Determination of Conductivity of Rock Samples using Fabricated Equipment

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The aim of this paper is to describe how simple and inexpensive equipment can be fabricated and used in the determination of thermal conductivity of rock samples. We used an experimental technique, known as the transient method of measuring thermal properties of rock samples, to investigate samples found in five different locations (Ewekoro, Ile-Ife, Igara, Ago-Iwoye and Abeokuta) in South Western regions of Nigeria. The rock samples are: limestone, dolerite, marble, gneiss, and granite. Although the samples are multi-mineral as revealed by photomicrograph, the thermal conductivity results obtained at values, 1.40, 1.50, 1.57, 1.75, and 2.94 W/m°C, are found to be consistent with those in literature and were obtained using highly expensive and sophisticated equipment not easily affordable in a developing nation.

1. Introduction

Nigeria lies between latitudes 5° and 14°N, and longitudes 3° and 14°E. Crystalline basement rocks of Precambrian age underlie about 50% of the country, which are un-conformably overlain by sedimentary rocks of Cretaceous to Recent age (Fig. 1). Samples used in this study were collected from South West Nigeria, comprising rocks of the Precambrian basement (granite, gneiss, dolerite and marble) and limestone from the Ewekoro Formation, which are generally considered to be of Paleocene age [1]. This region of the country is highly involved in geological exploration activities, especially borehole construction and well-logging.

It is a known fact that the heat form of transformation and transfer of the Earth’s inner energy determines such fundamental parameter as the temperature of depths, which in turn influences the physical properties of depths, their phase conditions, metamorphic processes and other fundamental properties of the Earth.

The local variations of the terrestrial heat flow are very small within the same geological region and consequently, the differences between geothermal gradients within areas of the same tectonic character are mainly due to the difference between the thermal conductivities of the rocks [5].

Investigations of the thermal properties of rocks using various methods such as the steady state divided bar technique, needle-probe method, transient methods etc., had been carried out by [10, 4, 9] and [6].

Transient method based on one-dimensional heat flow inside the sample was adopted in this study. It directly determines the thermal diffusivity, which is not important in its own right but offers a convenient, economical and accurate method for determining the thermal conductivity (the most important among the thermal properties of rocks).

This study, therefore, is aimed at determining: (i) the thermal parameters for each rock samples; (ii) the physical characteristics of the samples by visual inspection; (iii) relative ratio of mineral composition by thin section analysis; and thus

FIG. 1: Generalized geological map of Nigeria (after [7]).

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compile thermal conductivity data for the rock samples used.

2. Materials and method

2.1 Theory of method

The theory of thermal conduction describing the 1-D single-slab is given by [3] and [6]. They consider a slab that is at zero temperature initially, insulated at the surface \( x = 0 \) and has a constant heat flux introduced at the surface \( x = a \) at time \( t = 0 \). They showed that the temperature at a distance \( x \) within the slab and at time \( t \) (after the introduction of the constant heat at \( x = a \)) (Fig. 2) is given by:

\[
T = \frac{F}{\rho c a} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right) + \frac{Fa}{K} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right)
\]

\[= \frac{F}{\rho c a} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right) + \frac{Fa}{K} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right)
\]

(1)

Also:

\[
T(x, t) = \frac{F}{\rho c a} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right) + \frac{Fa}{K} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right)
\]

(2a)

\[= \frac{F}{\rho c a} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right) + \frac{Fa}{K} \left( \frac{3x^3 - a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n x}{n^2} e^{-\alpha n x/a} \right)
\]

(2b)

where \( \alpha \) is the thermal diffusivity, \( K \) is the thermal conductivity and, \( a \), is the thickness of the slab.

\[
F = \text{constant heat flux}
\]

\[
X=a
\]

\[
T=0 \text{ at } t=0
\]

\[
X=0
\]

Insulated initially \( \left( \frac{dT}{dx} = 0 \right) \)

FIG. 2: Diagram showing constant heat flux \( F \) at the top surface of one slab.

If the measurement is made at the base of the slab \( (x = 0) \), the expression for temperature becomes:

\[
T(a, t) = \frac{F}{\rho c a K} \left( \frac{3a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n a}{n^2} e^{-\alpha n a/a} \right)
\]

\[= \frac{F}{\rho c a K} \left( \frac{3a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n a}{n^2} e^{-\alpha n a/a} \right) + \frac{Fa}{K} \left( \frac{3a^3}{6a^2} - \frac{2}{P} \sum_{n=1}^{\infty} \frac{\cos \alpha n a}{n^2} e^{-\alpha n a/a} \right)
\]

(3)

For times large relative to \( \frac{a^2}{\alpha} \), the transient terms are negligible and the temperature versus time behaviour becomes linear. The intercept \( t_i \) on \( T = 0 \) axis is:

\[
t_i = \frac{a^2}{6\alpha^2}
\]

(4)

where, \( a \), is the thickness of the sample.

2.2 Heating device

A Nickel-Chromium 22 swg (standard weight gain) wire is used to design the heat source (coiled wire element) at the Department of Physics, Olabisi Onabanjo University, to supply a constant heat flux when placed at about 1cm above the exposed surface of the sample. The coiled wire (heat source) placed at about 1cm above the top surface of the sample amount to an `oven effect' [3] and as such the problems of thermal contact resistance are avoided at the top surface, hence the necessity to provide a fairly smooth surface.

The constant heat flux was maintained with the help of a transformer, which steps the 220V down to 12 - 15V and through which a constant current flows. Once a constant current is flowing, the wire will radiate near constant heat flux [3]. A stabilized low voltage and steady current power supply unit
powered the heater. The circuit diagram of the supply unit is as shown in Fig. 3.

The circuit is made up of an operational amplifier which is used as a variable voltage reference by wiring it as a voltage follower and applying a suitable reference to its input. The op-amp has very high input impedance when used in the ‘follower’ mode and thus draws near-zero current from the impedance and can supply several milliamps, the output loading cause little change in the output voltage value. The circuit then was made to act as a high-current regulated voltage (power) supply by wiring the current-boosting transistor networks into its output.

In this circuit, the available current is boosted by the Darlington-connected $Q_1$ and $Q_2$ pair of transistors. The circuit gain is fully variable from unity to $x10$ via $RV_1$ and the ZD1 pre-regulator network which enhances the stability of the $3V$ reference input to the op-amp. The circuit also incorporates an automatic overload protection. Here, $R_6$ senses the magnitude of the output current and when it exceeds the maximum current the resulting volt drop starts to bias $Q_3$, thereby shunting the base drive current of $Q_1$ and automatically limiting the circuit’s output current.

The output of the supply was set at 15V by $RV_1$ and thus there was a constant 15 stabilized voltage at the output.

![Circuit diagram of a stabilized low voltage and steady current power supply unit](image)

**FIG. 3**

### 2.3 Estimation of thermal conductivity

Thermal conductivity, a basic physical property of rocks, varies with changes in the rock composition. It is inversely proportional to thermal gradient and a temperature (gradient) log of a well in thermal equilibrium shows the actual variation of geothermal gradient (i.e., thermal conductivity) with lithology [8]. However, in this study, the expression in equation (4) is used to find the thermal diffusivity directly from a series of temperature versus time measurements. In practice, temperature is plotted against time (Fig. 4a-e), the intercept $t_i$ is read from the resulting graph and the thermal diffusivity is calculated using equation (4).

The relationship between thermal conductivity $K$ and diffusivity $\alpha$ is given as

$$K = \rho C \alpha$$  \hspace{1cm} (5)

where, $\rho$ is the density and $C$ is the specific heat. Equation (5) forms the basis for measuring thermal properties using this technique.

The specific heat and density are measured, whereas the thermal conductivity is calculated using equation (5).

### 2.4 Sample preparation and temperature measurement

A block of approximately 4 cm x 4 cm x 1 cm is cut and lightly polished to obtain its surface smooth. Surfaces other than the top are thermally insulated with very low thermal conductivity material (glass wool). The temperature is measured with a K–type thermocouple probe centred at the base of the sample rock at 10 second interval for 150 seconds. The K–type thermocouple plug is connected to a digital multi-meter (AVD890G). The K–type thermocouple probe has a temperature range of $–50^\circ C$ to $+400^\circ C$ with an accuracy of $\pm 0.75\%$ of $rgd$, and a resolution of $1^\circ C$.

A thin leaf of aluminium foil of 0.002cm (or 0.003cm) thickness is placed above the insulation layer and beneath the rock sample to improve the thermal contact to produce a uniform basal temperature since the major sources of error is the base contact between the rock sample, the K–type thermocouple probe and the basal insulation.

In effect, the aluminium foil, which is a much better thermal conductor than the rock sample or insulation, will distribute the basal temperature around the temperature probe “almost instantaneously”. However, the error due to the use of aluminium foil at the base of the sample slab is less than 1% and as such it can be neglected. The experimental set up is as shown in Fig. 4.

Thermocouple is capable of detecting 0.05 degree centigrade change above the ambient temperatures. Thermocouple wires usually 25 – 59 x $10^{-4}$m diameter are independently spot welded (in intrinsic arrangement). Thus, the small–diameter thermocouple of low conducting material attached
to a specimen of high conductivity and high diffusivity material yields the fastest response time.

Several precautions are required in the use of this sensor. For example, it must be focused at the centre of the base of the sample and the sample must be well insulated.

The measured temperatures are reduced by subtracting the initial ambient temperature effectively, making the measurements relative to zero initial temperature.

This reduced temperature is plotted against time, and the linear segment and intercept time \( t_i \) are then identified to determine the thermal diffusivity. In order to obtain a representative average value for a large volume of rock, it is always necessary to measure a number of conductivities from different samples of the same rock type. Hence, all the thermal properties were determined for 4 slabs per each rock sample and the mean values obtained except for dolerite (where we had only 3 slabs available). In this way, a more reliable data of mean conductivities was obtained.

![Diagram showing the experimental configuration for a single sample rock (one-slab solution).](image)

**FIG. 4:** Diagram showing the experimental configuration for a single sample rock (one-slab solution).

### 3. Results and discussion

#### 3.1 Rocks analysed

Plates 1-5 give the Photomicrograph of rock samples used which specifies different features of the samples.

**Limestone:**

Plate 1: Photomicrograph showing the typical texture of the limestone sample (Crossed polars (XP)). The length of the photograph represents 4.55 mm.
Dolerite:

Plate 2: Photomicrograph showing the fine-grained and porphyritic texture of the dolerite (Crossed polars (XP)). The length of the photograph represents 4.55 mm.

Marble:

Plate 3: Photomicrograph showing the coarse-grained and granoblastic texture of a marble sample (Crossed polars (XP)). The length of the photograph represents 4.55 mm.

Gneiss:

Plate 4: Photomicrograph showing the coarse-grained and foliated texture of the Gneiss (Crossed polars (XP)). The length of the photograph represents 4.55 mm.

Granite:

Plate 5: Photomicrograph showing the coarse-grained and the granular texture of granite sample (Crossed polars (XP)). The length of the photograph represents 4.55 mm.

The reliability and the accuracy of this method was ascertained through a comparison of the results obtained from this study with those obtained by [5]. The linear relationship of the temperature-time plot (Fig. 4a-e), which gives the intercept on the time axis and from which the thermal diffusivity is determined for the samples, agrees well with [6].

Graphs of Temperature against Time for the Samples

Fig. 4a: Graph of temperature-time plots for marble.
The physical characteristics through visual examination and the mineral content estimation from the thin section analyses (Plates 1-5) are summarized on Table 1, whereas, the density, thermal diffusivity, specific heat capacity, and thermal conductivity of five samples are summarized in Table 2.

All of the samples are multi-mineral and their densities ranges are (2.04-2.29E+03), (2.2-2.33E+03), (2.39-2.56E+03), (2.30-2.73E+03) and (2.04-2.24E+03) kg/m$^3$ for limestone, dolerite, marble, gneiss and granite, respectively.

The thermal conductivity values of the samples compare well with the published data [5], but with a little variation which could arise from the fact that the samples were cut perpendicular to the fabric that is said to give much lower values of conductivity than samples that were cut parallel to the fabric [2].

4. Conclusion

It can be seen that the values obtained here are consistent with the mineral composition and their relative abundances. Rocks are, as a rule, poor conductors of heat and have a comparatively narrow range of values of thermal conductivity (0.1-7 W/m$^0$C), which agrees favourably with this study. The differences observed in the present results and other published works could be traced back to two factors:

(i) Differences in the relative abundances of the mineral compositions;

(ii) Fabrics of the rocks.

The accuracy of thermal diffusivity in this study is strongly related to the strict adherence of the above-described experimental procedures. Errors of temperature and thickness measurement on the samples are approximated to be from 1-2% and that
of thermal conductivity about ±15%. However, the thermal conductivity values obtained will help in the study of the original thermal condition on the surface of the study area, and the increase in temperature with depth can also be determined by the terrestrial heat flow and the thermal conductivity of rocks.

Moreover, if the mean thermal conductivity cannot be accurately predicted, even the most sophisticated and appropriate modelling techniques for analysing thermal histories and maturation levels may fail when applied to real basins.

References

Table 1: Summary of the description of materials used.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Colour</th>
<th>Grain Size</th>
<th>Fabric</th>
<th>Mineral content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>Light grey</td>
<td>Fine</td>
<td>Isotropic</td>
<td>Calcite (97%), quartz (3%)</td>
</tr>
<tr>
<td>Dolerite</td>
<td>Dark grey</td>
<td>Fine</td>
<td>Foliated</td>
<td>Olivine &amp; pyroxene (45%), Plagioclase (40%), opaque ore &amp; calcite (15%)</td>
</tr>
<tr>
<td>Marble</td>
<td>Light grey</td>
<td>Coarse</td>
<td>Isotropic</td>
<td>Calcite (99%), plagioclase and opaque ore (&lt;1%)</td>
</tr>
<tr>
<td>Gneiss</td>
<td>Dark grey</td>
<td>Coarse</td>
<td>Foliated</td>
<td>Quartz (40%), plagioclase (30%), opaque ore &amp; sphene (5%)</td>
</tr>
<tr>
<td>Granite</td>
<td>Light grey</td>
<td>Coarse</td>
<td>Isotropic</td>
<td>Quartz (30%), microcline (35%), Plagioclase (30%)</td>
</tr>
</tbody>
</table>

Table 2: Thermal properties of samples and comparison results [5].

<table>
<thead>
<tr>
<th>Samples</th>
<th>Density (kg/m³)</th>
<th>Thermal Diffusivity (m²/s)</th>
<th>Specific heat capacity (J/kg°C)</th>
<th>Thermal conductivity (W/m°C)</th>
<th>This study</th>
<th>Kappelmayer &amp; Haenel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>2375.7</td>
<td>6.59E-07</td>
<td>978.5</td>
<td>1.40</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>Dolerite</td>
<td>2496.0</td>
<td>9.06E-07</td>
<td>973.6</td>
<td>1.50</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>Marble</td>
<td>2454.7</td>
<td>9.60E-07</td>
<td>664.6</td>
<td>1.57</td>
<td>2.65</td>
<td></td>
</tr>
<tr>
<td>Gneiss</td>
<td>2553.7</td>
<td>9.32E-07</td>
<td>735.4</td>
<td>1.75</td>
<td>2.08</td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>2114.0</td>
<td>1.78E-07</td>
<td>781.0</td>
<td>2.94</td>
<td>2.95</td>
<td></td>
</tr>
</tbody>
</table>

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