersion parameter, β_4=4x10^{-6} ps^4/m. Nonlinear coefficient determines the rate at which the nonlinear refractive index changes with power. This causes variation in the fiber's response to electromagnetic waves, hence varying the number and intensity of FWM products while the fiber length increases the effective interaction length thus improving the gain as shown in Fig. 3. Fig. 4 shows that gain increases with an increase in nonlinear coefficient while the bandwidth reduces with an increase in nonlinear coefficient. Improved FWM products as a result of increased nonlinear coefficient (Fig. 2) are responsible for the increased FOPA gain while the bandwidth reduces because of the dispersive differences between the fiber material constituents and the electromagnetic wave.

High nonlinearity enhances efficient nonlinear interaction in the fiber. A low value of β_4 helps in attaining phase matching (k = 0), which is a condition for FWM to occur (Eqn. (6)).

Fig. 5 relates the phase shift at different wavelengths. The third order dispersion parameter, β_3, distorts the gain flatness around the zero dispersion wavelength because of the instantaneous phase mismatch which due to pump phase modulation as given in Eqn. (9). This is because of the phase transition from positive to negative value at 1520 nm and 1580 nm on the total phase mismatch.

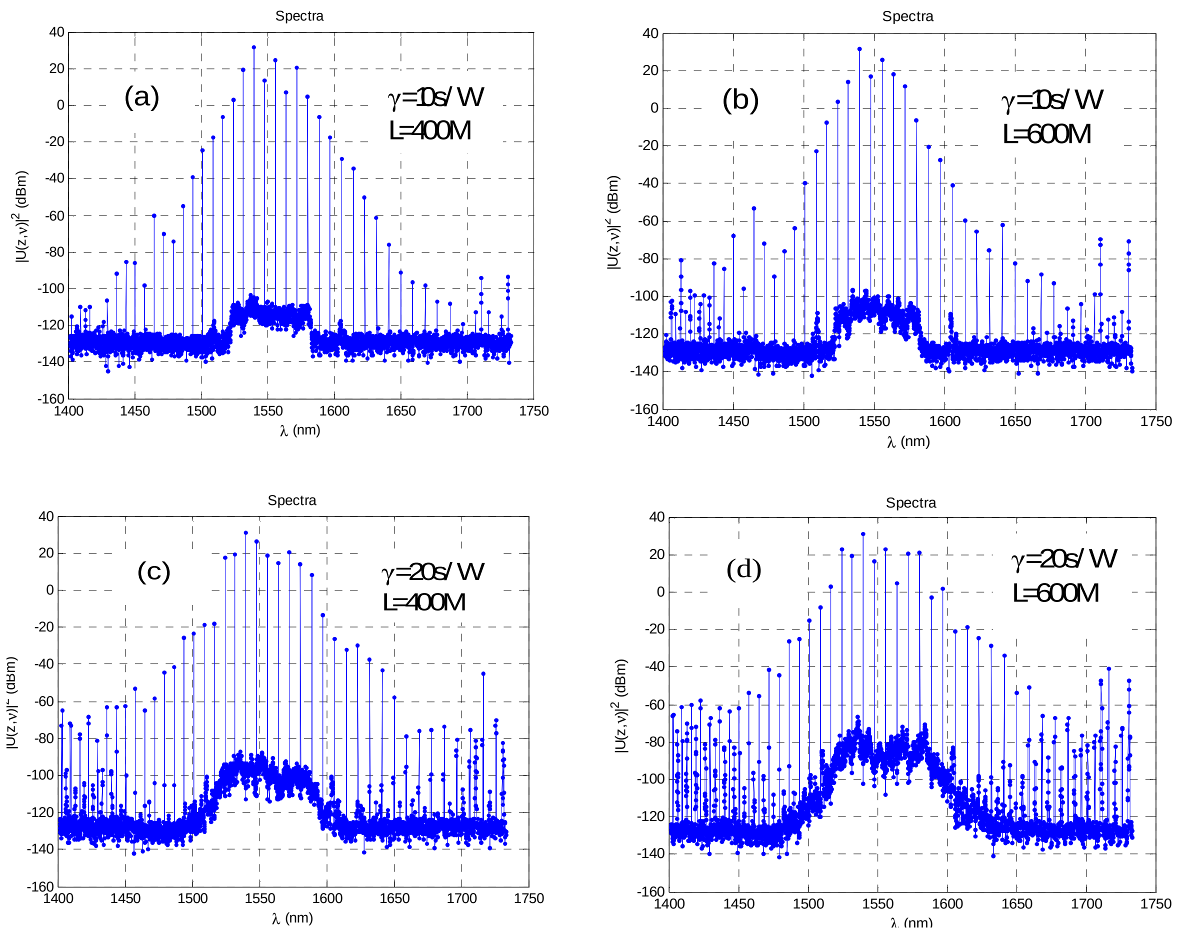

Fig.2: Power variation against wavelength at a) γ=10 s/W, L= 400 m, b) γ=10 s/W, L=400 m, c) γ=20 s/W L=400 m and d) γ=20 s/W, L=600 m.

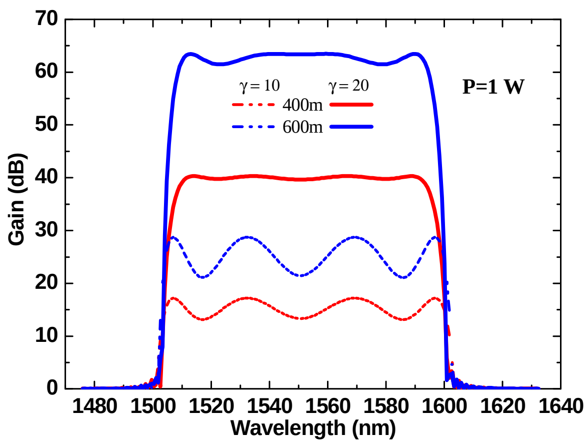

Fig.3: Graph of gain against wavelength at different values of γ for 400 m and 600 m fibre lengths.

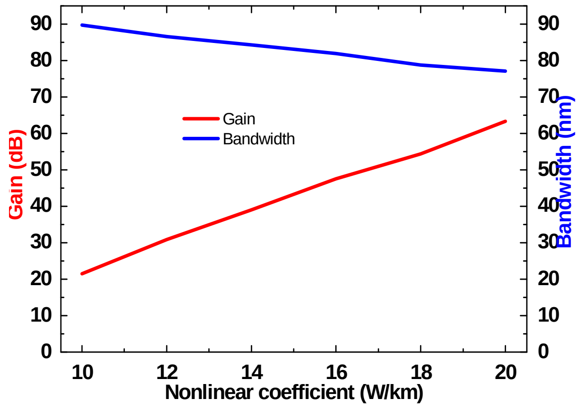

Fig.4: Graph summarizing the relation between Gain, Bandwidth and Nonlinear coefficient.

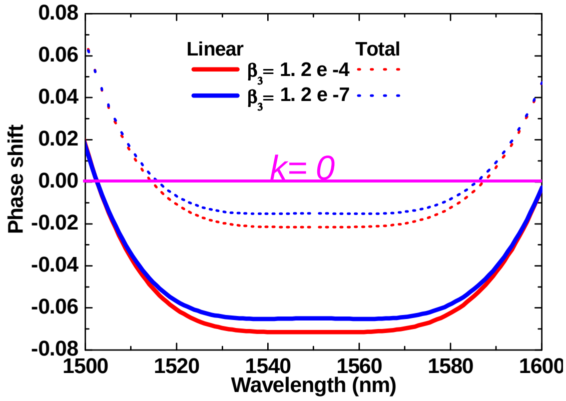

Fig.5: Graph of phase shift against wavelength at different values of β_3.

Both the gain flatness and magnitude are affected as the β_4 is varied as shown in Fig 6. This is because of the phase matching (k = 0) that need to be attained for four-wave mixing to occur. When phase matching is achieved, the amplitude contributions add up constructively, and high power conversion efficiency is achieved [11,16]. Otherwise, the direction of energy transfer changes periodically according to the change in the phase relation between the interacting waves.

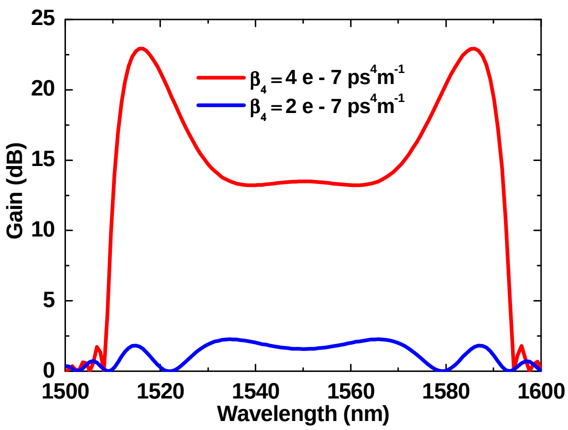

Fig.6: FOPA gain at different values of fourth order dispersion parameter, β_4.

The energy then oscillates between the waves rather than being transferred in a constant direction in the fiber. At β_4 = 2e-7 ps^4/m, the total phase mismatch is large and hence low gain. This is confirmed by the negative phase shift in Fig. 7.

Fig. 8 shows that polarization dependent gain variation due to D_p = 0.025 ps/km$^{1/2}$ is caused by the phase matching variation due to β and γ. It shows that PMD induces large fluctuations in the signal power that can affect the system performance. In fact, due to polarization sensitivity the gain is increased as the bandwidth is reduced and vice versa. This confirms the trade-off between bandwidth and gain in Fig. 4. Therefore, FOPA gain optimization requires polarization to be taken into consideration.

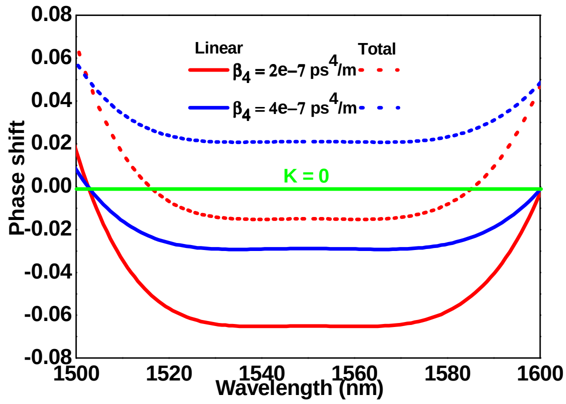

Fig.7: Graph of phase shift against wavelength for different values of β_4.

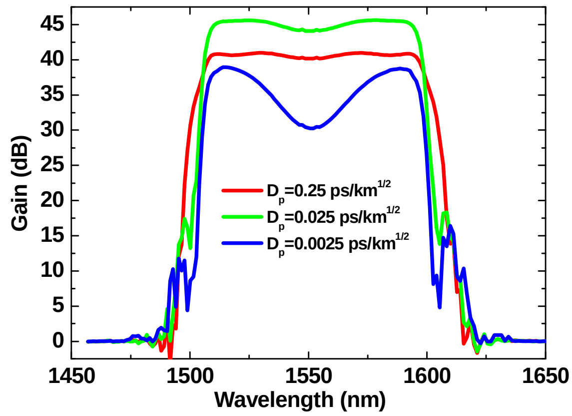

Fig.8: Graph of gain at different Polarization Mode Dispersion (PMD) coefficients, D_p.

5. Conclusion

We have analyzed the gain of a FOPA theoretically. A net gain of 40 dB and 62 dB is reported over a bandwidth >100nm on 400 m and 600 m fiber length, respectively. An increase in the nonlinear coefficient improves the flatness of the gain. The FOPA gain is increased by the significant increase in the intensity and number of FWM products brought about by the nonlinear coefficient and the fiber length increase. Higher order dispersion parameters affect the phase shifting thus directly influencing the gain. The third order dispersion limits the gain of the signals close to ZDW and the gain bandwidth is strongly limited by the fourth-order dispersion because of phase matching condition. PMD induces fluctuations altering the gain. These results should help in improving the data integrity in DWDM systems.

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