

Bianchi Type-I Universe with Bulk Viscosity in General Relativity

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Exact solutions are obtained for anisotropic Bianchi type-I model with bulk viscosity, variable gravitational constant G and cosmological constant Λ . To get the deterministic solution of field equations, we took the vacuum density Λ proportional to the Hubble parameter H (Schutzhold 2002; H. A. Borges and S. Carneiro, 2005). The vacuum energy density Λ is found to be positive and a decreasing function of time, which is supported by the results from recent supernovae Ia observations. Some physical and geometrical properties of the model are also discussed. We have found an expanding universe and isotropy at late times.

1. Introduction

In the area of bulk viscous cosmology, Kalligas, Wesson, and Everitt [1], Beesham [2], Arbab and Beesham [3] have investigated FRW model as well as anisotropic Bianchi type-I cosmological models with variable gravitational constant G and cosmological constant Λ . Around the same time Arbab [4,5] has considered a viscous cosmological model with variable G , Λ , and shown that the universe become isotropic in the course of expansion. We wish here to derive some results in Bianchi type-I cosmology with viscous fluid where G and Λ vary with time using slightly different method from that of Arbab [6].

In order to solve Einstein's field equations, we normally assume a form for the matter content or suppose that the space time admits killing vector symmetries. Solutions to field equations may also be generated by a law of variation of cosmological term Λ . The phenomenological Λ -decay scenarios have been considered by a number of authors [7-10]. Chen and Wu [11] considered Λ varying as R^{-2} , Curvalho and Lima [12] generalized it by taking $\Lambda = \alpha R^{-2} + \beta H^2$ where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters on the basis of quantum field estimation in the curved expanding background. Schutzhold [13,14] proposed a vacuum density proportional to the Hubble parameter. This leads to a vacuum energy density decaying as $\Lambda \approx m^3 H$, where $m \approx 150$ MeV is the energy scale of the chiral phase

transition of QCD. Borges and Carneiro [15] have considered an isotropic and homogeneous flat space filled with matter and cosmological term proportional to Hubble parameter H . Recently, we have presented a perfect fluid cosmological model with variable G and Λ in FRW model and Bianchi type-I model [16-18].

In this paper, we study Bianchi type-I cosmological model with bulk viscosity, time dependent gravitational constant G and cosmological constant Λ . We obtain solution of Einstein's equations assuming that cosmological term proportional to the Hubble parameter. The paper is organized as follows: the basic equations of the model are given in Sec. 2 and the solution is given Sec. 3. The model is discussed in Sec. 4 and the results are included in Sec. 5.

2. Model and Field Equations

The spatially homogeneous and anisotropic Bianchi type-I space time is described by the line-element

$$ds^2 = dt^2 - A^2(t) dx^2 - B^2(t) dy^2 - C^2(t) dz^2 \quad (1)$$

The special volume of this model is given by

$$V = ABC \quad (2)$$

We define $R = (ABC)^{1/3}$ as the average scale factor so that the Hubble parameter of anisotropic model may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (3)$$

And,

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$$H = \frac{1}{3}(H_1 + H_2 + H_3) \quad (4)$$

Where, $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors in the directions of x, y, and z, respectively. Here and elsewhere a dot stands for ordinary time derivative of the concerned quantity.

We assume that the cosmic matter is represented by the energy momentum tensor of an imperfect fluid given as

$$T_{ij} = (\rho + p^*) u_i u_j - p^* g_{ij} \quad (5)$$

Where, ρ and p are energy density, isotropic pressure and u_i is the four velocity vector of the fluid satisfying the relation $u_i u^i = -1$

Here,

$$p^* = p - 3\eta H \quad (6)$$

Where, η is the coefficient of viscosity

The bulk viscosity takes the form

$$\eta = \eta_0 \rho^n \quad (7)$$

$$\eta_0 \geq 0, \quad (n \text{ is a constant})$$

We assume that the matter content obeys an equation of state,

$$p = (\gamma - 1) \rho, \quad 0 \leq \gamma \leq 2 \quad (8)$$

The Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G(t) T_{ij} - \Lambda(t) g_{ij} \quad (9)$$

For the metric (Eqn. (1)) and energy momentum tensor (Eqn. (5)) in co-moving system coordinates, the field equation (Eqn. (9)) yields

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G \rho - \Lambda \quad (10)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} = 8\pi G p^* - \Lambda \quad (11)$$

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} = 8\pi G p^* - \Lambda \quad (12)$$

$$\frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = 8\pi G p^* - \Lambda \quad (13)$$

In view of the vanishing of divergence of Einstein's tensor, we have

$$8\pi \dot{G} \rho + \dot{\Lambda} + 8\pi G \left[\dot{\rho} + (\rho + p^*) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] = 0 \quad (14)$$

From Eqns. (10)-(13) one obtains

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = 4\pi G (\rho + 3p^*) - \Lambda \quad (15)$$

The conventional energy conservation splits Eqn. (14) into following two equations

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (16)$$

$$\dot{\Lambda} + 8\pi \dot{G} \rho = 72\pi \eta G H^2 \quad (17)$$

Using Eqn. (3) in Eqn. (16), we obtain

$$\dot{\rho} + 3\gamma \rho H = 0 \quad (18)$$

3. Solution of the Field Equations

The system of Eqns. (7), (8), (10)-(13), and (17) supply only seven equations in eight unknowns ($A, B, C, \rho, p, \eta, G$ and Λ), therefore, one extra equation is needed to solve the system completely. For this purpose we take the cosmological term proportional to the Hubble parameter. This variation law for vacuum density has initially been proposed by Schutzhold [13, 14].

Recently, Borges and Carneiro [15] have considered a cosmological term proportional to H . Thus we take the decaying vacuum energy density

$$\Lambda = aH \quad (19)$$

Where, a , is a positive constant.

For the constants G and ρ ($\gamma = 0$), Eqn. (17) gives

$$H = k_1 e^{bt} \quad (20)$$

Where, k_1 and b are constant.

Therefore,

$$R = k_2 \exp[k_1 \exp(bt)] \quad (21)$$

Where, k_2 is a constant.

This extra or “hyper-inflationary” solution has been obtained by Barrow [19]. During this epoch Λ grows enormously. This solution does not hold for the isotropic case [4]. It is shown, however, by Beesham [2] that de-Sitter type solutions do not exist for either constant or non-constant energy density.

Using Eqns. (7), (18), (19) in Eqn. (17), one gets

$$8\pi\dot{G}\rho + \frac{a}{3\gamma} \left(\frac{\dot{\rho}^2}{\rho^2} - \frac{\ddot{\rho}}{\rho} \right) = \frac{8\pi\eta_0}{\gamma^2} \left(\frac{\dot{\rho}}{\rho} \right)^2 \rho^n G \quad (22)$$

If we assume the density to have the form

$$\rho = c_1 t^k \quad c_1, k \text{ constants} \quad (23)$$

The Eqn. (22) becomes

$$\dot{G} - \frac{k^2 \eta_0}{c_1^{1-n} \gamma^2} t^{kn-k-2} G = \frac{-akt^{-k-2}}{24\pi c_1 \gamma} \quad (24)$$

This equation admits a power law solution of the form

$$G = c_2 t^l, \quad c_2, l \text{ constants} \quad (25)$$

It follows from Eqn. (24) that $k = \frac{-1}{(1-n)}$ and

$$l = \frac{n}{1-n}.$$

This type of result has already been obtained by Arbab [4] for the isotropic case. Hence Eqns. (23), (25) and (18) become

$$\rho = c_1' t^{\frac{n}{1-n}}, \quad c_1' \text{ constant} \quad (26)$$

$$G = c_2' t^{\frac{n}{1-n}}, \quad c_2' \text{ constant} \quad (27)$$

$$H = \frac{1}{3\gamma(1-n)t} \quad (28)$$

and

$$\Lambda = \frac{a}{3(1-n)\gamma} t^{-1} \quad (29)$$

We now consider the anisotropy energy σ defined by

$$8\pi G\sigma = \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right)^2 \quad (30)$$

Using Eqn. (16) and Eqn. (10), the above equation becomes

$$8\pi G\sigma = \frac{2}{\gamma^2} \left(\frac{\dot{\rho}}{\rho} \right)^2 + 48\pi G\rho + 6\Lambda \quad (31)$$

From Eqn. (29), we see that the anisotropy energy (σ) becomes

$$8\pi G\sigma = c_3 t^{-2} + c_4 t^{-1} \quad (32)$$

c_3 const, c_4 const

$$c_3 = \frac{2}{\gamma(1-n)^2}, \quad c_4 = 8\pi G$$

The vacuum energy ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{\Lambda}{8\pi G} = \frac{a}{24\pi\gamma c_2' (1-n) t^{\frac{1}{1-n}}} \quad (33)$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{1}{24\pi\gamma^2 c_2' (1-n)^2 t^{\frac{2n}{1-n}}} \quad (34)$$

The ratio between matter density and critical density is now given by

$$\Omega = \frac{\Lambda}{\rho} = 24\pi\gamma^2 c_1' c_2' (1-n)^2 t \quad (35)$$

The deceleration parameter q for the model is

$$q = -1 - \frac{\dot{H}}{H^2} = 3\gamma(1-n) - 1 \quad (36)$$

The bulk viscosity of the model is

$$\eta = \eta_0 t^{-n/(1-n)} \quad (37)$$

4. Discussion

We observe that in the presence of bulk viscosity, matter density $\rho = \infty$ at $t = 0$ when $c_1 > 0$ and $n < 1$. The model starts expanding with a big-bang at $t = 0$ and the expansion in the model decreases as time increases. The expansion scalar, shear scalar, diverges at initial singularity, and as t increases the spatial volume increases but the expansion scalar decreases. Thus the rate of expansion slows down with an increase in time. Also $\sigma, \eta, \rho_v, \rho_c$ and Λ decrease as time increases. As $t \rightarrow \infty$, the scale factor and volume become infinite, whereas $\rho, \sigma, \eta, \rho_v, \rho_c$ and Λ tend to zero. Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \rightarrow 0$, therefore, the model approaches isotropy at late times.

In the absence of bulk viscosity the reality condition satisfied. The expansion scalar is infinite at $t = 0$. Thus, the model starts expanding with a big-bang at $t = 0$ and the expansion in the model decreases as time increases, whereas $\rho, \theta, \Lambda, \rho_c, \rho_v$ vanish for large values of t . In this case also $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \rightarrow 0$, therefore, the model approaches isotropy. The deceleration parameter $q < 0$ represents an accelerating universe.

5. Conclusion

In this paper, we have considered the solution of Einstein's field equations in Bianchi Type-I space-time with bulk viscosity. For the model, we observe that initially, at $t = 0$, quantities like expansion scalar θ and shear scalar σ diverge. As t increases, θ and σ decrease and this finally vanish at large t . This agrees with the result obtained by Back et al. [20], Adams et al. [21], Adox et al. [22] and Yilmaz [23]. Further, we observe that the bulk viscosity η and the energy density ρ have initial singularities. However, as t increases both parameters ρ and η vanish asymptotically. In the derived model of the universe, the vacuum energy density Λ is a decreasing function of time and approaches a small positive value as time increases (i.e., the present epoch). The value of Λ (cosmological constant) for the model is found to be small and positive, which is supported by the observations (Garnavich et al. [24,25]; Perlmutter et al. [26-28]; Riess et al. [29-31]; and Schmidt et al. [32]).

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