Cosmic Strings and Domain Walls with Kaluza-Klein Metric in Self-Creation Theory of Gravitation

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We have considered the Kaluza-Klein metric in the context of cosmic strings and domain walls in Barber's second self-creation theory. For complete determinacy, a relation between the metric potentials is assumed. Various solution of the field equation are obtained and studied in several cases. Some physical and kinematical properties are studied in each case.

1. Introduction

Recently, Venkateswarlu and Sreenivas [1] studied the solutions of axially symmetric thick domain walls and cosmic strings in Barber's second self-creation theory. In continuation of our attempt to understand more on Barber's second self-creation theory, we have considered cosmic strings and domain walls with Kaluza-Klein metric in this paper.

Kaluza-Klein [2,3] have introduced a fundamental theory on higher dimensional models in which the extra dimension in FRW metric is considered to unify Maxwell's theory of electromagnetism and Einstein's gravity theory. It is also believed that the interaction of particles may be well explained by the higher dimensional cosmology. The study of higher dimensional space-time is of great importance to describe the early stages of evolution of the universe.

The study of higher dimensional cosmology has physical relevance to the early times before the universe has undergone compactification transitions. Chodos and Detweller [4], Witten [5], Marchiano [6] Applequist et al. [7], Venkateswarlu and Pavan [8], Reddy and Naidu [9], Reddy et al. [10], Reddy et al. [11], Ranjit et al. [12], Adhav et al. [13] and Venkateswarlu et al. [14] are some of the authors who have studied various aspect of Kaluza-Klein space-time in general relativity theory and in alternative theories of gravitation.

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2. Metric and Field Equations

We consider the five dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + dy^2 + dz^2\right] + A^2(t)d\mu^2 \tag{1}$$

Where, $R(t)$ is the scale factor and the fifth coordinate μ is assumed to be the space-like coordinate.

We may also use a correspondence to general relativity and define equivalent pressure and density (cf. Soleng [15]) as

$$p_{eq} = \frac{p_1}{\phi}, \; \rho_{eq} = \frac{\rho}{\phi}$$

The field equations, in Barber's second self-creation theory, are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi} T_{ij} \tag{2}$$

and

$$\Box \phi = \frac{8\pi}{3} \eta T \tag{3}$$

Where, all the symbols have their usual meaning as in general relativity.

2.1. Domain walls

The phase transitions in the early universe, due to spontaneous breaking of a discrete symmetry, could have produced the topological defects such as domain walls, strings and monopoles. Hill, Schramm and Fry [16] have suggested that light domain walls of large thickness may have been produced during the late time phase transitions such as those occurring after the decoupling of matter and radiation. Recently, the studies of the thick domain walls and space times associated with them have received considerable attention due to their application in structure formation in the Universe.

There are two approaches for the study of thick domain walls. In the first approach, one studies the field equations as well as the equations of domain wall treated as the self-interacting scalar field. In the second approach, one assumes the energy momentum tensor and then the field equations are solved. One way is to solve gravitational field equations with an energy-momentum tensor describing a scalar field Ψ with self interactions contained in a potential $V(\psi)$ given by

$$\psi_i \psi_j - g_{ij}\left[\frac{1}{2}\psi_k \psi^k - V(\psi)\right] \quad (4)$$

The second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho\,(g_{ij} + \omega_i\,\omega_j) + p_1\,\omega_i\,\omega_j \quad (5)$$

together with $\omega_i\,\omega^j = -1$.

Here ρ is the energy density of the domain walls, p_1 is the pressure in the direction normal to the plane of the domain walls and ω_i is the unit space-like vector. Using co-moving coordinate system, the non-vanishing component of T_{ij} can be obtained as

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = \rho,\, T_5^5 = -p_1 \quad (6)$$

The field equations (2) and (3) with the help of Eqns. (4)-(6) can be written as

$$\frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} + 2\frac{\ddot{R}}{R} = -\frac{8\pi}{\varphi}\rho \quad (7)$$

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\ddot{R}}{R} = -\frac{8\pi}{\varphi}\rho \quad (8)$$

$$3\frac{\dot{R}\dot{A}}{RA} + 3\frac{(\dot{R})^2}{R^2} = \frac{8\pi}{\varphi}p_1 \quad (9)$$

$$\ddot{\varphi} + \dot{\varphi}\left[\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right] = \frac{8\pi}{3}\eta(4\rho - P_1) \quad (10)$$

Where, the over head dot denotes ordinary derivative with respect to *t*.

2.1.1. Solutions to the field equations

The set of field Eqns. (7) to (10) are four equations in five unknowns. We assume the following condition to get a deterministic solution of the field equations:

$$A = R^m \quad (11)$$

Using Eqn. (11) in Eqns. (7)-(9), the solution of the field equations for self-gravitating or stiff domain walls (i.e., $p_1 = \rho$) is given by

$$R = \left[(m+3)(ct+d)\right]^{1/(m+3)} \quad (12)$$

$$A = \left[(m+3)(ct+d)\right]^{\frac{m}{m+3}} \quad (13)$$

Where, *c* and *d* are constants of integration.

By making use of Eqns. (12) and (13), now Eqn. (10) in this case reduces to

$$\frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{c}{(ct+d)}\right) = \frac{q}{(ct+d)^2} \quad (14)$$

Where, $q = 3\dfrac{(m+1)c^2\eta}{(m+3)^2}$.

The above equation is a second order differential equation with variable coefficient and hence its solution is

$$\varphi = c_1(ct+d)^{\frac{\sqrt{q}}{c}} + c_2(ct+d)^{\frac{-\sqrt{q}}{c}} \quad (15)$$

Where, c_1 and c_2 are constants of integration.

The kinematical and geometrical parameters of the model are

$$\frac{\rho}{\varphi} = \frac{P_1}{\varphi} = \frac{3c^2}{8\pi}\left[\frac{(m+1)}{(m+3)^2(ct+d)^2}\right] \quad (16)$$

$$\theta = \frac{c}{(ct+d)} \quad (17)$$

$$q=\frac{-a\,\ddot{a}}{\dot{a}^{2}}=2 \tag{18}$$

$$V=\frac{R^{3}r^{2}\sin\theta A}{\sqrt{\left(1-kr^{2}\right)}}=(m+3)(ct+d)r^{2}\sin\theta$$

$$\sigma=\frac{1}{\sqrt{3}}\left[\left(\frac{\dot{R}}{R}-\frac{\dot{A}}{A}\right)\right]=\frac{1}{\sqrt{3}}\left[\left(\frac{(1-m)c}{(m+3)(ct+d)}\right)\right] \tag{19}$$

It is observed that the above solution is similar to the solution obtained by Soleng [15] in Barber's second self-creation theory of gravitation when the source of gravitation field is a perfect fluid. It can be seen that one of the modes of the scalar field is expanding while the other one is diminishing for $m > -1$. If $m = -1$, the scalar field becomes constant and the solution in this theory reduces to Einstein's theory. However the extra dimension contracts as $t\to\infty$ where as the scale factor $R(t)$ expands forever in the case of $m = -1$. It is also noticed that $\rho = p_1 = 0$ when $m = -1$. Thus the self gravitating (or stiff) domain walls do not exist in Barber's second self-creation theory.

It is also observed that the stiff domain walls do exist for all t for $m > -1$. It can be seen that $V\to\infty, \theta\to 0, \sigma\to 0$, as $t\to\infty$. Since $q>0$, the model decelerates in its usual way.

2.2. Cosmic strings

The total energy momentum tensor for a cloud of massive strings can be written as

$$T_{ij}=\rho u_i u_j-\lambda x_i x_j \tag{20}$$

Here ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and $\rho=\rho_p+\lambda$, ρ_p being the energy density of the particles. The velocity u^i describes the 5-velocity, which has components (1, 0, 0, 0, 0) for a cloud of particles and x^i represents the direction of string, which will satisfy

$$u^i u_i=-x^i x_i=1 \quad \text{and} \quad u^i x_i=0 \tag{21}$$

In co-moving coordinate system, the non-vanishing component of T_{ij} can be obtained as

$$T_1^1=T_2^2=T_3^3=0, T_4^4=\lambda, T_5^5=\rho \tag{22}$$

For the weak, strong and dominant energy conditions, one finds that $\rho>0$ and $\rho_p\ge 0$ and the sign of λ is unrestricted. We consider x^i to be along the X_4-axis so that $x^i=(0, 0, 0, A^{-1}, 0)$.

The field Eqns. (2) and (3) for the metric in Eqn. (1) along with Eqns. (20) and (21) can be expressed as

$$\frac{\dot{R}^{2}}{R^{2}}+\frac{2\dot{R}\dot{A}}{RA}+\frac{\ddot{A}}{A}+2\frac{\ddot{R}}{R}=0 \tag{23}$$

$$3\frac{\dot{R}^{2}}{R^{2}}+3\frac{\ddot{R}}{R}=\frac{8\pi}{\varphi}\lambda \tag{24}$$

$$3\frac{\dot{R}\dot{A}}{RA}+3\frac{\dot{R}^{2}}{R^{2}}=\frac{8\pi}{\varphi}\rho \tag{25}$$

$$\ddot{\varphi}+\dot{\varphi}\left[\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right]=\frac{8\pi}{3}\eta(\rho+\lambda) \tag{26}$$

Where, the overhead dot denotes derivative with respect to time.

2.2.1. Solutions to the field equations

Eqns. (23)–(26) are set of four equations in five unknowns R, A, λ, ρ and φ. Thus for complete determinacy of the set once again we use the condition given by Eqn. (11).

Now, field Eqns. (23)-(25) together with Eqn. (11) admit the solution

$$R=\left[(1-m)(c_1 t+d_1)\right]^{\frac{1}{1-m}} \tag{27}$$

$$A=\left[(1-m)(c_1 t+d_1)\right]^{\frac{m}{1-m}} \tag{28}$$

Where, the arbitrary constants c_1 and d_1 satisfy

$$c_1\left(2m^2+3m+1\right)=0 \tag{29}$$

Eqn. (26) can be written as

$$(c_1 t+d_1)^2\frac{\ddot{\varphi}}{\varphi}+(c_1 t+d_1)\frac{\dot{\varphi}}{\varphi}K_1=K_2$$

Where, $K_1 = \frac{(m+3)c_1}{(1-m)}$, $K_2 = \frac{2\eta(m+1)^2 c_1^{\ 2}}{(1-m)^2}$.

The solution of the above equation is given by

$$\varphi = c_3 t^{\alpha_1} + c_4 t^{\alpha_2} \tag{30}$$

Where, $\alpha_1 = \frac{(1+m)}{(1-m)}\left[-1+\sqrt{2\eta+1}\right]$ and $\alpha_2 = \frac{(1+m)}{(1-m)}\left[-1-\sqrt{2\eta+1}\right]$. C_3 and C_4 are integration constants.

The tension density (λ), energy density (ρ), scalar expansion (θ), shear scalar (σ) and proper volume (*V*) of the model are

$$\frac{\rho}{\varphi} = \frac{\lambda}{\varphi} = \frac{3}{8\pi}\left[\frac{c_1^{\ 2}(1+m)}{(1-m)^2(c_1 t + d_1)^2}\right] \tag{31}$$

$$\theta = \frac{3\dot{R}}{R} + \frac{\dot{A}}{A} = \frac{(m+3)c_1}{(1-m)(c_1 t + d_1)}$$

$$V = R^3 A = \left[(1-m)(c_1 t + d_1)\right]^{\frac{m+3}{1-m}}$$

$$q = -3\theta^{-2}\left[\dot{\theta} + \frac{1}{3}\theta^2\right] = \frac{-4m}{(m+3)} \tag{32}$$

$$\sigma = \sqrt{\frac{2}{3}}\left[\frac{c_1}{c_1 t + d_1}\right]$$

It is evident from Eqn. (31), only geometric strings do exist in Barber's self-creation cosmology.

Now, from Eqn. (29) we have the following three cases:

Case(i): $c_1 = 0$

Case (ii): $2m^2 + 3m + 1 = 0$

Case(iii): $c_1 = 0$ and $2m^2 + 3m + 1 = 0$.

Case (i): In this case, the scale factors *R(t), A(t)* and $\varphi(t)$ become constant with $\lambda = \rho = 0$. Thus we conclude that the model reduces to a flat vacuum model in general relativity.

Case (ii): When $2m^2 + 3m + 1 = 0$ we have either *m* = -1 or $m = \frac{-1}{2}$.

Sub Case (1): If *m* = -1, the solution can be expressed as

$$R = \left[2(c_1 t + d_1)\right]^{\frac{1}{2}}$$

$$A = \left[2(c_1 t + d_1)\right]^{\frac{-1}{2}} \tag{33}$$

and the scalar field, the tension density, and the energy density are given by

$\varphi(t)$ = constant

$\rho = \lambda = 0$.

Thus the cosmic strings do not exist and the solution in this case also reduces to vacuum model general relativity.

Now the metric (1) can be written as (after suitably defining the constants)

$$ds^2 = -dt^2 + T\left[dx^2 + dy^2 + dz^2\right] + T^{-1} d\mu^2 \tag{34}$$

Sub Case (2): When, $m = \frac{-1}{2}$, the solution of the field equations take the form

$$R = \left[\frac{3}{2}(c_1 t + d_1)\right]^{\frac{2}{3}}$$

$$A = \left[\frac{3}{2}(c_1 t + d_1)\right]^{\frac{-1}{3}} \tag{35}$$

The scalar field is given by

$$\varphi = c_3 t^{m_1} + c_4 t^{m_2} \tag{36}$$

Where, $m_1 = \frac{-1+\sqrt{(1+2\eta)}}{3}$, and $m_2 = \frac{-1-\sqrt{(1+2\eta)}}{3}$.

Through proper choice of coordinates and constants, the metric in Eqn. (1) can be written as

$$ds^2 = -dt^2 + T^{\frac{4}{3}}\left[dx^2 + dy^2 + dz^2\right] + T^{-\frac{2}{3}} d\mu^2 \tag{37}$$

Eqn. (37) represents a five dimensional Kaluza-Klein geometric string model in Barber's second self-creation cosmology. The physical and kinematical parameters of the model (Eqn. (37)) are

$$\frac{\rho}{\varphi} = \frac{\lambda}{\varphi} = \frac{1}{12\pi}\frac{1}{T^2}$$

$$\theta = \frac{5}{3}\frac{c_1}{T}$$

$$V = T^{\frac{5}{3}} \tag{38}$$

$$\sigma = \sqrt{\frac{2}{3}\frac{c_1}{T}}$$

$$q = \frac{4}{5}$$

It is observed that the model (Eqn. (37)) starts expanding with time forever. The expansion in the model decreases with increase in time. So the model expands with a big-bang at $T = 0$ and halts at $T = \infty$. The tension density and energy density vanish as $T \to \infty$ and tend to infinity as $T \to 0$. Since $q > 0$, there is no inflation at any stage. It is further noticed that the extra dimension contracts, whereas the other scale factor *R(t)* expands indefinitely as time progresses. Thus, for $m = \frac{-1}{2}$, the extra dimension is amenable for contraction in geometric strings in this theory.

3. Conclusions

We have studied Kaluza-Klein metric with cosmic strings and domain walls in Barber's second self-creation cosmology. It is found that the extra dimension becomes unobservable in the case of thick domain walls and cosmic strings for some specific values of *m*, whereas the scale factor $R(T)$ expands forever. It is observed that the geometric string model is accelerating one and expands with a big-bang. The results obtained in this paper are new and different from that of Mohanty et al. [17].

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