

Kantowski-Sachs Model with Decaying Cosmological Term

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Einstein field equations with variable cosmological constant are considered in the presence of a perfect fluid for Kantowski-Sachs cosmological model by assuming that the cosmological term is proportional to the Hubble parameter $\Lambda \propto H$ and the scalar of expansion is proportional to shear $\theta \propto \sigma$, which leads to a relation between the metric potentials $A=B^n$, for n a constant. The corresponding physical interpretations of these cosmological solutions are also discussed.

1. Introduction

There are many open questions in theoretical and experimental cosmologies, which are still under consideration by cosmologists and need to be answered. One of the fundamental and challenging problems in cosmology is the cosmological constant (Λ) problem. Einstein considered the introduction of Λ -term into his field equations as an error. But later on researchers found that the Λ -term is very useful for describing many cosmological observations. Most of the astrophysical work continued on the assumption that Λ is a constant. For example, gravitational lens statistics [1], CMB anisotropies [2], CMB intensity [3], structure formation [4,5], cluster abundance [6], and similar other works assumed Λ a constant. At the same time the studies of Bart [7], Peebles and Ratra [8], Moffat [9], Frieman et al. [10], Abramo et al. [11], and many others have shown the existence of an effective cosmological term that decays with time. The discussions of Ratra and Peebles [12], Dolgov [13-15], and Sanni and Starobiusky [16] on the problem of cosmological constant and on cosmology with a time-varying cosmological constant which revealed that in the absence of any interaction with matter or radiation, a solution of Einstein's equations and the assumed equation of covariant conservation of stress energy with a time varying Λ can be found for these solutions, the conservation of energy requires a decrease in the energy density of matter or radiation.

Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by several authors [17-25]. Some authors have argued for the

dependence, $\Lambda \sim t^{-2}$. Keeping in mind the dimensional consideration in the spirit of quantum cosmology, Chen and Wu [26] considered Λ varying as R^{-2} and Carvalho and Lima [27] generalized it by taking $\Lambda = \alpha R^{-2} + \beta H^{-2}$, where R is the scale factor of Robertson-Walker metric, H is the Hubble parameter $\propto$ and β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved, expanding background. Schützhold [28, 29] recently proposed a vacuum density proportional to the Hubble parameter that leads to a vacuum energy density decaying as $\Lambda \approx m^3 H$, where $m \approx 150$ Mev is the energy scale of the chiral phase transition of QCD. In a recent paper, Borges and Carnerio [30] have considered an isotropic and homogenous flat space filled with matter and cosmological term proportional to H , obeying the equation of state.

It is well known that the exact solutions of general theory of relativity for homogeneous space-times belong to either the Bianchi type or to Kantowski-Sachs type [31]. Weber [32,33] has done quantitative study of the Kantowski and Sachs [34] cosmological models. Larcoz [35], Gron. [36], Matravers [37], Krori et al. [38], Darowski [39], Li and Hao [40] have also studied cosmological models for the Kantowski-Sachs space-time. Singh and Agrawal [41] discussed Kantowski-Sachs type models in Saez and Ballester scalar tensor theory. Recently, Pradhan and Yadav [42] and Tiwari et al. [43] have done Kantowski-Sachs model with variable G and Λ .

In this paper, we consider the space-time of Kantowski-Sachs model in a general form with variable Λ . We apply the equation $\Lambda = aH$ [30] and scalar of expansion proportional to the shear scalar $\theta \propto \sigma$ [44].

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2. The Kantowski-Sachs Model

We consider the Kantowski-Sachs metric in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

Where, A and B are functions of the cosmic time t alone. Einstein's field equations with variable Λ and $8\pi G = 1$ in suitable units are

$$R_{ij} - \frac{1}{2} R_l^l g_{ij} = T_{ij} + \Lambda(t) g_{ij} \quad (2)$$

The energy momentum tensor for a perfect fluid is

$$T_{ij} = (\rho + p) v_i v_j - p g_{ij} \quad (3)$$

Where, ρ is the energy density of cosmic matter and p is its pressure.

Einstein's field equation (2) for the metric (Eqn. (1)) leads to

$$2 \frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B} \right)^2 + \frac{1}{B^2} = p - \Lambda \quad (4)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = p - \Lambda \quad (5)$$

$$2 \frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B} \right)^2 + \frac{1}{B^2} = -\rho - \Lambda \quad (6)$$

Where, the overhead dots denote the ordinary differentiation with respect to t.

From Eqns. (4) and (5), we have

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}}{B} \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) + \frac{1}{B^2} = 0 \quad (7)$$

There are only three independent equations in five unknowns A, B, p, ρ and Λ , only two more equations are needed to solve the system completely. We assume the scalar of expansion to be proportional to the shear scalar, $\theta \propto \sigma$ [44], which leads to a relation between metric potentials

$$\rho = - \left[\frac{2(n+1)}{T^2(n^2-1)} + \frac{a(n+2)}{3\sqrt{n^2-1}} \frac{\sqrt{T^{2n+2} + c_1(n^2-1)}}{T^{n+2}} + \frac{c_1(2n+1)}{T^{2n+4}} \right] \quad (15)$$

$$A = B^n \quad (8)$$

and cosmological term proportional to H [30], that is

$$\Lambda = aH \quad (9)$$

Where, a and n are constants.

Using Eqns. (8) and (7), we have

$$\frac{\ddot{B}}{B} + (n+1) \frac{\dot{B}^2}{B^2} + \frac{1}{(1-n)} \frac{1}{B^2} = 0, \quad n \neq 1 \quad (10)$$

Integrating Eqn. (10), we obtain

$$\dot{B} = \frac{\sqrt{B^{2(n+1)} + c_1(n^2-1)}}{B^{(n+1)} \sqrt{n^2-1}} \quad (11)$$

Where, c_1 is a constant of integration. With the help of Eqns. (8) and (11), the line element in Eqn. (1) reduces to

$$ds^2 = \frac{B^{2(n+1)}(n^2-1)}{B^{2(n+1)} + c_1(n^2-1)} dB^2 - B^{2n} dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (12)$$

By suitable transformation of coordinates, the line element in Eqn. (12), reduces to

$$ds^2 = (n^2-1) \frac{T^{2(n+1)}}{T^{2(n+1)} + c_1(n^2-1)} dT^2 - T^{2n} dr^2 - T^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (13)$$

For the model (Eqn. (13)), the geometrical and physical parameters can easily be obtained. The expressions for the cosmological constant $\Lambda(t)$, energy density ρ and pressure p are given by

$$\Lambda = \frac{a(n+2)}{3\sqrt{n^2-1}} \frac{\sqrt{T^{2n+2} + c_1(n^2-1)}}{T^{n+2}} \quad (14)$$

$$p = \frac{n^2}{(n^2-1)T^2} + \frac{a(n+2)}{3\sqrt{n^2-1}} \frac{\sqrt{T^{2n+2} + c_1(n^2-1)}}{T^{n+2}} - \frac{c_1(2n+1)}{T^{2n+4}} \quad (16)$$

The expansion scalar θ and shear σ for the model (Eqn. (13)) are

$$\theta = \frac{(n+2)}{\sqrt{n^2-1}} \frac{\sqrt{T^{2n+2} + c_1(n^2-1)}}{T^{n+2}} \quad (17)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}(n^2-1)} \frac{\sqrt{T^{2n+2} + c_1(n^2-1)}}{T^{n+2}} \quad (18)$$

We observe that the model has singularity at $T = 0$. The model starts with ρ, θ, σ and Λ all being infinite and continues to expand until $T = \infty$. For this model, the scale factors are zero at $T = 0$, which shows that the space-time exhibits point type singularity. Since $\sigma/\theta = \text{constant}$, the model does not approach isotropy for large value of T . Therefore, the model describes a continuously expanding, shearing, non-rotating universe with the big-bang start. In this model, we observe that the cosmological term Λ is infinite initially, gradually decreases and becomes zero at late times.

In the special case of $c_1 = 0$, from Eqn. (11) the line element in Eqn. (1) reduces to

$$ds^2 = dt^2 - \frac{t^{2n}}{(n^2-1)^n} dr^2 - \frac{t^2}{n^2-1} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (19)$$

After a suitable transformation, Eqn. (19) reduces to

$$ds^2 = dT^2 - T^{2n} dr^2 - T^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (20)$$

The physical and geometrical parameters of the model (Eqn. (20)) are

$$\Lambda = \frac{a(n+2)}{3T} \quad (21)$$

$$\rho = - \left[\frac{n(n+2)}{T^2} + \frac{a(n+2)}{3T} \right] \quad (22)$$

$$p = \frac{n^2}{T^2} + \frac{a(n+2)}{3T} \quad (23)$$

The shear σ and expansion scalar θ are given by

$$\sigma = \frac{(n-1)}{\sqrt{3}T} \quad (24)$$

$$\theta = \frac{(n+2)}{T} \quad (25)$$

In this case, since $\sigma/\theta = \text{constant}$, the model does not approach isotropy. We can obtain the deceleration parameter $q = (1-n)/(n+2)$, which shows that the deceleration parameter is constant. The model of constant deceleration parameter has been considered by Berman and Som [45]. The Hubble parameter H reads as

$$H = \frac{n+2}{3T} \quad (26)$$

Which, can also be written as

$$H = \frac{n-1}{3qT} \quad (27)$$

For the present phase p , we obtain

$$T_\rho = \frac{1-n}{3q_p H_p} \quad (28)$$

It is evident that negative q_ρ would increase present age of the universe.

The energy density tends to zero as T tends to infinity. We also observe that the spatial volume is zero at $T=0$. Thus, the singularity exists at $T=0$ in the model. The cosmological term $\Lambda(t)$ varies as the age of the universe and tends to zero as $T \rightarrow \infty$.

The deceleration parameter is constant for all time for $n=1$, $T_0 H_0 = 1$, this is within the current limits for the universe age $0.8 < T_0 H_0 < 1.3$ and is in good agreement with the best observation. $T_0 H_0 \sim 1$ [46].

In the model (Eqn. (20)), we observe that $\Lambda \sim T^{-1}$, which is in accordance with the main dynamical laws one finds in the literature proposed for the decay of Λ . The dynamical law $\Lambda \sim T^{-1}$ has been considered by several authors, e.g., Lau [47], Berman [25], Beesham [48], Lopez et. al.

[49], Bertolomi [50], Eendo et al. [51], and Conuto et al. [52], just to mention a few.

3. Conclusion

We have obtained exact solutions of the field equations for Kantowski-Sach space-time with variable cosmological constant $\Lambda(t)$ in the presence of a perfect fluid. In general, the space-time exhibits “Point type singularity” at initial stage and cosmological term varies as the age of universe. In the model, cosmological term Λ is infinite at the beginning and it decreases to become zero at late times. Deceleration parameter is constant for all times. In the model, we obtained, $\Lambda \sim T^{-1}$, which is in accordance with the main dynamical laws of for the decay of Λ .

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