Five Dimensional Bianchi Type-III Domain Walls and Cosmic Strings in Bimetric Theory

P. K. Sahoo* and B. Mishra

Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad, India

Five dimensional Bianchi type-III thick domain walls and cosmic strings are considered in Rosen’s bimetric theory of gravitation. It is observed that, in this theory, thick domain walls and cosmic strings do not exist. Hence, only vacuum model can be obtained.

1. Introduction

Rosen [1] has modified the theory of general relativity by introducing a second metric tensor $f_{ij}$, besides the Riemannian metric tensor $g_{ij}$ at each point of the space-time. Accordingly, at each point of the space-time, one has two line elements

$$ds^2 = g_{ij}dx^idx^j$$

and

$$dσ^2 = f_{ij}dx^idx^j$$

The tensor $g_{ij}$ describes gravitation and interacts with matter. The background metric $f_{ij}$ has no direct physical significance but appears in the field equations. Therefore it interacts with $g_{ij}$ but not directly with matter. One can regard $f_{ij}$ as giving the geometry that would exist if there were no matter.

One can rewrite all the quantities occurring in general relativity so that $\{ \}$ is to be replaced by $\Delta$, partial derivative by $f$-covariant derivative and $\sqrt{-g}$ by $\kappa = \sqrt{\sqrt{2}}$, where $g = \det(g_{ij})$ and $f = \det(f_{ij})$. The advantage of this formalism is that it imparts tensor character to quantities which in the usual form of the theory do not have.

Using this bimetric formalism Rosen [1] formulated a bimetric theory of gravitation, which satisfies the principle of covariance and equivalence. The field equations of the present theory have a simpler structure. Using the conventional variational principle Rosen [1] obtained the field equations in bimetric theory of gravitation, which read as

$$N^i_j - \frac{1}{2}N\delta^i_j = -8\pi\kappa T^i_j$$

Where

$$N^i_j = \frac{1}{2}f^{ab}(g^{hi}g_{hj;i})_{;b}$$

and

$$N = N^i_i$$

A vertical bar (|) denotes the covariant differentiation with respect to $f_{ij}$ and $T^i_j$ is the usual stress tensor of the matter fields.

The purpose of Rosen’s bimetric theory is to get rid of the singularities that occur in general relativity and appear in the big-bang cosmological models. Therefore, there has been lot of interest in cosmological models related to Rosen’s bimetric theory of gravitation. Several aspects of the bimetric theory of gravitation have been studied and investigated [2-10]. Recently Sahoo and Mishra [11] have studied axially symmetric space-time with strange quark matter attached to string cloud in bimetric theory.

In particular, Reddy et al. [12,13] have studied plane symmetric and axially symmetric domain walls and cosmic strings in bimetric theory. Recently Sahoo and Mishra [14] have studied string cloud and domain walls with quark matter for plane symmetric space time in this theory. Inspite of the fact that a lot of work has been done, in this direction, it is evident from the literature that there is need for further investigation which may unravel some of the hidden secrets of the theory.

In this paper, we have shown that higher dimensional Bianchi type-III cosmological models representing domain walls and cosmic strings do not exist in Rosen’s bimetric theory of gravitation. Hence a vacuum model is obtained.

*sahoomaku@rediffmail.com
2. Domain Walls

At the very early stage of the evolution of the universe, it is generally assumed that during the phase transition the symmetry of the universe is broken spontaneously. It can give rise to topological stable defects such as strings, domain walls and monopoles [15]. Of all these cosmological structures, cosmic strings and domain walls have created the most interest. The domain walls have become important in recent years from cosmological standpoint when a new scenario of galaxy formation has been proposed [16]. Also [17-23] are some of the authors who have investigated several aspects of domain walls.

A thick domain wall can be viewed as a soliton-like solution of the scalar field equations coupled to gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field \( \phi \) with self-interactions contained in a potential \( V(\phi) \) given by

\[
T_{ij} = \phi_i \phi_j - g_{ij} \left[ \frac{1}{2} \partial_i \phi \partial_j - V(\phi) \right] \tag{4}
\]

The second approach is to assume the energy momentum tensor in the form

\[
T_{ij} = \rho (g_{ij} + \omega_i \omega_j) + p \omega_i \omega_j; \quad \omega_i \omega^j = -1 \tag{5}
\]

Where, \( \rho \) is the energy density of the wall, \( p \) is the pressure in the direction normal to the plane of the wall and \( \omega_i \) is a unit space-like vector in the same direction [21]. Here, we use the second approach and show that Bianchi type-III thick domain walls do not survive in the frame work of Rosen’s bimetric theory of gravity.

We consider the Bianchi type-III metric in the form

\[
ds^2 = -dt^2 + e^{2\alpha} dx^2 + e^{2(\beta + x)} dy^2 + e^{2\gamma} dz_1^2 + e^{2\delta} dz_2^2 \tag{6}
\]

with the convention \( x^0 = t, x^1 = x, x^2 = y, x^3 = z_1, x^4 = z_2 \) and \( \alpha, \beta, \gamma \) and \( \delta \) are functions of time \( t \) only.

The flat space-time corresponding to the metric (Eqn. (6)) is

\[
ds^2 = -dt^2 + dx^2 + dy^2 + dz_1^2 + dz_2^2 \tag{7}
\]

In the co-moving coordinate system, we have from (5)

\[
T_{00}^0 = T_{22}^3 = T_{33}^4 = \rho; \quad T_{11}^1 = -p; \quad T_{ij}^0 = 0, \quad i \neq j \tag{8}
\]

(here pressure is taken in the direction of \( x \)-axis) the quantities \( \rho \) and \( p \) depend on \( t \) only.

The field equations (3) for the Bianchi type III metric (Eqn.(6)) with the help of Eqns. (7) and (8) becomes

\[
\begin{align*}
\alpha_{00} - \beta_{00} - \gamma_{00} - \delta_{00} &= -16\pi \kappa p \\
\alpha_{00} - \beta_{00} + \gamma_{00} + \delta_{00} &= -16\pi \kappa p \\
\alpha_{00} + \beta_{00} - \gamma_{00} + \delta_{00} &= -16\pi \kappa p \\
\alpha_{00} + \beta_{00} + \gamma_{00} - \delta_{00} &= -16\pi \kappa p \\
\alpha_{00} + \beta_{00} + \gamma_{00} + \delta_{00} &= -16\pi \kappa p
\end{align*} \tag{9-13}
\]

Where, the suffix 0 hereafter, denotes ordinary differentiation with respect to time \( t \).

From Eqns. (9)-(13), we obtain

\[
\alpha_{00} = \beta_{00} = \gamma_{00} = \delta_{00} \tag{14}
\]

Using (14), (9)-(13) give

\[
p = 0 \quad \text{and} \quad \rho = 0 \tag{15}
\]

This shows that in Rosen’s bimetric theory the geometry of five dimensional Bianchi type-III space-time does not admit thick domain walls. Hence, the vacuum solutions of the field equations in bimetric theory can be written as

\[
\alpha = \beta = \gamma = \delta = k_1 t + k_2 \tag{16}
\]

Where \( k_1 \) and \( k_2 \) are constants of integration.

Thus in view of Eqn. (16), the metric in Eqn. (6) takes the form

\[
ds^2 = -dt^2 + e^{k_1 t + k_2} (dx^2 + e^{2x} dy^2 + dz_1^2 + dz_2^2) \tag{17}
\]

With a suitable coordinate transformations \((x, y, z, t) \rightarrow (X, Y, Z, T)\) where \( T = k_1 t + k_2 \), the above metric can be written as

\[
ds^2 = -dT^2 + e^T (dX^2 + e^{2X} dY^2 + dZ_1^2 + dZ_2^2) \tag{18}
\]

Einstein’s theory of gravitation is a beautiful geometric theory of gravitation, which gives a unified description of the metric. The bimetric theory suggested by Rosen does not have any effect on the generality of Einstein’s system. It is interesting to note that, the model (18) is conformally flat and free from singularity. At \( T = 0 \), the model reduces to flat one.
3. Cosmic Strings

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [24,25]. Moreover, the investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations, which lead to formation of galaxies [26,17]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein’s equations. The general treatment of strings was initiated by Letelier [27,28] and Satchel [29]. Letelier [27] has obtained the solution to Einstein’s field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Further, Letelier [28] solved Einstein’s field equations for a cloud of massive strings and obtained cosmological models in Bianchi type-I and Kantowski-Sachs spacetimes. Tikekar and Patel [30] have discussed some Bianchi type VI and IX space times. Pradhan [35-39] and Tripathi et al. [40,41] have studied several string cosmological models in Bianchi type II, VI, VIII and IX space times. Pradhan [35-39] and Tripathi et al. [40,41] have studied several string cosmological models in different contexts. Cosmic strings seem to be the best candidate for being observed and also the presence of strings in the early universe do not contradict present day observation.

We consider the five dimensional Bianchi type III cosmic string dust source with energy momentum tensor [28]

\[ T^i_j = \rho u^i u_j - \lambda x^i x_j \]  

(19)

with

\[ u^i u_i = -1 = -x^i x_i \quad and \quad u^i x_i = 0 \]  

(20)

Where, \( \rho \) is the rest energy density of the cloud of strings with massive particle attached to them, \( \rho = \rho_0 + \lambda \), \( \rho_0 \) being the rest energy of particles attached to this strings and \( \lambda \) the tension density of the system of strings. As pointed out by [28], \( \lambda \) may be positive or negative. The unit time like vector \( u^i \) is the flow vector and the unit space like vector \( x^i \) specifies the direction of the string.

In the co-moving coordinate system, taking the string in \( x \)-direction, Eqn. (19) takes the form

\[ T^1_1 = -\lambda; \quad T^2_2 = T^3_3 = T^4_4 = 0; \]

\[ T^0_0 = -\rho; \quad T^i_j = 0 \quad i \neq j \]  

(21)

The quantities \( \rho \) and \( \lambda \) are functions of \( t \) only.

The Rosen’s bimetric field equations (3) for the metric (Eqn. (6)) with the help of eqns. (7), (19), (20) and (21) take the form

\[ \alpha_{00} - \beta_{00} - \gamma_{00} - \delta_{00} = -16\pi \kappa \lambda \]  

(22)

\[ \alpha_{00} - \beta_{00} + \gamma_{00} + \delta_{00} = 0 \]  

(23)

\[ \alpha_{00} + \beta_{00} - \gamma_{00} + \delta_{00} = 0 \]  

(24)

\[ \alpha_{00} + \beta_{00} + \gamma_{00} - \delta_{00} = 0 \]  

(25)

\[ \alpha_{00} + \beta_{00} + \gamma_{00} + \delta_{00} = 16\pi \kappa \rho \]  

(26)

From the field equations (22)-(26), we obtain

\[ \alpha_{00} = \beta_{00} = \gamma_{00} = \delta_{00} \]  

(27)

and

\[ \lambda = 0 \quad and \quad \rho = 0. \]  

(28)

This means that five dimensional Bianchi type III cosmic strings do not exist in bimetric theory of gravitation. Hence in this case also, we get the vacuum model as given in Eqn. (18).

Hence a five dimensional isotropic and singularity free cosmological vacuum model is obtained, therefore, one can conclude that in Rosen’s bimetric theory of relativity, the only possible solutions of five dimensional Bianchi type III cosmological model in presence of cosmic string is a vacuum solution.

4. Conclusion

We have shown that five dimensional Bianchi type III cosmological models representing domain walls and cosmic strings do not exist in bimetric theory of gravitation. However, at the early stages of evolution of the universe the appearance of domain walls and strings was the leading cause of the formation of galaxies. Hence, it is concluded that Rosen’s bimetric theory of gravitation does not in any way help to describe the early era of the universe.
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References


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