The Relationship Between the Optomechanical Doppler Force and the Magnetic Vector Potential

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Photons are carriers of the electromagnetic force and propagate the electrostatic and magnetic vector potentials. While the electrostatic potential (φ) is agreed to be related to the work needed to move a unit charge in an electrostatic field, the meaning of the magnetic vector potential ($\vec{A}$) has been enigmatic. Here I show that the product of the time derivative of the magnetic vector potential and the charge (ze) of a moving particle can be considered to be the electromagnetic analog of the optomechanical Doppler force that prevents charged particles from accelerating beyond the speed of light. As the velocity of a charged particle approaches the speed of light, the time rate of change of the magnetic vector potential increases and produces a self-induced electrodynamical field that opposes the electrostatic field that accelerates the charged particle. Although magnetism is generally considered to be a relativistic aspect of electricity, here I show that by taking the magnetic vector potential into consideration, one can understand the electrodynamics of moving bodies in terms of real and absolute time.

1. Introduction

The Doppler Effect is readily perceived when there is relative motion. Curiously, standard textbooks rarely, if ever, include the Doppler effect as a primary consideration in the study and description of relative motion. My work incorporates the Doppler Effect from the beginning. When expanded to second order, its inclusion makes it possible to unify many aspects of mechanics and electrodynamics that are usually treated separately.

In previous papers [1,2], I developed a second order relativistic wave equation that describes the propagation of light waves between inertial frames moving relative to each other at velocity (v). This relativistic wave equation, which differs from Maxwell's by including the Doppler effect *a priori*, is given by:

$$\frac{\partial^2\psi}{\partial t^2} = cc' \frac{\sqrt{c - v\cos\theta}}{\sqrt{c + v\cos\theta}} \nabla^2\psi \tag{1}$$

or

$$\frac{\partial^2\psi}{\partial t^2} = cc' \frac{1 - \frac{v}{c}\cos\theta}{\sqrt{1 - \frac{v^2\cos^2\theta}{c^2}}} \nabla^2\psi \tag{2}$$

Where, ψ represents any of the scalar or vectorial electromagnetic fields or potentials, v is the magnitude of the velocity and θ is the angle between the velocity of a particle and the velocity of a photon. θ = 0 radians when the movements of a photon and a particle are parallel and $\theta = \pi$ radians when the movements of a photon and a particle are antiparallel. At v = 0, this relativistic wave equation reduces to Maxwell's wave equation.

The absolute and relative aspects of the photon's speed are represented by c and c', respectively. The parameter c is equal to the square root of the reciprocal of the product of the electric permittivity (ε_o) and the magnetic permeability (μ_o) of the vacuum as given by Maxwell's relation. On the other hand, c', which depends on the relative velocity of the source and observer, gives the ratio of the angular frequency (ω_{source}) of the source in its inertial frame to the angular wave number ($k_{observer}$) observed in any inertial frame. At any relative velocity between the source and the observer:

$$c' \frac{1 - \frac{v}{c}\cos\theta}{\sqrt{1 - \frac{v^2\cos^2\theta}{c^2}}} = c = 2.99 \times 10^8 \text{ m/s} \tag{3}$$

Substitution of c' with $\frac{\omega_{source}}{k_{observer}}$ gives a relativistic dispersion relation that characterizes the Doppler effect:

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$$\omega_{observer} = ck_{observer} = \omega_{source}\ \frac{1-\ \frac{v}{c}\cos\theta}{\sqrt{1-\frac{v^2\cos^2\theta}{c^2}}} \quad (4)$$

or

$$k_{observer} = k_{source}\ \frac{1-\ \frac{v}{c}\cos\theta}{\sqrt{1-\frac{v^2\cos^2\theta}{c^2}}} \qquad (5)$$

that must be satisfied when the general plane wave solution has the form:

$$\psi = \psi_o e^{i\left(\vec{k}_{observer}\cdot\vec{r} - \omega_{source}\frac{1-\ \frac{v}{c}\cos\theta}{\sqrt{1-\frac{v^2\cos^2\theta}{c^2}}}t\right)} \qquad (6)$$

I used the second order relativistic wave equation given above to model the interaction between a moving charged particle and the photon gas through which it moves [1]. As a result, I interpreted the observations that particles with a charge cannot exceed the vacuum speed of light because such particles interact asymmetrically with the photon gas that makes up the environment through which the particles move.

The spatially asymmetrical interaction between a moving particle and the photons results in a temperature- and velocity-dependent, self-induced counterforce because the photons that collide with the front of the moving particle are blue-shifted and thus have more linear momentum than the photons that collide with the back of the moving particle, which are red-shifted as a result of the Doppler effect (Fig. 1). Consequently, the optomechanical Doppler force acts asymmetrically and is directed antiparallel to the velocity of the particle [1]. A photon gas is present at all temperatures greater than absolute zero [3], and, according to the Third Law of Thermodynamics, absolute zero is unattainable [4]. I conclude that the optomechanical Doppler force, which is a contact force, is a ubiquitous and ever present force on moving particles with a charge. Furthermore, the optomechanical Doppler effect has increasing significance as the velocity of the particle approaches the speed of light.

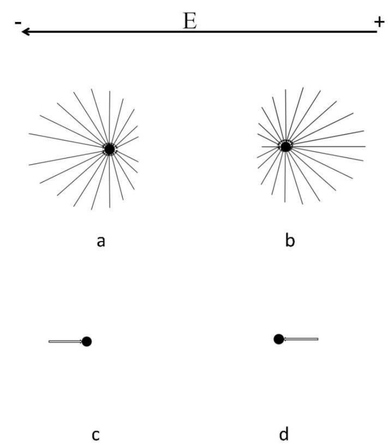

Fig.1: The individual momentum vectors that represent the transfer of momentum from the photon gas to a) a positively-charged particle moving in an electric field towards the left and b) a negatively-charged particle moving towards the right. The net momentum vector represents the transfer of momentum from the photon gas to the c) positively-charged and d) negatively-charged particle as a result of the optomechanical Doppler force. The length of the net vector increases as the velocity increases.

2. Results and Discussion

The optomechanical Doppler force is an induced force that is catalyzed by a charged particle moving through a photon gas. In the presence of an applied force, the optomechanical counterforce prevents the velocity of such a particle from exceeding the speed of light. After the removal of the applied force, the optomechanical counterforce causes the particle to decelerate [1,5]. Being an induced force, the optomechanical Doppler force cannot exist in free space in the absence of a moving particle. Given that a particle must be charged in order to interact with the photon gas through which it moves, I will specifically discuss the electrostatic force as the applied force used to accelerate a charged particle. An applied electric field provides an electrostatic force that can be used to accelerate a particle with charge ze according to the following equation:

$$\vec{F}_{applied} = ze\vec{E}_{applied} \qquad (7)$$

Where, $\vec{E}_{applied}$ is the applied electric field (in V/m), $\vec{F}_{applied}$ represents the constant applied electrostatic force, e is the elementary charge (1.6×10^{-19} C) and z is the valence of the particle. When one takes into consideration the opposition to the movement of the charged particle by the optomechanical Doppler force [1], Newton's Second Law becomes:

$$\vec{F}_{applied} + \vec{F}_{Doppler} = \vec{F}_{effective} = m_o \frac{d\vec{v}}{dt} \quad (8)$$

Where, $\vec{F}_{Doppler}$ represents the optomechanical Doppler force [1] antiparallel to $\vec{F}_{applied}$ and $\vec{F}_{effective}$ represents the effective force that accelerates the charged particle with an acceleration that is less than the acceleration that would be caused by $\vec{F}_{applied}$ alone. The optomechanical Doppler force can be interpreted to produce an electric field across the moving particle with a polarity that opposes the applied electric field, thereby reducing the applied electric field to an effective electric field ($\vec{E}_{effective}$) in the vicinity of the extended charged particle according to the following equation:

$$\vec{F}_{effective} = ze\vec{E}_{effective}$$

$$= ze\left(\vec{E}_{applied} + \vec{E}_{Doppler}\right) \quad (9)$$

Where, $\vec{E}_{Doppler}$ can be measured as the difference between the effective electric field and the applied electric field at various temperatures.

As shown in previous papers [1,6], the optomechanical Doppler force is given by:

$$\vec{F}_{Doppler} = \frac{dn}{dt}\frac{h}{2\lambda_o}\frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (10)$$

Where, λ_o is the peak wavelength of a photon gas with a blackbody distribution that depends only on the absolute temperature T, h is Planck's constant, and $\frac{h}{\lambda_o}$ is the momentum of a photon with the peak wavelength. The collision rate between a charged particle with constant mass (m_o) moving at speed (v) and the photons that make up the photon gas through which it moves is given by $\frac{dn}{dt}$. The collision rate is the product of the photon density (ρ), the cross sectional area [1, 6-9] of the photons ($\sigma = \frac{\lambda^2}{2\pi}$), and the speed of the particle. c is the speed of light in a vacuum and dt, in the derivative, is an invariant duration of time. By defining the product of ρ and σ as the linear photon density (ρ_L), replacing h with $\frac{e^2}{\varepsilon_o c\alpha}$ using the definition of the fine structure constant (α), substituting μ_o for $\frac{1}{\varepsilon_o c^2}$, and replacing λ_o with $\frac{w}{T}$ using Wien's displacement law, we get:

$$\vec{F}_{Doppler} = \frac{\rho_L T e^2 \mu_o}{2w\alpha}\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (11)$$

This expression of the optomechanical Doppler force shows explicitly that the counterforce depends on the temperature of the photon gas as well as electrodynamic properties, including the velocity of the moving particle, the square of the charge of the moving particle, the magnetic permeability of the vacuum, and the strength of the interaction between a charged particle and a photon, which is given by the fine structure constant [1]. The counterforce that resists the acceleration of the particle vanishes when the temperature of the photon gas vanishes, or when the charge and/or the velocity of the particle goes to zero.

Since the linear photon density (ρ_L, in m^{-1}) is solely a function of temperature, it can be written as $\frac{\sigma_B w^3}{\pi h c^2}T$, where σ_B is the Stefan-Boltzmann constant (5.6704 × 10^{-8} J m^{-2} s^{-1} K^{-4}) and w is the Wien constant (2.89784 × 10^{-3} m K). By combining the constants [1], the optomechanical Doppler force experienced by a univalent particle, where $z = \pm 1$, can be expressed exclusively in terms of temperature and velocity:

$$\vec{F}_{Doppler} = -s[2.82 \times 10^{-39}\, N\, s^2\, M^{-2}\, K^{-2}]T^2\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (12)$$

Where s = 1, when the absorbed photons are re-emitted or scattered isotropically and s = 2, when the absorbed photons are re-emitted or reflected in the same direction in which they were absorbed [1]. Taking into consideration the counterforce provided by the optomechanical Doppler force, Newton's Second Law for a univalent particle can then be written like so:

$$\vec{F}_{effective} = \vec{F}_{applied} - s\Omega T^2\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_o\frac{d\vec{v}}{dt} \quad (13)$$

Where, $\Omega = 2.82 \times 10^{-39}\, N\, s^2\, M^{-2}\, K^{-2}$ and the time rate of change of momentum $\frac{d\vec{p}}{dt}$ is then given by:

$$\vec{F}_{effective} = \vec{F}_{applied} + \vec{F}_{Doppler} = \frac{d\vec{p}}{dt} \quad (14)$$

Where, $\vec{p} = m_o\vec{v}$ represents the momentum of a univalent particle moving through a vacuum at temperature (T). m_o indicates a constant and invariant quantity of matter that makes up a body [10], and $d\vec{p}$ indicates the product of a constant and invariant quantity of matter and its change in velocity [11]. In the absence of the velocity-dependent optomechanical Doppler force, the time rate of change of momentum would be linear with respect to the applied force and Eqn. (14) would reduce to Newton's Second Law. The nonlinear relationship between the applied force and the time rate of change of the apparent momentum ($\frac{d\vec{p}\prime}{dt}$) is typically explained by considering the factors that make up the time rate of change of the apparent momentum such as m or t to be velocity-dependent variables.

$$\vec{F}_{applied} = \frac{d\vec{p}}{dt} - \vec{F}_{Doppler} = \frac{d\vec{p}\prime}{dt} \qquad (15)$$

2.1. Interpretations of apparent momentum

While I interpret the difference between the temporal derivative of the apparent momentum ($\vec{p}'$) and the temporal derivative of the momentum ($\vec{p}$) to be a result of the optomechanical Doppler force, historically, the difference has been attributed to the relativity of mass that resulted from the self-induced magnetic effects produced by a moving charge [12]. As a result, the apparent momentum has been interpreted to be equal to $(m + \delta m)\vec{v}$, where $m + \delta m$ is the apparent mass and δm is the electromagnetic mass or the hydrodynamical mass, terms coined by Walter Kaufmann [13] and Charles Galton Darwin [14], respectively, to indicate the apparent electromagnetic or hydrodynamical increase in the mass of a particle as it moved through the luminous aether or any viscous medium. Wilhelm Wien championed the complete replacement of the Newtonian mechanical world view with the electromagnetic world view and considered the mass of a charged particle to be exclusively due to δm, where δm was a consequence of self-induction. Max Abraham and Hendrik Lorentz each constructed a geometrical model of the electron that would use electrodynamics to explain the self-induced change in mass with velocity. While Abraham's rigid sphere model appeared to be confirmed in 1903 by Walter Kaufmann's experiments on the relationship between mass and velocity, later experiments supported Lorentz's contractile model of the electron [15-19].

The apparent variation of the mass of moving bodies was later explained by the Special Theory of Relativity [20]. According to the Special Theory of Relativity, the difference between the temporal derivative of the apparent momentum and the temporal derivative of the momentum is due to the relativity of time between two different inertial frames and is independent of the size, structure, and charge of the electron [21]. I claim that the interpretations of relative mass or relative time are a consequence of neglecting the environment through which the charged particle with constant and invariant mass moves [1,2,5,6,22]. Unless one denies the presence of the black body radiation that exists as long as $T > 0$, and the ubiquitous Doppler effect, the optomechanical Doppler force must by necessity provide a resistance to acceleration. The optomechanical Doppler effect may also account for the electromagnetic phenomena that depend on the relative motion of a conductor and a magnet—the very phenomena whose apparent asymmetry was resolved by Einstein with the Special Theory of Relativity in terms of the relativity of time after he noticed asymmetries in Maxwell's electrodynamics that did "*not seem to attach to the phenomena*'" when it came to "*the electrodynamic interaction between a magnet and a conductor*" [20].

The apparent variation in the mass of moving bodies has also been explained by quantum electrodynamics (QED), which posits that the increase in the effective mass of an electron is a consequence of the dressing or renormalizing of the electron as it interacts with the virtual photons in the environment characterized by the quantum electrodynamical vacuum. The renormalization is introduced as a result of a moving electron being considered to be a mathematical point [23,24] that produces an electromagnetic field around itself which in turn acts back upon the electron resulting in an increase in δm. However, δm becomes infinite as a consequence of the electron being considered to be a mass without extension. The infinite mass term is neutralized by using a renormalization procedure based on a perturbation expansion involving powers of the fine structure constant [25], a measure of the interaction between a charged particle and a photon. However, the fact that the electron has an anomalous magnetic moment [26] indicates that the electron may not be a mathematical point and thus "*the electron may have size and structure!*" Currently, quantum chromodynamics (QCD), which is known as the Standard Model of Physics, considers only mathematical point particles and fields, and it posits that it is the Higgs field that composes the environment through which the massless point particles move, gives mass (δm) to the point-like elementary particles that would be massless in its absence [27,28].

If the electron does have a size and a structure, perhaps the expansion terms involving powers of the fine structure constant are related to the physical size and structure of an electron itself. The movement of a charged particle will produce a magnetic field ($\vec{B}$, in Tesla where T = Wb/m^2) that can be characterized by magnetic flux lines (Φ_B, in Webers where Wb = Vs) or a magnetic vector potential ($\vec{A}$, in Vs/m). Since a retarded magnetic vector potential would interact first with those parts of an extended charged body that are nearer and would interact later with those parts that are farther, knowledge of the size and structure of a charged particle would make it possible to quantitatively derive the retarded magnetic vector potential.

2.2. The relationship between the optomechanical Doppler force and the magnetic vector potential

Michael Faraday [29] considered that the "*physical lines of magnetic force [may] correspond (in having a real existence) to the rays of light*." If Faraday's conjecture be valid, then the optomechanical Doppler force, which is a manifestation of the spatially asymmetrical interaction between charged particles and photons, the carriers of the electromagnetic force, may be related to the spatial density of magnetic flux lines. The spatial density of the magnetic flux lines ($\frac{d\Phi_B}{d\vec{S}}$) is related to the magnetic induction [30-36], (which is also known as the magnetic induction field [37], or the magnetic flux density [38], or the magnetic field ($\vec{B}$) [39-46]), and the curl of the magnetic vector potential:

$$\frac{d\Phi_B}{d\vec{S}} = \vec{B} = \vec{\nabla} \times \vec{A} \tag{16}$$

Where, $\vec{S}$ is any plane through which the magnetic flux lines produced by the moving charge pass through. The plane is established by the velocity vector and any radial vector. The magnetic vector potential is easier to work with than the magnetic flux lines or the magnetic field since the magnetic vector potential, which is orthogonal to the magnetic field lines, is approximately parallel to the trajectory of the moving charge allowing the approximation by one-dimensional equations. The magnetic vector potential at a given point in space and time is defined as:

$$\vec{A} = \left(\frac{\mu_o ze}{4\pi}\right)\frac{\vec{v}}{r\prime} \tag{17}$$

Where, $\vec{v}$ is the velocity of an extended charged body and r' is the distance between a point on an extended charged body where $\vec{A}$ is determined at a given time and another point on the extended moving charge which served as the source of $\vec{A}$ at a previous point in time consistent with the second order Doppler effect. When the magnetic flux lines in a given region of space are close together, the magnetic field and the magnetic vector potential in that space are large and when the magnetic flux lines are widely separated, the magnetic field and the magnetic vector potential in that space are small.

The magnetic induction force exerted by a charge moving through a coil, as measured by the magnitude and duration of deflection of a galvanometer needle, is greater than the force exerted by a static or frictional charge on a capacitor measured with a Coulomb torsion balance electrometer. The ratio of electromagnetic and the electrostatic units of charge, as measured through their ponderomotive action by Wilhelm Weber and Rudolf Kohlrausch was equal to a constant, c, which was then known as the ratio of the units and is now known as the speed of light [30,47-50]. Since electromagnetic properties are propagated from a moving charged body at the speed of light [30,51,52], the magnetic flux lines produced by a moving charged object will, by necessity, be Doppler shifted (Fig. 2).

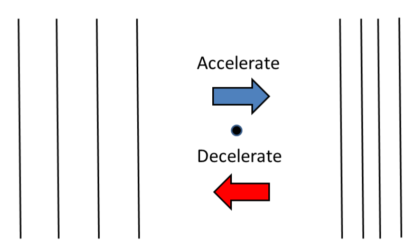

Fig.2: The density of a portion of the magnetic flux lines around a current-carrying wire that is perpendicular to the paper. The density of magnetic flux lines in front of the wire increases as the wire accelerates and decreases as a wire decelerates. The density of magnetic flux lines behind the wire decreases as the wire accelerates and increases as the wire decelerates.

Moreover, in a charged body moving at constant velocity, the spatial density of the magnetic flux lines will be greater at the front and lesser at the back of a moving charged particle (Fig. 3). The angular-dependence of the spatial frequency of magnetic flux lines can be obtained from the relativistic dispersion relation:

$$k_{observer} = k_{source} \frac{1-\frac{v}{c}\cos\theta}{\sqrt{1-\frac{v^2\cos^2\theta}{c^2}}} \tag{18}$$

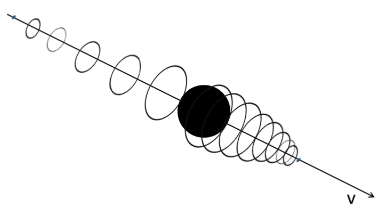

Fig.3: An illustration of the magnetic flux lines produced by a charged particle moving towards the bottom right corner. The magnetic flux lines are drawn assuming that the magnetic flux lines propagate through the vacuum at the vacuum speed of light and that the distance between the magnetic flux lines are Doppler shifted according to Eqn. (18). The sense of rotation of the lines depends on the sign of the charge of the moving particle.

Because the relativistic Doppler effect is second order with respect to velocity, the average angular wave number ($\bar{k}_{observer}$) of the magnetic flux lines produced in front of and behind the center of a moving particle is velocity dependent and does not become velocity independent as it would if the Doppler effect were only first order with respect to velocity [53]. The average angular wave number ($\bar{k}_{observer}$) of magnetic flux lines produced by a moving charged particle is:

$$\bar{k}_{observer} = \frac{1}{2}k_{source}\left[\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right] = \frac{k_{source}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{19}$$

Where, k_{source} is the angular wave number of magnetic flux lines produced by a vibrating charged particle at rest. The arrival time of the average Doppler-shifted magnetic flux lines to a point in space is equivalent to the retarded time of the magnetic vector potential. As long as the moving charged particle has extension, it will also experience an increase in the density of magnetic flux lines and an increase in the magnetic vector potential in a velocity-dependent manner (Fig. 4).

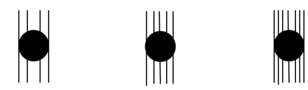

Fig.4: Diagram of the average number of magnetic flux lines produced by an extended charged particle moving at a given velocity. The faster the extended charged particle moves, the greater the number of magnetic flux lines produced and the greater the number of magnetic field lines experienced by the extended charged particle. The greater the number of magnetic flux lines, the greater the magnetic field and the greater the magnetic vector potential.

Since movement of a charged particle results in a change in the temporal derivative of the magnetic flux lines, an extended moving charged particle produces and experiences a velocity-dependent electromotive force (Ɛ, in V). Since the functional form of the velocity dependence of the angular wave number and the magnetic flux lines are the same, the electromotive force is given by:

$$\mathcal{E} = \frac{d\Phi_B}{dt\sqrt{1-\frac{v^2}{c^2}}} \tag{20}$$

or more simply:

$$\mathcal{E} = \frac{d\bar{\Phi}_B}{dt} \tag{21}$$

Where, $\bar{\Phi}_B$ represents the average number of magnetic flux lines produced and experienced by an extended charged particle—the details of which depend on the size and structure of the charged particle. Such a velocity-dependent electromotive force produced and experienced along the length of an extended moving charged particle will reduce the effectiveness of the electrostatic field applied to accelerate the charged particle. Moreover, the fact that the average number of magnetic flux lines per unit time crossed by a moving piece of wire increases with the velocity of the wire, makes the production of a motional electromotive force (Ɛ) intelligible in terms of Faraday's Law. Taking the temporal derivative of Eqn. (16), we get:

$$\frac{d\Phi_B}{d\vec{S}\,dt} = \frac{d\vec{B}}{dt} = \vec{\nabla} \times \frac{d\vec{A}}{dt} \quad (22)$$

The vector form of Faraday's Law states that:

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \quad (23)$$

and since

$$-\frac{d\Phi_B}{d\vec{S}\,dt} = -\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \frac{d\vec{A}}{dt} \quad (24)$$

then, after integration and within a constant of integration,

$$\vec{E} = -\frac{d\vec{A}}{dt} \quad (25)$$

and after multiplying both sides by the charge ze of the extended moving particle, we get:

$$\vec{F}_{induced} = ze\vec{E} = -ze\frac{d\vec{A}}{dt} \quad (26)$$

Where, $ze\vec{E}$ is an electrodynamic reaction force or induced force ($\vec{F}_{induced}$) produced by an extended moving charged particle. Equating the electromagnetic reaction force or self-force to the optomechanical Doppler force, we see that the optomechanical Doppler force is equivalent to the negative of the product of the charge of the particle (ze) and the time rate of change of the magnetic vector potential ($\frac{d\vec{A}}{dt}$):

$$\vec{F}_{Doppler} = -ze\frac{d\vec{A}}{dt} \quad (27)$$

Assuming that the above relationship be true, then the velocity and temperature dependence of the temporal derivative of the magnetic vector potential would be given by the following equation:

$$\frac{d\vec{A}}{dt} = \frac{s\Omega}{ze}T^2\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (28)$$

and the magnetic vector potential at a constant temperature would be given by:

$$\vec{A} = \frac{s\Omega}{ze}\,T^2\int\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}}\,dt \quad (29)$$

which indicates that the magnetic vector potential would vanish at absolute zero and there would be no self-inductance resisting the flow of charges.

Eqn. (27) shows that the product of the charge of an extended moving particle and the temporal derivative of the magnetic vector potential quantifies an induced electrodynamic force that resists the movement of an extended charged particle. To Maxwell, who took the dynamics in electrodynamics seriously [30,54,55], the magnetic vector potential was a quantity that represented "*the fundamental quantity in the theory of electromagnetism*." J. J. Thomson [56] also considered the magnetic vector potential to represent "*the momentum due to the magnetic force*" and that it "*represents a most important physical property of the system*" However, subsequently, the magnetic vector potential lost favor. To Oliver Heaviside [57], the "*very artificial nature*"' of the magnetic vector potential "*obscures and complicates many investigations*" and he "*went to the root of the evil, and cured it*," by rewriting Maxwell's equations without the gauge-dependent vector potential. Since Heaviside's time, the magnetic vector potential has been considered to be merely a mathematical device used to help in calculations but had no physical meaning or reality of itself. Heinrich Hertz [58] saw the magnetic vector potential as "*magnitudes which serve for calculation only*." According to Andrew Gray [31], "*The use of the vector-potential is sometimes convenient as an analytical expedient. But it is not a physical quantity which can be observed experimentally....*"

An understanding of the quantitative relationship between the magnetic vector potential and the optomechanical Doppler force may help provide constraints for the size and structure of a charged particle and an understanding of the relationship between its momentum and apparent momentum. In terms of the movement of charged particles, the optomechanical Doppler force treatment and electromagnetic treatment are incomplete yet complementary treatments. In the optomechanical Doppler force treatment, the radius of the charged particle is an outsider, while in the electromagnetic treatment; the radius of the photon is an outsider. Fig. 5 shows the Doppler-shifted magnetic flux lines produced by a single indivisible moving charge which suggests that a polarity or spatial asymmetry must be introduced into the model. The bipolarity or spatial asymmetry can be introduced in the field and/or in the charged particle itself. In the next section I will discuss the experiments that led Maxwell [30] to state that the magnetic vector potential was a quantity that represented "*the fundamental quantity in the theory of electromagnetism*".

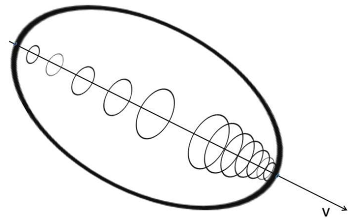

Fig.5: An illustration of how the magnetic flux lines produced by a charged particle moving towards the bottom right corner can be modeled as an extended and bipolar electron. If this illustration represented a static image of an ``induced'' electron captured in an instant of time, the flux lines would indicate that the front of the electron was paramagnetic ($\mu > \mu_o$) and a good conductor while the back of the electron was diamagnetic ($\mu < \mu_o$) and a poor conductor. The magnetic flux lines may differentially act back upon the front and back of the bipolar particle causing an additional shape change. A spinning electron would also produce magnetic flux line parallel to the axis of spin.

2.3. Faraday's electro-tonic state

Knowing that static electricity induced a redistribution of the charges in a nearby object [59], Michael Faraday [60] wondered if a current of moving charges in one stationary wire coil could likewise induce a current of moving charges in another stationary wire coil. Faraday found that it could, but only when the primary current increased or decreased. An increase in the primary current produced a secondary current whose direction was contrary to the circulation of the primary current while a decrease in the primary current produced a secondary current that flowed with the same sense as the primary current. Faraday also found that the inductive effect disappeared when the primary current in the stationary wire was steady although a steady primary current could induce a secondary current when the two wires were moved relative to each other. The direction of the induced current was contrary to the direction of the primary current when the two wire coils approached each other, and it had the same sense as the primary current when the two wire coils receded from each other. Faraday called the production of an induced current, *volta-electric induction*. Soon Faraday discovered that a moving magnet could also induce a current in a wire coil when it was introduced into or withdrawn from the loop and that the direction of the current depended on the N-S orientation of the magnet being introduced or withdrawn. Due to the "*similarity of action, almost amounting to identity*," between temporal and spatial variations in the primary electric currents and movements of common magnet, Faraday called the production of an induced current by a magnet, *magneto-electric induction*. The production of a secondary current by a variation of either a primary current or magnetic field, which was independently demonstrated by Joseph Henry [61,62], is now known as electromagnetic induction.

Faraday [63] could visualize the magnetic flux lines with iron filings, and he concluded that electromagnetic induction occurs when a wire experiences either an increase or a decrease in the number of magnetic flux lines as a result of a variation in the current in the primary wire, or as a result of the relative motion between the two wires or between a magnet and a wire. Each of these causes can be seen as changing the number of magnetic flux lines experienced by the secondary wire. Heinrich Lenz [64-66] proposed a generalized rule for determining the direction of the current induced in a wire that was independent of the method of induction. According to Lenz's Law, the induced current flowed in a direction such that the magnetic flux lines produced by the secondary current opposed the change in the magnetic flux lines that induced the secondary current. Lenz's Law described a governing effect in terms of neutralizing the magnetic flux lines.

Franz Neumann [66] combined Faraday's and Lenz's observations on induction into one mathematical equation that is known as Faraday's Law of Induction. Newman wrote, "*From the law of Lenz: the action which the inducing current, or magnet, exercises upon the induced conductor, if the induction results from the movement of the latter, is always in the nature of a check on its motion.*" James Clerk Maxwell also saw Lenz's Law as describing a neutralizing mechanical effect. According to Maxwell [30], "*the direction of the secondary current is such that the mechanical action between the two conductors is opposite to the direction of motion, being a repulsion when the wires are approaching, and an attraction when they are receding.*"

Faraday [67] considered that "*the induced electric current excited in bodies moving relatively to magnets, is made dependent on the intersection of the magnetic curves by the metal*." He went on to say "*….I cannot resist the impression that there is some connected and correspondent effect produced by this lateral action of the elements on the electric stream.... An action of this kind, in fact, is evident in the magnetic relations of the parts of the current.*

But admitting...the magnetic forces to constitute the power which produces such striking and different results at the commencement and termination of a current, still there appears to be a link in the chain of effects, a wheel in the physical mechanism of the action, as yet unrecognized. If we endeavour to consider electricity and magnetism as the results of two forces of...a peculiar condition of matter, exerted in determinate directions perpendicular to each other, then it appears to me, that we must consider these two states or forces as convertible into each other in a greater or smaller degree; i.e., that an element of an electric current has not a determinate electric force and a determinate magnetic force constantly existing in the same ratio, but that the two forces are...convertible by a process...at present unknown to us [68]."

Faraday [60] observed that induction exposes something novel and wrote "*whilst the wire is subject to either volta-electric or magneto-electric induction, it appears to be in a peculiar state; for it resists the formation of an electrical current in it....I have...ventured to designate it as the electro-tonic state*." Faraday [60] went on to say that "*The current of electricity which induces the electro-tonic state in a neighbouring wire, probably induces that state also in its own wire....*" That is, Faraday postulated the simultaneous existence of the conducting and electro-tonic state, or self-induction. Maxwell [69] called the electro-tonic state, "*the fundamental quantity in the theory of electromagnetism.*" In concluding a paper entitled, *On Faraday's Lines of Force*, Maxwell [70] wrote "*By a careful study of the laws of elastic solids and of the motions of viscous fluids, I hope to discover a method of forming a mechanical conception of this electro-tonic state adapted to general reasoning*." In a following paper, Maxwell [71] "*stated the mathematical relations between this electrotonic state and the lines of magnetic force...and also between the electrotonic state and electromotive force*."

Maxwell [30] took dimensional analysis seriously in exploring the relationships between seemingly unrelated quantities and introduced the equivalent terms: electromagnetic momentum and electrokinetic momentum to describe Faraday's electro-tonic state. Maxwell [71] wrote "*It appears...that if we admit that the unresisted part of electromotive force goes on as long as it acts, generating a self-resistant state of the current, which we may call (from mechanical analogy) its electromagnetic momentum...then induction of currents...may be proved by mechanical reasoning. What I have called the electromagnetic momentum is the same quantity which is called by Faraday the electrotonic state of the circuit, every change of which involves the action of an electromotive force, just as a change of momentum involves the action of a mechanical force*." In order to characterize Faraday's electro-tonic state, Maxwell gave it a number of designations, including the electrotonic state (without a hyphen), the electromagnetic momentum, and electrokinetic momentum. This emphasized the relationship between the electro-tonic state and momentum. Maxwell also used the magnetic vector potential to characterize the Faraday's lines of force ("magnetic flux lines") produced by a moving charge. According to Maxwell [30], "*The conception which Faraday had of the continuity of the lines of force precludes the possibility of their suddenly starting into existence in a place where there were none before. If, therefore, the number of lines which pass through a conducting circuit is made to vary, it can only be by the circuit moving across the lines of force, or else by the lines of force moving across the circuit. In either case a current is generated in the circuit. The number of the lines of force which at any instant pass through the circuit is mathematically equivalent to Faraday's earlier conception of the electrotonic state of that circuit...The total electromotic force acting around a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it*."

2.4. The magnetic vector potential

Maxwell [30] related the magnetic vector potential to the magnetic field:

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{30}$$

According to Maxwell [30], the time rate of change of the magnetic vector potential represented the electromotive intensity (V/m = N/C) that a charged particle placed at the point *x, y, z* would experience if the primary current were suddenly stopped. It follows that the product of ze and the time rate of change of the magnetic vector potential represents a force that a charged particle placed at the point *x*, *y*, *z* would experience if the primary current were suddenly stopped. This induced force ($\vec{F}_{induced}$) is antiparallel to the flow of current and is thus a reaction force or counter force and an electrodynamic way of describing the optomechanical Doppler force. The magnetic vector potential can be interpreted to act directly on an extended moving charge that generates it in such a way that it reduces the forward velocity of the charge. The counterforce and the time rate of change of electrokinetic momentum ($\vec{p}_{electrokinetic}$) caused by the magnetic vector potential is given by the product of the charge of the extended moving object and the electromotive intensity, which is the temporal derivative of the magnetic vector potential:

$$\vec{F}_{induced} = \frac{d\vec{p}_{electrokinetic}}{dt} = -ze\frac{d\vec{A}}{dt} \qquad (31)$$

The equation of motion for a particle whose velocity is governed by an induced force characterized by the magnetic vector potential is given by:

$$\vec{F}_{applied} = m_o\frac{d\vec{v}}{dt} + ze\frac{d\vec{A}}{dt} \qquad (32)$$

Which reduces to Newton's Second Law when the magnetic vector potential or its time rate of change vanish. The time rate of change of the apparent momentum ($\frac{d\vec{p}\prime}{dt}$) is given by:

$$\vec{F}_{applied} = \frac{d(\vec{p} + \vec{p}_{electrokinetic})}{dt}$$

$$= \frac{d(m_o\vec{v} + ze\vec{A})}{dt} = \frac{d\vec{p}\prime}{dt} \qquad (33)$$

Note that $ze\vec{A}$ is also incorporated into the canonical momentum used in the Schrödinger wave equation for charged particles. Substituting $-\vec{F}_{induced}$ for $ze\frac{d\vec{A}}{dt}$, we get:

$$\vec{F}_{effective} = \vec{F}_{applied} + \vec{F}_{induced} = m_o\frac{d\vec{v}}{dt} \qquad (34)$$

Where, $\vec{F}_{effective}$ is the effective force that results in the acceleration of a charged particle.

During the acceleration of a particle, the applied force is dissipated by electromagnetic friction, which can be modeled in terms of the temperature- and velocity-dependent optomechanical Doppler force ($\vec{F}_{Doppler}$) or the analogous induced electrodynamic force ($\vec{F}_{induced}$) given by the negative of the product of the charge and the time rate of change of the magnetic vector potential (Fig. 6).

As a result of the unavoidable production of the optomechanical or electromagnetic friction by a moving particle, the speed of the particle does not increase linearly with applied potential energy (PE) because a portion of the energy is used to create the

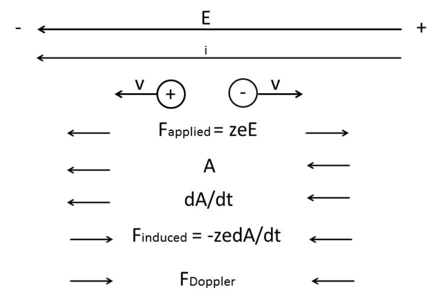

Fig.6: An illustration of the relative directions of the vectors that represent the current ($\vec{\imath}$), the velocity ($\vec{v}$), the magnetic vector potential ($\vec{A}$), the applied force ($\vec{F}_{applied}$), the induced force ($\vec{F}_{induced}$) and the optomechanical Doppler force ($\vec{F}_{Doppler}$) for a positively charged and negatively charged particle moving in an electric field ($\vec{E}$).

magnetic vector potential. Assuming that the potential energy is not transformed into the momentum of a particle but into its apparent momentum, then $PE = \frac{\vec{p}\prime^2}{2m_o}$. As a consequence of the creation of the magnetic vector potential, the potential energy needed to accelerate a particle of constant mass (m_o) from rest to velocity (v) is given by:

$$PE = \frac{\vec{p}\prime^2}{2m_o} = \frac{(m_o\vec{v} + ze\vec{A})^2}{2m_o}$$

$$= \frac{1}{2}m_o\vec{v}^2 + ze\vec{v}\vec{A} + \frac{(ze\vec{A})^2}{2m_o} \qquad (35)$$

Letting the kinetic energy (KE) of the particle be equal to $\frac{1}{2}m_o\vec{v}^2$, we see that only a portion of the potential energy is transformed into kinetic energy. The remainder is dissipated or transformed into a frictional loss described by first and second order terms that depend on the charge and the magnetic vector potential.

$$PE = KE + ze\vec{v}\vec{A} + \frac{(ze\vec{A})^2}{2m_o} \qquad (36)$$

According to the hypothesis that the optomechanical Doppler force and the product of the charge and the time rate of change of the magnetic vector potential are two equivalent ways of describing the same thing, at low velocities and/or low temperatures, where the magnetic vector potential becomes negligible, the magnitude of the energy necessary to accelerate a particle with constant mass to speed v approaches the classical value of $\frac{1}{2}m_o\vec{v}^2$. By equating the product of the charge and the time rate of change of the magnetic vector potential to the optomechanical Doppler force, we get an absolute zero of the magnetic vector potential at the absolute zero of temperature. This allows one to establish a gauge for the magnetic vector potential and to quantify the efficiency of energy transformations that involve electricity and magnetism [72].

The magnetic vector potential does not propagate instantaneously but serves as a means to describe the time-delayed influence of a moving particle on the surrounding field and the subsequent time-delayed influence of the surrounding field on the moving particle. The magnetic ($\vec{B}$) and electric ($\vec{E}$, in V/m) fields that make up the emitted radiation are related to the magnetic vector potential through the following relations:

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{37}$$

and

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \tag{38}$$

Where, φ represents the scalar electrostatic potential (in V).

During the acceleration of a charged particle, the energy partitioned into the magnetic vector potential $(ze\vec{v}\vec{A} + \frac{(ze\vec{A})^2}{2m_o})$ is unavailable to accelerate the particle and the frictional loss is emitted as Doppler-shifted electromagnetic radiation, an example of which is synchrotron radiation. One could test the equivalence of the induced electrodynamic force characterized by the magnetic vector potential and the temperature-dependent optomechanical Doppler force by checking the temperature dependence of synchrotron radiation. If the equivalence holds, the synchrochron radiation generated at 300 K should be ten thousand times greater than at 3 K.

2.5. Aharonov-Bohm effect

Apparently Heaviside [57] did not completely "*murder*" the "*artificial*" and "*useless*" magnetic vector potential after he reformulated Maxwell's equations exclusively using fields [73]. Eventually, Ehrenberg and Siday [74] and Aharonov and Bohm [75] independently questioned the post-Maxwellian idea that the magnetic vector potential was artificial and useless and devised an experiment that could test its significance. They devised an electron interferometer that could spatially separate the magnetic vector potential from the magnetic field and thus test the influence of the magnetic vector potential on the electrodynamics of moving bodies without complications introduced by the magnetic field. This can be accomplished with a solenoid, which produces a magnetic field that is uniform and concentrated within the solenoid but weak and divergent outside of the solenoid [76,77]. On the other hand, the magnetic vector potential produced by a solenoid extends outside of the solenoid, falling off with the inverse of the distance. The magnetic vector potential influences the apparent momentum of an electron when it is self-induced ($\vec{A}_e$) as described above; or, when it is produced by a solenoid ($\vec{A}_s$). In the electron interferometer, the phase of an electron is influenced by the magnetic vector potential produced by the solenoid, according to the following formula:

$$e\vec{A}_e = \hbar\vec{\nabla}\theta \tag{39}$$

Where, θ is the local space-time dependent phase of a quantum mechanical wave, and $\hbar\vec{\nabla}\theta = \vec{k}$, which is the angular wave vector.

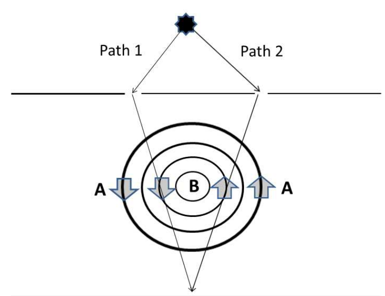

Fig.7: Diagram of an electron interferometer used to demonstrate the Aharonov-Bohm effect. A solenoid is used to generate a magnetic vector potential and a magnetic field. The magnetic field is concentrated into a uniform magnetic field within the solenoid while the magnetic field is weak and divergent outside the solenoid. On the other hand, the magnetic vector potential extends outside of the solenoid, falling off with distance. The magnetic vector potential has the same sense as the current used to create the magnetic field and the magnetic vector potential. If the current is clockwise, the magnetic flux lines point into the paper and the magnetic vector potential is clockwise. If the current is counterclockwise (as shown), the magnetic flux lines point away from the paper, and the magnetic vector potential circulates counterclockwise.

Indeed, when an iron whisker was placed between the two paths of an electron interferometer, originally made out of a modified Phillips EM100 electron microscope [78], such that the magnetic vector potential was parallel to one electron path and antiparallel to the other, the magnetic vector potential changed the interference pattern. Subsequent experiments by Möllenstedt and Bayh [79] using a solenoid (Fig. 7) demonstrated the predicted shift in the interference pattern confirming that the magnetic vector potential had a real meaning and a physical existence [80-83]. However, skeptics of the reality of the magnetic vector potential interpreted the results of the Aharonov-Bohm experiment in terms of a nonlocal interaction between the magnetic field and the electrons [84-87].

According to the optomechanical Doppler force interpretation of the magnetic vector potential, we do not have to choose between a mysterious potential that is local but cannot exert a real physical force and a real physical field that exerts a physical force but is nonlocal. Interpreting the Aharonov-Bohm effect using the optomechanical Doppler force, I find a change in the apparent momentum of the electrons resulting from the decelerating and accelerating forces caused by the magnetic vector potential produced by the solenoid. In one path, in which the magnetic vector potential produced by the solenoid is parallel to the self-induced magnetic vector potential, the magnetic vector potential produced by the solenoid adds to the apparent momentum of the electron. In the other path, the magnetic vector potential produced by the solenoid is antiparallel to the self-induced magnetic vector potential and subtracts from the apparent momentum of the electron.

$$\text{Path 1:} \quad \frac{\vec{p}_{apparent}}{m_o} = \vec{v} + \frac{ze\vec{A}_e}{m_o} + \frac{ze\vec{A}_s}{m_o} \qquad (40)$$

$$\text{Path 2:} \quad \frac{\vec{p}_{apparent}}{m_o} = \vec{v} + \frac{ze\vec{A}_e}{m_o} - \frac{ze\vec{A}_s}{m_o} \qquad (41)$$

The interference between two electron beams in an interferometer depends on the difference in the speed of the two beams and as the magnetic vector potential is increased, the interference fringe moves left or right in proportion to $(\frac{2ze\vec{A}_s}{m_o})$. By analogy with the optomechanical Doppler force, I predict that the magnetic vector potential produced by the solenoid will be temperature dependent, and it, along with the shift in the interference bands will vanish at absolute zero, where the magnetic vector potential vanishes along with the photon density.

3. Conclusions

In his study of induction, Faraday described a state of tension, which he called the electro-tonic state. Maxwell saw the electrotonic state as an electrokinetic momentum that could be described mathematically by the time rate of change of the magnetic vector potential. I have drawn an analogy between the product of the charge of an extended moving body and the time rate of change of the magnetic vector potential, and the optomechanical Doppler force, which is a contact force that characterizes the interaction between a charged particle and its environment consisting of a photon gas composed of the carriers of the electromagnetic force. Using this analogy, I have shown that the time rate of change of the magnetic vector potential characterizes the work done on a moving extended charged particle to resist its motion in an electrostatic field. By clarifying the relationship between the magnetic vector potential and its ability to do work on a charged particle, I have shown that magnetism is not just "*a relativistic aspect of electricity*" [42] but is necessary to understand the electrodynamics of moving bodies in real and absolute time. Moreover, according to the optomechanical Doppler effect interpretation of the magnetic vector potential, the Aharonov-Bohm effect is not a mysterious quantum effect, but may be a consequence of the change in the apparent momentum of the electrons that results from the local action of the magnetic vector potential produced by the solenoid.

While the magnetic vector potential came out of classical obscurity and gained some status in explaining the Aharonov-Bohm effect, its importance skyrocketed in terms of local gauge invariance [88]. Perhaps the optomechanical Doppler force analogy will help relate the magnetic vector potential to the canonical momentum used in quantum mechanics and to other quantum effects such as superconductivity.

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