

## New Agegraphic Model of Dark Energy with Varying Gravitational Constant in Flat Dvali-Gabadadze-Porrati Brane World Model

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In the context of DGP universe, the new agegraphic model in flat FRW universe is studied. Considering the variation of gravitational constant, EoS parameter and G-corrected deceleration parameter  $q$  for both  $\varepsilon = 1$  and  $\varepsilon = -1$  branches of the DGP model are obtained. Assuming non-linear EoS parameter for phantom liquid, the conditions of cosmic acceleration have been established. Moreover, the correspondence between new agegraphic model of dark energy and quintessence and chaplyng gas models has been discussed.

### 1. Introduction

The explanation of the accelerated expansion of the universe represents one of the most fundamental questions in physics and cosmology today. This fact, initially obtained using supernovae type Ia as reference sources [1,2], is now supported by more recent supernova measurements [3] and confirmed by a number of independent measurements [3-6]. It is suggested that the main contribution in this scenario comes from dark energy (DE).

The simplest candidate of dark energy is a tiny positive cosmological constant  $\Lambda$  with a constant equation of state (EoS)  $\omega = -1$  [3, 4], the value of this cosmological constant is very small as compared to quantum field theory. However, as is well known, it is plagued with the cosmological constant problem (the fine-tuning problem) and the cosmological coincidence problem. Therefore, many alternative dark energy models such as the quintessence [7,8], phantom [9,10], quintom [11,12], K-essence [13,14], tachyon [15,16], and ghost condensate [17,18] have been proposed. However, none of these models is very successful.

Some models, which are constructed based on fundamental principles, are more preferred as they may exhibit some underlying features of dark energy. The two examples of such kind of dark energy models are the agegraphic [19,20] and holographic [21-23] models. Although many dynamical dark energy models have been suggested, the nature of dark energy is still unknown.

Holographic principle [23] attracted many scientists to solve problems of cosmological constant and DE. According to this principle, the degree of freedom in a bounded system should be finite and it scales with its boundary area but not with its volume.

Cohen et al. [22] proposed a relationship between ultraviolet (UV) and infrared (IR) cut-offs due to a limit set by the formation of a black hole i.e.,  $L^3 \rho_L \leq LM_p^2$ . Here  $\rho_L$  is the vacuum energy density associated with UV cut-off and  $L$  is the IR cut-off. The largest  $L$  allowed is the one saturating this inequality, thus

$$\rho_A = 3c^2 M_p^{-2} L^{-2} \quad (1)$$

For convenience, we introduce the numerical constants:  $3c^2$  and  $M_p^{-2} = (8\pi G)$ , the reduced Planck mass. Holographic dark energy model (HDE) has been used to investigate the cosmic evolution by different people [14].

In this paper, we will focus on what is called new agegraphic dark energy model (NADE), which takes into account the uncertainty relation of quantum mechanics together with gravitational effect of general relativity. What attracts us to study (NADE) is the fact that this model has the same number of parameters as the  $\Lambda$ -cold-dark-matter ( $\Lambda$ CDM) model, which is less than other dynamical dark energy models.

This paper is organized as follows. In Sec. 2, we describe the NADE and DGP models. In Sec. 3, we derive the mathematical formulas for some parameters in our interacting model. In Sec. 4, the acceleration condition for our model in phantom liquid case is established. In Sec. 5 correspondence

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with some scalar fields is considered. Finally in the last section conclusions are given.

## 2. Basic Equations

### 2.1. New agegraphic dark energy model (NADE)

The NADE model is constructed in the light of the Karolyhazay relation and the corresponding energy fluctuation of space time. The energy density of the new agegraphic dark energy is given by [19]

$$\rho_A = 3n^2 M_p^2 \eta^{-2} \quad (2)$$

Where,  $n$  is a numerical parameter and  $\eta$  is the conformal age of the universe, which has the form [20]

$$\eta = \int \frac{dt}{a} = \int \frac{da}{Ha^2} \quad (3)$$

Where,  $a$  is the scale factor and  $H=a'/a$  is the Hubble parameter. Here, we consider the derivative with the cosmic time. The dark energy density (Eqn. (2)) has the same form as the holographic dark energy density in Eqn. (1), just the conformal time is used as IR cut-off instead of future event horizon distance of the universe.

In order to assume the momentum energy conservation laws, the total energy density  $\rho$  must satisfy the following continuity equation [23]

$$\rho' + 3H(1 + \omega)\rho = 0 \quad (4)$$

Where,  $\omega = p/\rho$  is the total EoS parameter.

In most investigations, dark matter DM and DE are considered as independent substances. However, there is no reason to neglect interactions in the dark sector. Including the possibility of a coupling between DM and DE is the most general case and gives rise to a richer dynamics [24]. That way the continuity equation could be split in the following way

$$\rho_A' + 3H(1 + \omega_A)\rho_A = -Q \quad (5)$$

$$\rho_m' + 3H\rho_m = Q \quad (6)$$

Where,  $\rho_A$  and  $\rho_m$  are the energy densities of DE and DM, respectively, and  $Q$  representing the interaction term.

Assuming that the critical density  $\rho_c = 3 M_p^2 H^2$  [23], one can define the fractional energy density for DE, matter and curvature as follows

$$\Omega_A = \frac{8\pi G_A}{3H^2} \quad (7)$$

$$\Omega_m = \frac{8\pi G_m}{3H^2} \quad (8)$$

$$\Omega_k = \frac{k}{a^2 H^2} \quad (9)$$

Recent observation shows that the curvature fractional energy density  $\Omega_k = 0.02$  [25], that way one can assume that the space time is flat.

Using Eqn. (2), one can write Eqn. (6) as

$$\Omega_A = \frac{n^2}{\eta^2 H^2} \quad (10)$$

This leads to

$$\eta = \frac{n}{H\sqrt{\Omega_A}} \quad (11)$$

This is the conformal time in terms of fractional energy density of DE.

### 2.2. Dvali, Gabadadze and Porrati model (DGP)

Dvali, Gabadadze and Porrati (DGP) braneworld model has been considered as a model that could modify gravity because of the existence of extra dimensions. This model assumes that we are living on 4D brane world in a 5D extra-dimensional bulk. All particles and standard model forces are pinned onto the brane world, only gravity can explore into the 5D bulk. It appears in large scale as weakening of gravity on the brane. Moreover, the DGP model can solve the cosmological constant problem, because gravity in 5D bulk is screened into 4D brane [26].

Within the framework of the Friedmann-Robertson-Walker (FRW) cosmology, the line element for non-flat universe is given by [27]

$$ds^2 = -dt^2 + a^2(t) \times \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right) \quad (12)$$

Where, the sign function of the curvature parameter  $k$  takes values +1, 0 and -1 for positively curved, flat and negatively curved spatial section, respectively.

If we consider a non flat universe containing agegraphic dark energy and dark matter, the corresponding Friedmann equations [28]

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_A + \rho_m) \quad (13)$$

Using Eqns. (7) to (9), Eqn. 13 could be written as

$$\Omega_A + \Omega_m + \Omega_k = 1 \quad (14)$$

The modified Friedmann equation in the DGP model relates the evaluation of Hubble parameter to the matter-energy content. For a self-accelerating universe, one can write [29]

$$H^2 + \frac{k}{a^2} = \left( \sqrt{\left( \frac{8\pi G}{3} + \frac{1}{4r_c^2} \right)} + \varepsilon \frac{1}{2r_c} \right)^2 \quad (15)$$

Where,

$$r_c = \frac{M_p^2}{2M_5^3} \quad (16)$$

This is the cross-vector scale, which determines the transition from 4-dimension to 5-dimension and  $\varepsilon = +1$  and -1.

The modified Friedmann equation (15) could be written in terms of the red shift [1]

$$H^2(z) = H_0^2 \left\{ \Omega_k(1+z)^2 + \left( \sqrt{\Omega_{rc}} + \sqrt{\Omega_{rc} + \Omega_m(1+z)^3} \right)^2 \right\} \quad (17)$$

Where the red shift,  $1+z = \frac{a_0}{a}$ , the effect of a finite  $r_c$  compared with Hubble parameter, could be written as a dimensionless parameter

$$\Omega_{rc} = \frac{1}{4r_c^2 H^2} \quad (18)$$

By setting  $z=0$  in Eqn. (17), one finds the normalization condition

$$1 = \left( \sqrt{\Omega_{rc}} + \sqrt{\Omega_{rc} + \Omega_m} \right)^2 + \Omega_k \quad (19)$$

Where,  $\Omega_m$  and  $\Omega_k$  are given by Eqns. (8) and (9) respectively.

Due to the central role played by the crossover factor, it is important to recast the impact of  $r_c$  as an effective dark energy component.

By defining  $\Omega_A$  same way as  $\Omega_m$  the ordinary condition (Eqn. (14)) is applied and by comparing it and Eqn. (19), one can write the relation between density parameters  $\Omega_A$  and  $\Omega_k$  as [29]

$$\Omega_A = 2\sqrt{\Omega_{rc}(1-\Omega_k)} \quad (20)$$

### 3. Interacting Model in Flat Universe

Observations of the cosmic microwave background demonstrate that the universe is very nearly flat. So, from now we assume that we are dealing with flat space-time problem, thus both  $k$  and  $\Omega_k$  are set to be equal zero. The NADE energy density in Eqn. (2) could be written as

$$\rho_A = \frac{3n^2}{8\pi\eta} \eta^{-2} \quad (21)$$

By taking the derivative of Eqn. (21) with respect to the cosmic time and by using Eqn. (11), one finds

$$\rho_A = -\rho_A \left( \frac{G'}{G} + \frac{2H\sqrt{\Omega_A}}{na} \right) \quad (22)$$

Since  $G' = dG/dx$  (where  $x = \ln a$ ), we derive

$$\rho_A = -\rho_A H \left( \Delta G + \frac{2\sqrt{\Omega_A}}{na} \right) \quad (23)$$

Where,  $\Delta G = G' / G$ .

Assuming that the interaction term [30] in Eqn. (5) is taken to be

$$Q = 3d^2 H \rho_A \quad (24)$$

Where,  $d$  parameter is the coupling constant and the selection of its square leads to the condition of decay from DE to DM. We derive an expression for the EoS parameter in the interacting phenomenon of NADE with DM as

$$\omega_A = -1 + d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \quad (25)$$

From Eqn. (7), we obtain

$$\eta' = -\frac{n}{\sqrt{\Omega_A}} \left( \frac{H'}{H^2} + \frac{\Omega_A'}{2H\Omega_A} \right) \quad (26)$$

Using Eqn. (3), it is easily seen that  $\eta = 1/a$  and thus one can write Eqn. (25) as

$$\frac{1}{a} = -\frac{n}{\sqrt{\Omega_A}} \left( \frac{H'}{H^2} + \frac{\Omega_A'}{2H\Omega_A} \right) \quad (27)$$

Since  $\Omega_A' = \frac{d\Omega_A}{dx} = \frac{1}{H} \Omega_A'$  we derive

$$\Omega_A' = -2\Omega_A \left( \frac{H'}{H^2} + \frac{\sqrt{\Omega_A}}{na} \right) \quad (28)$$

For flat geometry ( $k=0$ ) the DGP Friedmann Eqn. (15) can be rewritten as

$$\begin{aligned} H^2 &= \left( \sqrt{\frac{8\pi G}{3} + \frac{1}{4r_c^2}} + \varepsilon \frac{1}{2r_c} \right)^2 \\ &= \left( \left( \frac{1}{4r_c^2} + \frac{\rho}{3M_p^2} \right) + \frac{\varepsilon}{2r_c} \sqrt{\left( \frac{1}{4r_c^2} + \frac{\rho}{3M_p^2} \right)} + \frac{1}{4r_c^2} \right) \end{aligned} \quad (29)$$

For the early universe,  $\frac{\rho}{4M_p^2} \gg \frac{1}{4r_c^2}$ , the ordinary Friedmann equation is recovered.

Depending on the sign of  $\varepsilon$ , the cosmological solution of Eqn. (29) has two different regimes:

I - For  $\varepsilon = +1$  (DGP(+)) [31] one finds

$$\Omega_A' = 2\Omega_A \left( 2\Delta G \left( 1 + \Omega_A - 3\sqrt{\Omega_{rc}} \right) + 3 \left( 1 - 2\sqrt{\Omega_{rc}} \right) + \Omega_A \left( 3d^2 + \frac{2\sqrt{\Omega_A}}{na} \right) - \frac{\sqrt{\Omega_A}}{na} \right) \quad (37)$$

For completeness, we also derive the deceleration parameter [23]  $q$  as

$$q = -1 - \frac{H'}{H^2} \quad (38)$$

$$H^2 = \frac{1}{r_c^2} + \frac{2\rho}{3M_p^2} \quad (30)$$

Adding Eqns. (5) and (6) we obtain

$$\rho' = -3H(\rho_m + \rho_A(1 + \omega_A)) \quad (31)$$

By taking the derivative for Eqn. (30) and by using Eqn. (31) we obtain

$$\begin{aligned} H' &= (16\pi M_5^3)^2 GG' - 8\pi\pi(\rho_m + \rho_A(1 + \omega_A)) \\ &\quad + \frac{8}{3}\pi(\rho_m + \rho_A)G' \end{aligned} \quad (32)$$

Using Eqns. (7), (8), (16) and (30), it is possible to write Eqn. (32) as

$$\frac{H'}{H^2} = \Delta G(1 - 3(\Omega_m + \Omega_A)) - 3(\Omega_m + \Omega_A(1 + \omega_A)) \quad (33)$$

Assuming flat universe Eqn. (19) takes the form

$$1 - \sqrt{\Omega_{rc}} = \sqrt{\Omega_{rc} + \Omega_m} \quad (34)$$

By re-arranging and expanding Eqn. (34), we get

$$\Omega_m = 1 - 2\sqrt{\Omega_{rc}} \quad (35)$$

Here we ignored the higher order terms. Inserting Eqns. (25), (35) in (33), we finally obtain

$$\begin{aligned} \frac{H'}{H^2} &= -2\Delta G \left( 1 + \Omega_A - 3\sqrt{\Omega_{rc}} \right) - 3 \left( 1 - 2\sqrt{\Omega_{rc}} \right) \\ &\quad - \Omega_A \left( 3d^2 + \frac{2\sqrt{\Omega_A}}{na} \right) \end{aligned} \quad (36)$$

From Eqn. (28) and by substituting Eqn. (36), it is possible to write

From Eqns. (36) and (38), one finds

$$q = -1 + 2 \Delta G \left( 1 + \Omega_A - 3\sqrt{\Omega_{rc}} \right) + 3 \left( 1 - 2\sqrt{\Omega_{rc}} \right) + \Omega_A \left( 3d^2 + \frac{2\sqrt{\Omega_A}}{na} \right) \quad (39)$$

The DGP(+) branch gives a solution for late times, even when no matter is present on the brane and thus one can consider this branch as a self-accelerating branch. To explain the observed acceleration, one requires,  $r_c \approx H_o^{-1}$ , where  $H_o$  is the present value of Hubble parameter. Recently, many observations have been conducted to confirm the self acceleration [32,33].

II - For  $\square = -1$ (DGP(-)) in Eqn. (29) and by expanding in terms of  $\rho r_c^2 / M_p^2 \ll 1$  one finds at lowest order

$$H = \frac{\rho r_c}{3M_p^2} \quad (40)$$

By using Eqn. (16) in Eqn. (40), we obtain

$$\Omega'_A = -2\Omega_A \left( \frac{3 \left( 1 - 2\sqrt{\Omega_{rc}} + \Omega_A \left( d^2 - \frac{1}{3} \left( \Delta G - 2 \frac{\sqrt{\Omega_A}}{na} \right) \right) \right)}{(1 - 2\sqrt{\Omega_{rc}} + \Omega_A)} + \frac{\sqrt{\Omega_A}}{na} \right) \quad (44)$$

In the same way as done earlier, we find the deceleration parameter as

$$q = -1 - \frac{3 \left( 1 - 2\sqrt{\Omega_{rc}} + \Omega_A \left( d^2 - \frac{1}{3} \left( \Delta G - 2 \frac{\sqrt{\Omega_A}}{na} \right) \right) \right)}{(1 - 2\sqrt{\Omega_{rc}} + \Omega_A)} \quad (45)$$

DGP(-) represents the normal branch of the DGP model that requires a dark energy component to accommodate the current observation.

The evaluation of the non interacting model can easily be done by setting the  $d$ -parameter equal to zero in Eqns. (36), (37), (39), (43), (44) and (45).

$$H = \frac{\rho}{6M_5^3} \quad (41)$$

Now, by taking the derivative of Eqn. (41) and then using Eqn. (31) and Eqn. (16), we get

$$H \cdot = 3r_c H^3 (\Omega_m + \Omega_A (1 + \omega_A)) \quad (42)$$

Thus, using Eqns. (7), (8), (25), (35) and (40) in the above equation, we get

$$\frac{H \cdot}{H^2} = \frac{3 \left( 1 - 2\sqrt{\Omega_{rc}} + \Omega_A \left( d^2 - \frac{1}{3} \left( \Delta G - 2 \frac{\sqrt{\Omega_A}}{na} \right) \right) \right)}{(1 - 2\sqrt{\Omega_{rc}} + \Omega_A)} \quad (43)$$

From Eqn. (28) and by substituting Eqn. (43), it is possible to write

#### 4. Phantom Liquid in DGP Space

##### 4.1. Non-linear equation of state of phantom liquid

We assume that our universe can be conveniently described by a field giving rise to phantom energy liquid, which obeys a non-linear equation of state

$$P = -\rho - f(\rho) \quad (46)$$

Where,  $f(\rho)$  is a non-linear function of  $\rho$ , which is assumed [32] to take the following form

$$f(\rho) = A\rho^\alpha \quad (47)$$

Where,  $\alpha$  is a real number and  $A$  is a coupling constant of dimension  $(mass)^{4-4\alpha}$ .

For DE dominated case, by substituting Eqns. (46), (47) in Eqn. (5) one finds

$$\rho_A = 3HA\rho_A^\alpha - 3d^2H\rho_A \quad (48)$$

Now, by inserting (48) in (23), we obtain

$$A = \left( d^2 - \frac{1}{3} \left( \Delta G + \frac{2\sqrt{\Omega_A}}{na} \right) \right) \rho^{1-\alpha} \quad (49)$$

This is the derived coupling constant value in interacting case. Setting  $d = 0$  in the above equation one finds

$$A = - \left( \frac{1}{3} \left( \Delta G + \frac{2\sqrt{\Omega_A}}{na} \right) \right) \rho^{1-\alpha} \quad (50)$$

The above Eqn. (50) represents the value of  $A$  for non-interacting case.

#### 4.2 Cosmic acceleration condition in DGP brane cosmology

The condition for accelerating expansion [31] is

$$H \cdot + H^2 > 0 \quad (51)$$

Assuming flat space and by taking the first derivative of Eqn. (13) with respect to the cosmic time and then inserting in Eqn. (51), one finds

$$-\frac{1}{6M_p^2}(\rho + 3p) > 0 \quad (52)$$

This is the acceleration condition for standard cosmology, which implies  $p < -\rho/3$ . This condition will be modified in DGP cosmology with varying gravitational constant, let us consider the two DGP branches.

Using  $\omega = p/\rho$  and by applying Eqn. (46) in Eqn. (4), one finds

$$\rho \cdot = 3HA\rho^\alpha \quad (53)$$

I - For DGP (+) model, by taking the derivative of Eqn. (30) yields

$$H \cdot = \frac{1}{r_c^2} \Delta G + \frac{A}{M_p^2} \rho^\alpha + \frac{1}{3M_p^2} \rho \Delta G \quad (54)$$

So, one can write

$$H \cdot + H^2 = \frac{1}{r_c^2} (\Delta G + 1) + \frac{A}{M_p^2} \rho^\alpha + \frac{1}{3M_p^2} \rho (\Delta G + 2) \quad (55)$$

Inserting this equation in acceleration expansion condition (Eqn. (51)), one finds

$$H \cdot + H^2 = \left( \frac{1}{r_c^2} (\Delta G + 1) + \frac{A}{M_p^2} \rho^\alpha + \frac{1}{3M_p^2} \rho (\Delta G + 2) \right) > 0 \quad (56)$$

That implies

$$\rho(2 + \Delta G) + 3A\rho^\alpha > -\frac{3M_p^2}{r_c^2} (1 + \Delta G) \quad (57)$$

The above is the cosmic acceleration expansion condition for non linear phantom liquid in DGP(+) universe.

II - For DGP (-) model, by taking the derivative of (40) yields

$$H \cdot = \frac{1}{72(M_5^3)^2} A\rho^{\alpha+1} \quad (58)$$

That way one can write

$$H \cdot + H^2 = \frac{1}{72(M_5^3)^2} A\rho^{\alpha+1} + \frac{1}{36(M_5^3)^2} \rho^2 \quad (59)$$

Inserting this equation in acceleration expansion condition (Eqn. (52)), one finds

$$H \cdot + H^2 = \left( \frac{1}{72(M_5^3)^2} A\rho^{\alpha+1} + \frac{1}{36(M_5^3)^2} \rho^2 \right) > 0 \quad (60)$$

That implies that

$$\rho^{\alpha-1} > -\frac{2}{A} \quad (61)$$

This is now the cosmic acceleration expansion condition for non linear phantom liquid in DGP(-) universe.

#### 5. Correspondence with Some Scalar Fields Models

In this section, the correspondence between NADE and two important scalar fields, namely quintessence and

chaplying gas model is established. In this study, we shall consider the contribution of the matter to complete the analysis. This correspondence could be achieved by comparing the NADE-density with those of the scalar field and consequently equating EoS of the scalar fields with that of NADE.

### 5.1. Agegraphic quintessence model

The basic idea of quintessence is that the DE is in the form of a time varying scalar field, which is slowly rolling down towards its potential minimum and it leads to late time acceleration of the universe, the quintessence is then described in terms of the following action [34]

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \quad (62)$$

The pressure density and energy density of the model are given by

$$p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (63)$$

$$\rho_q = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (64)$$

EoS parameter is given by

$$\omega_q = \frac{\frac{1}{2} \dot{\phi}^2 - 2V(\phi)}{\frac{1}{2} \dot{\phi}^2 + 2V(\phi)} \quad (65)$$

Then one gets,  $\omega_q < -1$ , for  $\frac{1}{2} \dot{\phi}^2 < 2V(\phi)$ .

By comparing the EoS parameter of the quintessence scalar field (Eqn. (65)) and that of new agegraphic model of dark energy (Eqn. (25)) and by using Eqn. (5), it is possible to write

$$\dot{\phi}^2 = (1 + \omega_A) \rho_A \quad (66)$$

$$V(\phi) = \frac{1}{2} (1 - \omega_A) \rho_A \quad (67)$$

Consequently, one finds

$$\dot{\phi}^2 = \left( d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right) \rho_A \quad (68)$$

$$V(\phi) = \left( 2 - d^2 + \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right) \frac{\rho_A}{2} \quad (69)$$

Using Eqn. (69), one finds

$$\dot{\phi} = \frac{1}{H} \sqrt{\left( d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right) \rho_A} \quad (70)$$

Finally, by integrating the above equation one can find the evaluation of the quintessence scalar field as

$$\phi(a) - \phi(a_0) = \int_{a_0}^a \frac{da}{aH} \sqrt{\left( d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right) \rho_A} \quad (71)$$

Where,  $a_0$  is the present value of the scale factor, which is the interaction evaluation of the model and hence the results.

### 5.2. Chaplying gas

This fluid also leads to the acceleration of universe at late times. Its simplest form [33] is given by

$$p = -\frac{A}{\rho} \quad (72)$$

Where,  $A$  is a positive constant. We recall that for tachyons  $p = -V^2(\phi)\rho$ . That way, one can say that chaplying gas is a special case of tachyons with constant potential [33]. Following string theory [27,35], the homogeneous chaplying gas can be described using the pressure and energy densities by

$$p_c = \frac{\sigma}{2} \dot{\phi}^2 - V(\phi) \quad (73)$$

$$\rho_c = \frac{\sigma}{2} \dot{\phi}^2 + V(\phi) \quad (74)$$

One can write Eqn. (10) as

$$\omega_c = \frac{p_c}{\rho_c} = -\frac{A}{\rho^2} \quad (75)$$

By comparing the EoS parameter of the chaplying gas (Eqn. (75)) and that of new agegraphic model of dark energy (Eqn. (25)), one can deduce

$$A = -\rho^2 \left( -1 + d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right) \quad (76)$$

Using Eqns. (73), (74) and (75) one finds

$$\varphi'^2 = \frac{1}{\sigma} (1 + \omega_A) \rho_A \quad (77)$$

By substituting Eqn. (25) in Eqn. (76) and using the fact that  $\varphi' = \varphi' H$  one obtains

$$\varphi'^2 = \frac{1}{H} \sqrt{\frac{\rho_A}{\sigma} \left( d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right)} \quad (78)$$

In the same way as in previous sections, one finds

$$\varphi(a) - \varphi(a_0) = \int_{a_0}^a \frac{da}{aH} \sqrt{\frac{\rho_A}{\sigma} \left( d^2 - \frac{1}{3} \left( \Delta G - \frac{2\sqrt{\Omega_A}}{na} \right) \right)} \quad (79)$$

The above equation gives the interacting chaplyng gas evolution.

## 6. Conclusion

In this work, we have investigated the dynamics of G-corrected interacting NADE in which the geometry of the universe is described by a 3-dimension brane embedded in 5-dimensional bulk space, namely the DGP flat model. The DGP brane world is like general relativity in early times but it just modifies the dynamics in late times. Using the expression of NADE energy density, the EoS parameter,  $\Omega_A$  and the deceleration parameter  $q$  are established for the DGP two regimes, the extra dimensional gravity in DGP(+) propose late times acceleration as a result of the gravitational leakage from 3-dimension to 5-dimension Hubble distance. While, the DGP(-) represents the normal branch of the DGP, it shows that the universe expands slower than standard cosmology. The universe acceleration condition is calculated for phantom liquid in DGP space also for DE dominated case using a formula for non linear coupling constant  $A$  is deduced.

Moreover, due to the importance of scalar field models, which are regarded as a source for DE, the correspondence with the model is established. This correspondence plays an important role in our understanding of how different candidates of DE can be related to each other. Scalar fields show an important characteristic in explaining phantom crossing.

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