Analytical Solutions of Schrödinger Equation with Generalized Hyperbolic Potential Using Nikiforov-Uvarov Method

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The exact solution of Schrödinger equation for the generalized hyperbolic potential are presented using the Nikiforov-Uvarov method. The wave function and energy equation are each obtained analytically for the s-wave bound state. It is shown that the result for this generalized hyperbolic potential reduces to the standard Rosen-Morse, Poschl-Teller and Scarf potentials as special cases. The wave function and the corresponding energy equation for these special cases are also discussed.

1. Introduction

The exact bound state solution of Schrödinger equation can be obtained only for a few cases. These exact solutions are very important to quantum physics that can be understood through such solutions [1-9]. Such solutions are also valuable tools in checking and improving models. Furthermore, numerical methods are being introduced to solve complicated problems for some limiting cases.

The hyperbolic potential plays a vital role in atomic and molecular physics since it can be used to model inter-atomic and inter-molecular forces [10-11]. Among the extensively studied cases are the Poschl Teller [12], the Rosen-Morse [13] and the Scarf potentials [14]. However, some of these hyperbolic potentials are exactly solvable, or quasi-exactly solvable, and their bound state solutions have been found [15].

The main aim of this paper is to present and study a generalized hyperbolic potential from which other hyperbolic potentials can be deduced as special cases. We shall show that deduced potentials are also exactly solvable and that their energy spectrum and wave function have properties closely related to those that characterized the hyperbolic potentials. Different methods have been adopted in solving the Schrödinger equation with hyperbolic potential. These include Nikiforov-Uvarov method (NU) [16], factorization method [17], asymptotic iteration method [18], shape invariant [19], and super symmetric quantum mechanics (SUSYQM) [20]. The method used here reproduces accurate analytical solutions for many differential equations that have important applications in physics. For example, it can be used for equations of Hermite, Laguerre, Legendre, Bessel, and Jacobi [21].

Motivated by the recent successes in obtaining the bound state solutions for Schrödinger equation with hyperbolic potentials [22], we attempt to study the bound state solutions for a generalized hyperbolic potential using Nikiforov-Uvarov method.

The plan of the article is as follows. In Sec. 2, we review the Nikiforov-Uvarov method and present the generalized hyperbolic potential and limiting cases in Sec. 3. Sect. 4 is devoted to bound state solutions of the radial Schrödinger equation. Results and discussion are presented in Sec. 5, while a brief conclusion is given in Sec. 6.

2. Review of the Nikiforov-Uvarov Method

The NU method is based on the solution of a generalized second order linear differential equation with special orthogonal functions. Therefore, a non-relativistic Schrödinger equation can be solved exactly using this method. For any given real or complex potential, the Schrödinger equation is reduced to a generalized equation of hyper-geometric type with an appropriate, $s = s(r)$ coordinate transformation. Thus, it can be written as follows:

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi' + \frac{\bar{\sigma}(S)}{\sigma^2(S)}\psi(s) = 0 \quad (1)$$

Where, $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials of second degree and $\bar{\tau}(s)$ is a first degree polynomial.

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In order to find the exact solution to Eqn. (1), we set the wave function as

$$\psi(s) = \varphi(s)\chi(s) \qquad (2)$$

Substituting Eqn. (2) into Eqn. (1) reduces Eqn. (1) into hyper-geometric type and given as

$$\sigma(s)\chi''(s) + \tau(s)\chi'(s) + \lambda\chi(s) = 0 \qquad (3)$$

Where, the wave function $\varphi(s)$ is defined as the logarithmic derivative [16]

$$\frac{\varphi'(s)}{\varphi(s)} = \frac{\pi(s)}{\sigma(s)} \qquad (4)$$

Where, $\pi(s)$, is at most a first degree polynomial.

Also, the hyper-geometric type function $\chi(s)$ in Eqn.(3) for a fixed n is given by the Rodriques relation

$$\chi_n(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \qquad (5)$$

Where, B_n is the normalization constant and the weight function, $\rho(s)$ must satisfy the condition [16]

$$(\sigma(s)\rho(s))' = \tau(s)\rho(s) \qquad (6)$$

With

$$\tau(s) = \bar{\tau}(s) + 2\pi(s) \qquad (7)$$

For the weight function $\rho(s)$ to be satisfied, it is necessary that the classical orthogonal polynomials $\tau(s)$ be equal to zero in an interval (a, b) and its derivative at this interval at $\sigma(s) > 0$ be negative. That is

$$\tau'(s) = 0 \qquad (8)$$

Thus, the function $\pi(s)$ and the parameter λ required for the NU-methods are defined as follows

$$\pi(s) = \frac{\sigma' - \bar{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \bar{\tau}}{2}\right)^2 - \bar{\sigma} + k\sigma} \qquad (9)$$

$$\lambda = k + \pi'(s) \qquad (10)$$

The k-values in the square root of Eqn. (9) are possible to evaluate if the expression under the square root must be the square of polynomials. This is possible if and only if its discriminant is zero. Therefore, the new eigenvalue equation for the Schrödinger equation becomes

$$\lambda = \lambda_n = -n\tau' - \frac{n(n-1)\sigma''}{2}, n = 0,1,2\ldots \qquad (11)$$

On comparing Eqns. (10) and (11), we obtain the energy eigenvalues.

3. Generalized Hyperbolic Potential

We define the generalized hyperbolic potential as

$$V_{a,b,c,d}(r) = aV_0 \tanh(\alpha r) + bV_1 \tanh^2(\alpha r) - cV_2 \sec h^2(\alpha r) + d \qquad (12)$$

Where, V_0, V_1 and V_2 are the depth of the potential, and *a*, *b*, *c* and *d* are real numbers.

The limiting cases of the generalized hyperbolic potential are as follows:

(i) Rosen-Morse

When we set: $b = d = 0$, we obtain the Rosen-Morse potential as

$$V_{a,c,0,o}(r) = aV_0 \tanh(\alpha r) - cV_2 \sec h^2(\alpha r) \qquad (13)$$

(ii) Poschl-Teller potential

Setting $a = 0, b = 0, c = -c, d = 0$ we get the Poschl-Teller potential as

$$V_{0,0,c,0}(r) = cV_2 \sec h^2(\alpha r) \qquad (14)$$

(iii) Scarf potential

The Scarf potential is obtain from the generalized potentials by setting $a = 0, c = 0, d = 0$, and we obtain

$$V_{0,b,0,0}(r) = bV_2 \tanh^2(\alpha r) \qquad (15)$$

The generalized hyperbolic potential Eqn. (12) and its special cases Eqn. (13), Eqn. (14) and Eqn. (15) are common models for inter-atomic and inter-molecular forces. The special cases, Eqns. (13)-(15), have been solved extensively by many authors [22-26]. We seek to find the exact solution to Eqn. (12) and then deduce the special cases from the general result.

4. Solution of the Radial Schrödinger Equation

The radial Schrödinger equation takes the form [27]

$$\frac{d^2R(r)}{dr^2}+\frac{2m}{\hbar^2}[E-V(r)]R(r)=0 \qquad (16)$$

Substituting Eqn. (12) into Eqn. (16) yields

$$\frac{d^2R}{dr^2}+\frac{2m}{\hbar^2}[E-aV_0\tanh(\alpha r)-bV_1\tanh(\alpha r) + cV_2\,\mathrm{sec}h^2(\alpha r)-d]R(r)=0 \qquad (17)$$

Now using an *ansatz* for the wave function in the following form [28-29]

$$R(r)=e^{-\alpha r/2}F(r) \qquad (18)$$

This reduces Eqn. (17) into the following differential equation

$$\frac{d^2F}{dr^2}+\frac{\omega dF}{dr}+\frac{2m}{\hbar^2}[E-\left(\omega/2\right)^2+a\tanh(\alpha r) - b\tanh^2\alpha r+c\,\mathrm{sec}h^2\alpha r-d]F(r)=0 \qquad (19)$$

Setting: $s=\tanh(\alpha r)$, we get

$$\frac{d}{dr}=(1-s^2)\frac{d}{ds}$$
$$\frac{d^2}{dr^2}=\alpha^2(1-s^2)^2\frac{d^2}{ds^2}+2\alpha^2 s(1-s^2)\frac{d}{ds} \qquad (20)$$

On substituting Eqn. (20) into Eqn. (19), we get the generalized hyperbolic type equation as

$$\frac{d^2F(r)}{ds^2}-\frac{2(s-\omega/2)}{(1-s^2)}\frac{dF}{ds} -\frac{1}{(1-s^2)^2}\left[-\beta^2s^2+aV_0\,s-\varepsilon^2\right]F(s)=0 \qquad (21)$$

Where, we have used the following dimensional quantities

$$\beta^2=\frac{2m}{\hbar^2\alpha^2}(cV_2+bV_1)$$
$$\varepsilon^2=\frac{2m}{\hbar^2\alpha^2}\left(\left(\frac{\omega}{2}\right)^2+E+d+cV_2\right) \qquad (22)$$

Now comparing Eqns. (1) and (21), we get

$$\sigma(s)=(1-s^2), \qquad \bar{\tau}(s)=-2s+\alpha,$$
$$\bar{\sigma}(s)=\beta^2s^2-as+\varepsilon^2 \qquad (23)$$

Inserting these polynomials into Eqn. (9), we get $\pi(s)$ function as

$$\pi(s)=\frac{-\alpha}{2}\pm\frac{1}{2}\sqrt{-4(k+\beta^2)^2s^2+4as+(\omega^2-4\varepsilon^2+4k)} \qquad (24)$$

According to the NU method, the expression in the square roots of Eqn. (24) must be the square of polynomial. Thus, we find new possible functions of $\pi(s)$ for each k-value as

$$\pi(s)=\frac{-\omega}{2}\pm\frac{1}{2}\begin{cases}\left(\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2}\right)S-\sqrt{\left(\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2\right)} \\ \quad \text{for } k=\dfrac{\varepsilon^2-\beta^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\sqrt{u^2-v^2}}{2} \\ \left(\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2-\delta^2}\right)S-\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2} \\ \quad \text{for } k=\dfrac{\varepsilon^2-\beta^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\sqrt{u^2-v^2}}{2}\end{cases} \qquad (25)$$

Where

$$\delta^2 = a^2\sqrt{\left(1-\left(\frac{\omega}{2\sqrt{a}}\right)^4\right)}$$

$$U^2 = \sqrt{\left(\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2\right)^2}$$

$$V^2 = a^2\left(1-\left(\frac{\omega}{2\sqrt{a}}\right)\right) \quad (26)$$

The polynomial of $\tau = \bar{\tau} + 2\pi$ has a negative derivative

$$\tau = -2s - \left(\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2}\right)S$$

$$+\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2} \quad (27)$$

Now, using $\lambda = k + \pi'(s)$ and its other definition $\lambda_n = -n\tau' - \frac{n(n-1)\sigma''}{2}$, we have

$$\tau'(s) = -2 - \sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2} \quad (28)$$

$$\lambda(s) = \varepsilon^2 - \beta^2 - \left(\frac{\omega\varepsilon}{2}\right)^2 - \sqrt{u^2-v^2} - \frac{1}{2}\sqrt{u^2-\delta^2} \quad (29)$$

$$\lambda_n = 2n + n\sqrt{u^2-\delta^2} + n(n-1) \quad (30)$$

Equating Eqn. (29) and Eqn. (30), we obtain

$$\varepsilon^2 - \frac{(n+1)}{2}\sqrt{u^2-\delta^2} - \sqrt{u^2-v^2} - \Sigma = 0 \quad (31)$$

Where, $\Sigma = \beta^2 + \left(\frac{\omega}{\alpha}\right)^2 + 2n + n(n-1)$.

Simplifying Eqn. (31) and after a little algebra, we obtain

$$\left[\left(\frac{(n+1)}{4\delta} - \frac{1}{2v}\right)\left(1-\left(\frac{\omega}{2}\right)^2\right)^2\right]\varepsilon^4$$

$$+2\left[\left(\frac{(n-1)\beta^2}{4\delta} - \frac{\beta^2}{2v}\right) - \frac{i}{2}\right]\varepsilon^2$$

$$+\left[\left(\frac{(n-1)}{4\delta} - \frac{1}{2v}\right)\beta^4 - \frac{(n+1)\delta}{2} - v - i\Sigma\right] = 0$$

(32)

Solving Eqn. (32) explicitly and using Eqn. (22), we obtain the energy spectrum for the Schrödinger equation for the generalized hyperbolic potential as

$$E = \frac{\hbar^2/2m}{\left(1-\left(\frac{\omega}{2}\right)^2\right)^2}\left[-2\beta^2 + \frac{i4\delta v}{(n+1)v-2\delta}\right]$$

$$\pm \frac{4\delta v\,\hbar^2/m}{((n+1)v-2\delta)\left(1-\left(\omega/2\right)^2\right)}\sqrt{\beta^2 - \frac{i2\delta v}{\left(1-\left(\omega/2\right)^2\right)(n+1)v-2\delta} - \frac{((n+1)v-2d)\beta^4}{4\delta v} + \frac{(n+1)\delta}{2} + v - i\Sigma - d - c - \left(\omega/2\right)^2}$$

(33)

In order to find the corresponding wave function, we first evaluate the weight function $\rho(s)$ using Eqn. (6) and then write

$$\frac{d}{ds}\left((1-s^2)\rho(s)\right) = (-2s - \mu s + v)\rho(s) \quad (34)$$

Where, $\mu=\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2}$ and $\nu=\sqrt{\beta^2+\varepsilon^2-\left(\frac{\omega\varepsilon}{2}\right)^2+\delta^2}$.

Solving the first order differential of Eqn. (34), we get

$$\rho(s)=(1+s)^{\mu+\nu/2}(1-s)^{\nu+\mu/2} \tag{35}$$

Similarly, by substituting the values of $\pi(s)$ and $\sigma(s)$ into Eqn. (4), we have

$$\frac{\varphi'(s)}{\varphi(s)}=\frac{1}{4}\frac{(\mu-\omega-\nu)}{(1+s)}-\frac{1}{4}\frac{(\omega+\mu+\nu)}{(1+s)} \tag{36}$$

after resolving into partial fraction. Integrating Eqn. (36), we obtain

$$\varphi(s)=\left((1+s)\right)^{\frac{\mu-\nu-\omega}{4}}\left((1-s)\right)^{\frac{\mu+\nu+\omega}{4}} \tag{37}$$

Inserting the calculated $\rho(s)$ and $\sigma(s)$ into the Rodrique relation of Eqn. (5), we obtain the other wave function as

$$\chi_n(s)=B_n(1+s)^{\frac{-(\mu+\nu)}{2}}(1-s)^{\frac{\mu-\nu}{2}}\frac{d^n}{ds^n}\left[(1+s)^{n+\frac{\mu+\nu}{2}}(1-s)^{n+\frac{\nu-\mu}{2}}\right] \tag{38}$$

Where, B_n is the normalization constant.

The polynomial solution of $\chi_n(s)$ can be expressed in terms of the Jacobi polynomials, which is one of the orthogonal functions, i.e., $\chi_n(s)\approx P_n^{\left(\frac{\mu+\nu}{2},\frac{\nu-\mu}{2}\right)}(s)$. Hence the radial wave function for the generalized polynomial becomes

$$F(s)=\chi_n(s)\varphi(s)$$
$$=N_n(1+s)^{\frac{(\mu-\nu-\omega)}{4}}(1-s)^{\frac{(\mu+\nu+\omega)}{4}}P_n^{\left(\frac{\mu+\nu}{2},\frac{\nu-\mu}{2}\right)}(s) \tag{39}$$

The total radial wave function of the Schrödinger equation for the generalized hyperbolic potential can be constructed in compact form using Eqns. (18) and (39) as

$$R(r)=N_n e^{-\frac{\omega r}{2}}(1+\tanh(\alpha r))^{\frac{\mu-\nu-\omega}{4}}(1-\tanh(\alpha r))^{\frac{\mu+\nu+\omega}{4}}P_n^{\frac{\mu+\nu}{2},\frac{\nu-\mu}{2}}(\tanh(\alpha r)) \tag{40}$$

5. Results and Discussion

We can make appropriate choices for the values of the parameters in the generalized hyperbolic potential to obtain the well known potentials as stated before in Sec. 3. In Fig. 1, we plotted the variation of the generalized potential with r for: $a=11$, $b=0.5$, $V_0=1MeV$, $V_1=0.5MeV$, $c=2$, $V_2=0.02MeV$, $d=0.02MeV$, and for $\alpha=1,2,3$, 4.

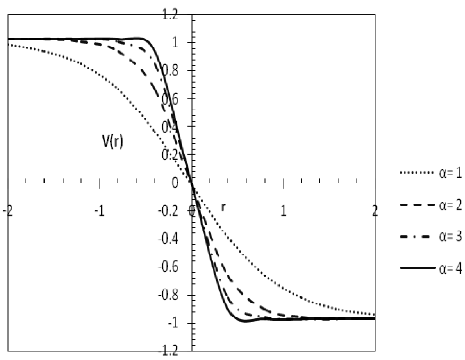

Fig1. A plot of generalized hyperbolic potential with r for a=1, 0.01, c=2, d=0.02, V_0=1Mev, V_1=0.5Mev, V_2=0.02MeV and α=1, 2, 3 and 4

5.1. Rosen-Morse potential

For, $b=d=0$, we obtain the Rosen-Morse potential as given in Eqn. (13). In Fig. 2, we plotted the variation of Rosen-Morse potential with r for: $a=-1$, $V_0=1MeV$, $c=2$, and $V_2=0.02MeV$, and for $\alpha=1,2,3$ and 4.

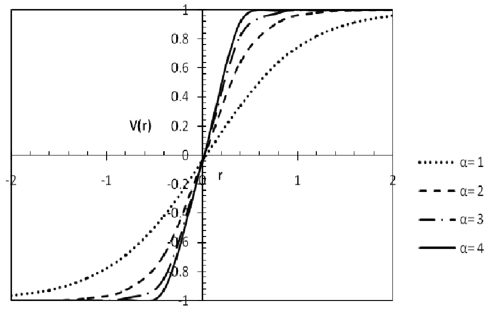

Fig2. Variation of Rosen-Morse potential with r for a= -1, b=0, c=2, d=0, V_0=1MeV, V_1=0.5MeV , V_2=0.02MeV with various parameter of α=1, 2, 3 and 4

The corresponding energy eigenvalues is obtained from Eqn. (33) as

$$E = \frac{\hbar^2/2m}{\left(1-\left(\frac{\omega}{2}\right)^2\right)^2}\left[-2\beta'^2+\frac{i4\delta v}{(n+1)v-2\delta}\right]$$

$$\pm\frac{4\delta v\,\hbar^2/m}{((n+1)v-2\delta)\left(1-\left(\omega/2\right)^2\right)}\sqrt{\beta^2-\frac{i2\delta v}{\left(1-\left(\omega/2\right)^2\right)(n+1)v-2\delta}-\frac{((n+1)v-2\delta)\beta'^4}{4\delta v}+\frac{(n+1)\delta}{2}+v-i\Sigma'-c-\left(\omega/2\right)^2} \quad (41)$$

Where $\beta'^2=\frac{2mc}{\hbar^2}$, $\varepsilon'^2=\frac{2m}{\hbar^2}\left(\left(\frac{\omega}{2}\right)^2+E+cV_2\right)$

and $\Sigma'=\beta'^2+\left(\frac{\omega}{2}\right)^2+2n+n(n-1)$.

The wave function of the Rosen-Morse potential is obtain from Eqn. (40) as

$$R(r)=N_n e^{-\frac{\omega r}{2}}(1+\tanh(\alpha r))^{\frac{\mu'-v'-\omega}{4}}(1-\tanh(\alpha r))^{\frac{\mu'+v'+\omega}{4}}P_n^{\frac{\mu+v}{2},\frac{v-\mu}{2}}(\tanh(\alpha r)) \quad (42)$$

Where $\mu'=\sqrt{\beta'^2+\varepsilon'^2-\delta^2}$ and $v'=\sqrt{\beta'^2+\varepsilon'^2-\delta^2}$.

5.2. Poschl Teller potential

For $a=b=d=0$ and $c=-c$, the generalized hyperbolic potential transforms to the Poschl-Teller potential given in Eqn.(14). In Fig. 3, we display the plot of Poschl-Teller potential versus r for $c=-2$, and $V_2=0.02MeV$. The corresponding energy equation and wave function for this potential are

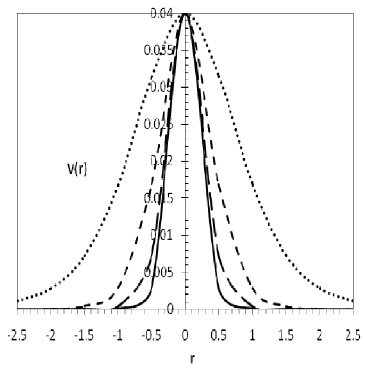

Fig3. A plot of Poschl-Teller potential with r for a= 0, b=0, c= -2, d=0, V_0=1MeV, V_1=0.5MeV , V_2=0.02MeV with various parameter of α=1, 2, 3 and 4

$$E=\frac{\frac{\hbar^2\beta'^2}{m}}{\left(1-\left(\frac{\omega}{2}\right)^2\right)^2}+c-\left(\frac{\omega}{2}\right)^2 \qquad (43)$$

$$R(r)=N_n e^{-\frac{\omega r}{2}}(1+\tanh(\alpha r))^{\frac{\overline{\mu}-\overline{\nu}-\overline{\omega}}{4}}(1-\tanh(\alpha r))^{\frac{\overline{\mu}+\overline{\nu}+\overline{\omega}}{4}}P_n^{\frac{\overline{\mu}+\overline{\nu}}{2},\frac{\overline{\nu}-\overline{\mu}}{2}}(\tanh(\alpha r)) \qquad (44)$$

Where, $\beta'^2=\frac{2mcV_0}{\hbar^2}$, $\overline{\mu}=\sqrt{\beta'^2+\varepsilon'^2-\left(\frac{\omega\varepsilon'}{2}\right)^2}$,

$\overline{\nu}=\sqrt{\beta'^2+\varepsilon'^2-\left(\frac{\omega\varepsilon}{2}\right)^2}$ and

$\varepsilon'^2=\frac{2m}{\hbar^2}\left(\left(\frac{\omega}{2}\right)^2+E-cV_2\right)$

5.3. Scarf potential

The generalized hyperbolic potential transforms to the standard Scarf potential, as given in Eqn. (15) when we set $a=c=d=0$. In Fig.4, we plotted the variation of this potential as a function of r for $b=0.05$ and $V_1=1MeV$ and $\alpha=1,2,3$ and 4. The corresponding energy equation and wave function for this potential are

$$E=\frac{\frac{-\hbar^2\overline{\beta}^2}{m}}{\left(1-\left(\frac{\omega}{2}\right)^2\right)^2}-\left(\frac{\omega}{2}\right)^2 \qquad (45)$$

$$R(r)=N_n e^{-\frac{\omega r}{2}}(1+\tanh(\alpha r))^{\frac{\overline{\overline{\mu}}-\overline{\overline{\nu}}-\omega}{2}}(1-\tanh(\alpha r))^{\frac{\overline{\overline{\mu}}+\overline{\overline{\nu}}+\omega}{2}}P_n^{\frac{\overline{\overline{\mu}}+\overline{\overline{\nu}}}{2},\frac{\overline{\overline{\nu}}-\overline{\overline{\mu}}}{2}}(\tanh(\alpha r)) \qquad (46)$$

Where, $\overline{\beta}^2=\frac{2mbV_1}{\hbar^2}$, $\overline{\overline{\mu}}=\sqrt{\overline{\beta}^2+\overline{\varepsilon}^2-\left(\frac{\omega\varepsilon'}{2}\right)^2}$,

$\overline{\overline{\nu}}=\sqrt{\overline{\beta}^2+\overline{\varepsilon}^2-\left(\frac{\omega\overline{\varepsilon}}{2}\right)^2}$ and

$\overline{\varepsilon}^2=\frac{2m}{\hbar^2}\left(\left(\frac{\omega}{2}\right)^2+E\right)$.

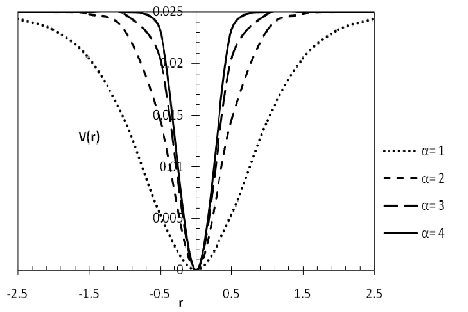

Fig 4. A plot of Scarf potential with r for a= 0, b=0.05, c=0, d=0, V_0=1MeV V_1=0.5MeV , V_2=0.02MeV with various parameter α=1, 2, 3 and 4

6. Conclusion

In this paper, the bound state solutions of the Schrödinger equation (SE) with a generalized potential have been investigated within the framework of the Nikiforov-Uvarov method. To the best of our knowledge, this potential has never been evaluated in compact form as we obtained above. We discussed the exact solution of the SE with this potential for S-wave bound states. We obtained explicitly energy eigenvalue equation and the eigen function of the SE equation for the generalized hyperbolic potential. As special cases, Rosen-Morse, Poschl-Teller and Scarf potentials were discussed.

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References

[1] A. N. Ikot, L. E. Akpabio and E. J. Uwah, EJTP, 8, **25**, 225 (2011).
[2] A. N. Ikot, A. D. Antia, L. E. Akpabio and J. A. Obu, Journal of Vect. Relativity, 6, **2**, 65 (2011).
[3] A. N. Ikot, L. E. Akpabio and J. A. Obu, Journal of Vectorial Relativity 6, **1**, 1 (2011).
[4] R. Sever, C. Tezcan, O. Yesiltas and M. Bucurget, Int. J. Theor. Phys. **47**, 2243 (2007).
[5] G. F. Wei, X. Y. Liu and W. L. Chen, Int. J. Theor. Phys. **48**, 1649 (2009).
[6] K. J. Oyewumi, E. O. Akinpelu and A. D. Agboola, Int. Theor. Phys. **47**, 1039 (2008).
[7] O. M. Al-Dossary, Int. J. Quant. Chem. 107, **10**, 2040 (2007).
[8] S. Ikhdair and R. Sever, Int. J. Theor. Phys. **46**, 2384 (2007).
[9] H. X. Quan, L. Guang, W. Z. Min, N. L. Bin and M. Yan, Commun. Theor. Phys. **53**, 242 (2010).
[10] A. S.Harberg, Int. J. Maths. Sci .2011(2011)1-9, Doi.10.1155/2011/3589
[11] S. Flugge, *Practical Quantum Mechanics*, Vol.1 (Springer, Berlin, 1994).
[12] Y. Xu, S. He and C. S. Jia, Phys. Scr. **81**, 045001 (2010).
[13] A. N. Ikot and L. E. Akpabio, App. Phys. Res. 2, **2**, 202 (2010).
[14] S. Meyur, EJTP, 8, **25**, 217 (2011).
[15] S. G. Roy, J. Choudhuruy, N. K. Sahkar, S. R. Karumuri and R. Bhattaharjee, EJTP, 7, **24**, 237 (2010).
[16] A. F .Nikiforov and U. B. Uvarov, *Special Functions of Mathematical Physics* (Birkhauser, Basel, 1988).
[17] S. H. Dong, *Factorization Methods in Quantum Mechanics* (Springer, Berlin, 2007).
[18] H. Ciftci, R. L. Hall and N. Saad, J. Phys. A. **36**, 11807(2003).
[19] M. R. Setare and O. Hatami, Chin. Phys. Lett. **25**, 3848 (2008).
[20] F. Cooper, A. Khare and U. Surhatme, Supersymmetry and Quantum Mechanics, Phys. Rept. **251**, 267 (1995).
[21] G. Chen, Phys. Lett. A **55**, 326 (2004).
[22] C. B. Compean and M. Kirchbach, J. Phys. A **39**, 547 (2006).
[23] K. J. Oyewumi and C. O. Akoshile, Euro. Phys. J. **A45**, 311 (2010).
[24] W. A. Yahya, K. J. Oyewumi, C. O. Akoshile and T.T. Ibrahim, J. Vect. Relativity 5, **3**, 27 (2010).
[25] Y. Xu, S. He and C. S. Jia, J. Phys. A: Math. Theor. **41**, 255302 (2008).
[26] W. C. Qiang and S. H. Dong, Phys. Lett. A. 3724789(2008).
[27] A. N. Ikot and L. E. Akpabio, IREPHY, 4, **4**, 224 (2010).
[28] S. D. Hernandez and David J. Fernandez C., Int. J. Theor. Phys. Doi.10.10007/s10735.01005333z (2010).
[29] C. B. Compean and M. Kirchbach, Euro. Phys. J. **A33**, 1 (2007).

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