

Interacting Two-fluid Viscous Dark Energy Models in Self Creation Cosmology

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We present a class of solutions of Barber (1982) field equations describing two fluid of the universe for a new class of Bianchi type models. In this model, one fluid is barotropic and the other is bulk viscous. The exact solutions of the field equations are obtained for a different form of the scale factor. Also, the behavior of some physical and kinematical parameters is discussed in detail.

1. Introduction

The most striking discovery of modern cosmology indicates that the current universe is not only expanding but also accelerating. Today, this behavior of the universe is confirmed by various independent observational data, including the Type Ia Supernovae (SNe Ia) Perlmutter et al. [1-4], fluctuation of cosmic microwave background radiation (CMBR) [5] and large scale structure (LSS) [6,7]. In addition, measurements of the cosmic microwave background (CMB) and the large scale structure (LSS) strongly indicate that our universe is dominated by a component with negative pressure, dubbed as dark energy. The dark energy model has been characterized in a conventional manner by the equation of state (EoS)

parameter $w_D = \frac{p_D}{\rho_D}$, which is not necessarily

constant, where ρ_D and p_D be the energy density and fluid pressure. The w_D lies close to -1 . If it would be equal to -1 (standard Λ CDM cosmology), a little bit more than -1 (the quintessence dark energy) or less than -1 (phantom dark energy), while the possibility $w_D \ll -1$ is ruled out by current cosmological data from SN Ia. The simplest candidate for the dark energy is a cosmological constant Λ , which has pressure $\rho_D = -p_D$.

Barber [8] proposed two self-creation cosmologies by modifying the Brans-Dicke theory [9] and General Relativity. These modified theories create the universe out of self-contained gravitational and matter fields. Brans [9] has pointed out that Barber's first theory is not only in

disagreement with experiments but it actually is inconsistent. Barber's second theory is a modification of general relativity. Various aspects of the self-creation theories have been investigated by Pimentel [10], Soleng [11,12], Reddy et al. [13], Singh [14], Maharaj et al. [15], Venkateswarlu et al. [16-18], Reddy [19], Pradhan et al. [20-22], and Katore et al. [23,24].

The introduction of viscosity into cosmology has been investigated from different viewpoints [25-31]. The astrophysical observations also indicate some evidences that cosmic media is not a perfect fluid [32] and the viscosity effect could influence the evolution of the universe [33-35]. The cosmological evolution of a two-fluid dilation model of dark energy was investigated by Amirhashchi et al. [36, 37]; Saha et al. [38] have studied the two-fluid scenario for dark energy in FRW universe in different context. Very recently Singh and Chaubey [39] have studied interacting dark energy in Bianchi type I space-time. The viscous dark tachyon cosmology in interacting and non-interacting cases in non-flat FRW universe was studied by Setare et al. [40]. Two-fluid cosmological models in Bianchi type-III and V space-time has been investigated by Adhav et al. [41,42]. Bianchi type-VI₀ model with a two-fluid source has been investigated by Coley et al. [43]. Pant et al. [44] examined two fluid cosmological models using Bianchi type-II space-time. Two fluid Bianchi type-I models are studied by Oli [45].

Incited by above discussions, in this paper we propose the evolution of the dark energy parameter within the framework of a Bianchi type-III, V, VI₀ and VI_h cosmological model filled with two fluid bulk viscous dark energy (in both interacting and non-interacting cases) by considering a constant deceleration parameter. Also, the cosmological

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implications of this two-fluid scenario are discussed in detail.

2. Metric and Field Equations

Consider the diagonal form of the metric of a general class of Bianchi type models as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 e^{-2mx} dz^2 \quad (1)$$

Where, the additional classes of Bianchi type models are as follows: type III corresponds to $m = 0$, type V corresponds to $m = 1$, type VI₀ corresponds to $m = -1$, and all other m give type VI_h, where $m = h - 1$, A, B and C are functions of t only with m is constant.

The Einstein field equation in Barber's second self-creation theory of gravitation is

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\phi^{-1} T_{ij} \quad (2)$$

Where, G_{ij} is the Einstein tensor and T_{ij} is the energy-momentum tensor.

The scalar field equation is defined as

$$\square\phi = \frac{8\pi}{3} \lambda T \quad (3)$$

Where, $\square\phi = \phi_{;k}^k$ the invariant d'Alembertian and T is the trace of the energy momentum tensor.

The energy-momentum tensor for the fluid sources is given by

$$T_j^i = (\rho + \bar{p}) U_i U^i - \bar{p} g_j^i \quad (4)$$

Where, T_j^i represent two-fluid energy momentum tensor of bulk viscous and dark energy, together with

$$U_i U^i = 1 \quad (5)$$

and

$$\bar{p} = p - \eta U_{;i}^i \quad (6)$$

Where, ρ , p , $\bar{p}$ and η are the energy density, isotropic pressure, effective pressure and the bulk viscosity coefficient, respectively. U^i is the four-velocity vector of the matter distribution defined as

$$U^i = \delta_4^i, i=1,2,3,4 \quad (7)$$

We assume that the coordinate to be co-moving, so that $U^i = (0,0,0,1)$.

For the universe filled with bulk viscous fluid, one find from Eqn. (4)

$$T_1^1 = T_2^2 = T_3^3 = -\bar{p}, T_4^4 = \rho \text{ and } T = \rho - 3\bar{p} \quad (8)$$

In a co-moving coordinate system, the field equation (2) for the metric in Eqn. (1) with the help of Eqn. (8) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{m}{A^2} = -8\pi\phi^{-1} \bar{p} \quad (9)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{m^2}{A^2} = -8\pi\phi^{-1} \bar{p} \quad (10)$$

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{B_4 A_4}{BA} - \frac{1}{A^2} = -8\pi\phi^{-1} \bar{p} \quad (11)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{(m^2 + m + 1)}{A^2} = 8\pi\phi^{-1} \rho \quad (12)$$

$$(m+1) \frac{A_4}{A} - \frac{B_4}{B} - m \frac{C_4}{C} = 0 \quad (13)$$

$$\phi_{44} + \phi_4 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = \frac{8\pi\eta}{3} (\rho - 3\bar{p}) \quad (14)$$

Where, the suffix 4 represents the derivative with respect to time t .

Since the twice contracted Bianchi identity $G_{;j}^j = 0$, assures the conservation of the energy-momentum tensor $T_{;j}^{ij} = 0$, we could be sure that the fluid would satisfy the energy-momentum conservation law.

The conservation equation yield

$$\rho_4 + (\rho + \bar{p}) \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad (15)$$

Where,

$$\bar{p} = p_m + \bar{p}_D \quad \text{and} \quad \rho = \rho_m + \rho_D \quad (16)$$

Here p_m and ρ_m are the pressure and density of barotropic fluid, and $\bar{p}_D$ and ρ_D are the pressure and density of dark fluid, respectively.

The equation of state (EoS) for barotropic (w_m) fluid and dark fluid (w_D) are, respectively, given by

$$w_m = \frac{p_m}{\rho_m} \quad (17)$$

$$w_D = \frac{\bar{p}_D}{\rho_D} \quad (18)$$

Now, we define some parameters for the general class of Bianchi type model that are important in cosmological observations. The average scale factor and spatial volume are

$$a = (ABC)^{\frac{1}{3}} \quad (19)$$

$$V = a^3 = (ABC) \quad (20)$$

The anisotropy parameter of the expansion is expressed in terms of mean and directional Hubble parameters as

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 \quad (21)$$

Where

$$H = (\ln a)^{\bullet} = \frac{a_4}{a} = \frac{1}{3} (H_1 + H_2 + H_3) \quad (22)$$

Where, H is the Hubble parameter, w_D , $H_2 = \frac{B_4}{B}$

and $H_3 = \frac{C_4}{C}$ are the directional Hubble parameters of x , y and z axes, respectively.

The kinematical parameters, which are of observational interest in cosmology, are the expansion scalar θ , the shear scalar σ^2 , and the deceleration parameter q , and defined as

$$\theta = u_{;m}^m = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) \quad (23)$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left\{ \left(\frac{A_4}{A} \right)^2 + \left(\frac{B_4}{B} \right)^2 + \left(\frac{C_4}{C} \right)^2 \right\} - \frac{\theta^2}{6} \quad (24)$$

$$q = -\frac{a_{44} a}{a_4^2} = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \quad (25)$$

The field equations (9) to (12) can be expressed in terms of q, H, σ^2 as

$$\bar{p} = H^2 (2q - 1) - \sigma^2 + \frac{(m^2 + m + 1)}{3A^2} \quad (26)$$

$$\rho = H^2 - \sigma^2 - \frac{(m^2 + m + 1)}{A^2} \quad (27)$$

3. Exact Solution of Field Equations

The Einstein field equations (9)-(12) are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. Berman [46] in the 1980's proposed a law of variation for Hubble parameter within the context of RW space-time in general relativity given as

$$q = \frac{-a a_{44}}{a_4^2} = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = -kt + n - 1 \quad (28)$$

Where, $k, n \geq 0$ are positive constants.

Solving Eqn. (28) for $k = 0$, one obtains two different forms of solutions for the scale factor as

$$a = (nlt + b)^{1/n}, \quad k = 0, \quad n > 0 \quad (29)$$

and

$$a = de^{lt}, \quad k = 0, \quad n = 0 \quad (30)$$

Where, l, b and d are the constants of integration.

Here we discuss two different physically viable decay laws for the context of cosmological models

Case I:

3.1. In this case we find value of metric potentials for $k = 0$, $n > 0$ and $C = V^\alpha$ [47] where α be any constant number, using Eqns. (13), (19), (29), we get

$$A = (nlt + b)^{\frac{(3+3m-3\alpha-3\alpha)}{n(m+2)}} \quad (31)$$

$$B = (nlt + b)^{\frac{(3+3m-3\alpha-6m\alpha)}{n(m+2)}} \quad (32)$$

$$C = (nlt + b)^{\frac{(3\alpha)}{n}} \quad (33)$$

From Eqn. (22), the average generalized Hubble parameter H is

$$H = \frac{l}{(nlt + b)} \quad (34)$$

Also from Eqns. (24) and (25) we get

$$\sigma^2 = \frac{X_1}{2(m+2)^2(nlt + b)^2} \quad (35)$$

Where,

$$X_1 = l^2 \left[(3+3m\alpha-3\alpha)^2 + (3+3m-3\alpha-6m\alpha)^2 + (m+2)^2(9\alpha^2-3) \right]$$

From the above Eqns. (34) and (35) it is observed that the Hubble parameter and shear scalar are the decreasing function of the time, t . Moreover, it diverges at t_0 where $t_0 = -b/nl$.

The deceleration parameter,

$$q = n - 1 \quad (36)$$

For $n > 1$, $q > 0$, therefore, the model represents a decelerating model of the universe. For $n \leq 1$, we get $-1 \leq q \leq 0$, which implies an accelerating model of the universe. The recent observation of type Ia supernovae [1-4] revealed that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 \leq q \leq 0$. It follows that solutions obtained in this model are consistent with the observation.

In the following section we deal with two cases: i) non interacting model; ii) interacting model.

3.1. i) For non-interacting two fluid model

We assume that two fluids do not interact. Therefore, the general form of conservation Eqn. (15) leads to bulk viscous and dark fluid separately as

$$(\rho_m)_4 + (\rho_m + p_m) \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad (37)$$

$$(\rho_D)_4 + (\rho_D + p_D) \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad (38)$$

The above Eqn. (37) is in the form of w_m , which is constant and hence Eqn. (37) is integrable. But Eqn. (38) is a function of w_D , which is an unknown time-dependent parameter and accordingly p_D and ρ_D are also function of w_D . Therefore, we cannot integrate Eqn. (38).

Integrating Eqn. (37) and using Eqn. (17), we get the energy density of barotropic fluid as

$$\rho_m = \rho_0 (nlt + b)^{\frac{-3}{n}(1+w_m)} \quad (39)$$

Where, ρ_0 is the constant of integration.

From Eqns. (26) and (27), we get the effective pressure and energy density as

$$8\pi\bar{p} = \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} \quad (40)$$

$$8\pi\rho = \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} \quad (41)$$

We obtain also the pressure and density for dark fluid as

$$8\pi\rho_D = \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0(nlt+b)^{\frac{-3}{n}(1+w_m)} \quad (42)$$

$$8\pi\bar{p}_D = \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0 w_m (nlt+b)^{\frac{-3}{n}(1+w_m)} \quad (43)$$

The equation of state (EoS) parameter for dark fluid is

$$w_D = \frac{\left\{ \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0 w_m (nlt+b)^{\frac{-3}{n}(1+w_m)} \right\}}{\left\{ \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0 (nlt+b)^{\frac{-3}{n}(1+w_m)} \right\}} \quad (44)$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{ \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0 w_m (nlt+b)^{\frac{-3}{n}(1+w_m)} - \frac{24\pi\xi}{(nlt+b)} \right\}}{\left\{ \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_0 (nlt+b)^{\frac{-3}{n}(1+w_m)} \right\}} \quad (45)$$

The expression for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{\rho_0}{3l^2} (nlt+b)^{\left(2-\frac{3}{n}(1+w_m)\right)} \quad (46)$$

$$\Omega_D = \frac{1}{24\pi} \left\{ \frac{3l^2}{(nlt+b)^{-1}} - \frac{X_1}{2(m+2)^2(nlt+b)^{-1}} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-3\right)}} - \rho_0 (nlt+b)^{2-\frac{3}{n}(1+w_m)} \right\} \quad (47)$$

3.1. ii) For interacting two fluid model

$$Q = 3H\sigma\rho_m \quad (50)$$

In this section, we consider the interaction between dark viscous and barotropic fluids. For this purpose we can write the continuity equation as

$$(\rho_m)_4 + (\rho_m + p_m) \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = Q \quad (48)$$

$$(\rho_D)_4 + (\rho_D + p_D) \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = -Q \quad (49)$$

Where, the quantity $Q > 0$ expresses the interaction between the fluids. Here we emphasize that the continuity Eqns. (48) and (49) imply that the interaction term Q should be proportional to a quantity with units of inverse of time $Q \propto 1/t$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. In our study, we consider Q of the form (Hassan Amirhashchi et al. [48])

Where, σ is the coupling constant.

Solving Eqn. (48) with the help of Eqn. (50), we get

$$\rho_m = \rho_{00} (nlt+b)^{\frac{-3}{n}(1+w_m-\sigma)} \quad (51)$$

Where, ρ_{00} are integration constants.

The density for dark fluid as

$$8\pi\rho_D = \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00} (nlt+b)^{\frac{-3}{n}(1+w_m-\rho)} \quad (52)$$

The effective pressure as

$$8\bar{\rho}_D = \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00}w_m(nlt+b)^{\frac{-3}{n}(1+w_m-\sigma)}$$

(53)

The equation of state (EoS) parameter for dark fluid is

$$w_D = \frac{\left\{ \frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00}w_m(nlt+b)^{\frac{-3}{n}(1+w_m-\sigma)} \right\}}{\left\{ \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00}(nlt+b)^{\frac{-3}{n}(1+w_m-\rho)} \right\}}$$

(54)

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{ \left(\frac{l^2(2n-3)}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} + \frac{(m^2+m+1)}{3(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00}w_m(nlt+b)^{\frac{-3}{n}(1+w_m-\sigma)} \right) - \frac{3(8\pi)l\xi}{(nlt+b)} \right\}}{\left\{ \frac{3l^2}{(nlt+b)} - \frac{X_1}{2(m+2)^2(nlt+b)} - \frac{(m^2+m+1)}{(nlt+b)^{\left(\frac{6+6m\alpha-6\alpha}{n(m+2)}-1\right)}} - 8\pi\rho_{00}(nlt+b)^{\frac{-3}{n}(1+w_m-\rho)} \right\}}$$

(55)

Fig. 1 depicts the plots of energy density of the fluid (ρ) versus time (t) in interacting and non-interacting fluids of the universe. In both interacting and non-interacting modes, we observe that in early universe the viscous fluid density ρ having positive value, decreases with time and at late time it approaches to small constant value.

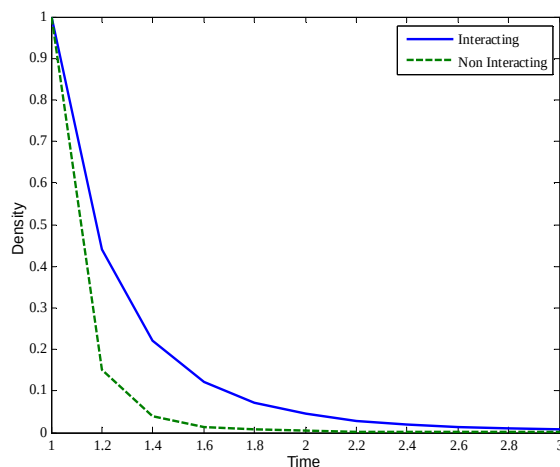


Fig.1: The plot of barotropic fluid density ρ_m versus time (t) in both interacting and non-interacting fluid.

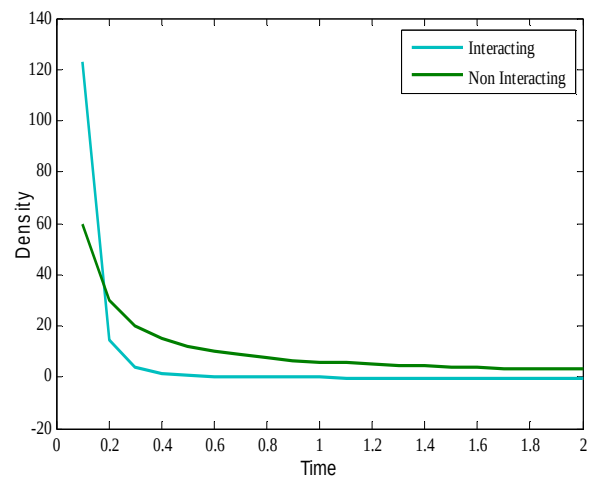


Fig.2: The plot of density for dark fluid ρ_D versus time (t) in both interacting and Non-interacting fluid.

Fig. 2 depicts the energy density of dark energy (ρ_D) for Bianchi type-III, Bianchi type-V and Bianchi type-VI₀ models versus time t . From this figure, we observe that ρ_D , in both non-interacting and interacting cases, it is a decreasing function of

time and it approaches a small positive constant value at late time and never go to infinity. In both cases, Bianchi type universe is free from big rip.

The physical quantities i.e., the average scale factor and spatial volume, expansion scalar and anisotropic parameter are

1. Average scale factor $(a) = (nlt + b)^{\frac{1}{n}}$.
2. Spatial volume $(V) = (nlt + b)^{\frac{3}{n}}$.
3. Expansion scalar $(\theta) = \frac{3l}{(nlt + b)}$.
4. Anisotropic parameter $(A_m) = X_4 = \text{constant}$.

It is observed that the spatial volume is zero at $t = \frac{-b}{nl} = t_0$ and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = t_0$ with an infinite rate of expansion. The scale factors also vanish at $t = t_0$ and hence the model has a point singularity at the initial epoch. Hubble's parameter and shear scalar diverge at the initial singularity. The anisotropy parameter is constant. The universe exhibits the power law expansion after the big bang impulse. As t increases the scale factor and spatial volume increase but the expansion scalar and shear scalar decrease. Thus the rate of expansion slows down with increase in time.

Case II:

3.2. In this case, we find value of metric potentials for $k = 0$, $n = 0$ and $C = V^\alpha$ [47], where α is the any constant number, then from Eqns. (13), (19) and (30), we get

$$A = d^{\frac{(3+3m\alpha-3\alpha)}{(m+2)}} e^{\left(\frac{3+3m\alpha-3\alpha}{m+2}\right)lt} \quad (56)$$

$$B = d^{\frac{(3+3m-3\alpha-6m\alpha)}{(m+2)}} e^{\left(\frac{3+3m-3\alpha-6m\alpha}{m+2}\right)lt} \quad (57)$$

$$C = d^{3\alpha} e^{3\alpha lt} \quad (58)$$

From Eqn. (22) the average generalized Hubble parameter H is

$$H = l \quad (59)$$

Also from Eqn. (24), we get

$$\sigma^2 = \frac{l^2}{2} \left(\frac{(3+3m\alpha-3\alpha)^2 + (3+3m-3\alpha-6m\alpha)^2}{(m+2)^2} \right) + \frac{3l^2}{2} (\alpha^2 - 1) \quad (60)$$

From the above Eqns. (59) and (60) it is observed that the Hubble parameter and shear scalar are constant.

The deceleration parameter,

$$q = -1 \quad (61)$$

For $n = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $dH/dt = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore, the solutions presented in this model are consistent with the observations.

In the following section we deal with two cases: i) non interacting model; ii) interacting model.

3.2. i) For non-interacting two-fluid model

Proceeding just as in Sec. 3 for Case-I (i) we again obtain the expression for the density of barotropic fluid as

$$\rho_m = \rho_0 (de^{lt})^{-3(1+w_m)} \quad (62)$$

Where ρ_0 is the constant of integration.

From Eqns. (26) and (27), we get the effective pressure and energy density as

$$8\pi\bar{p} = X_3 + \frac{(m^2 + m + 1)}{3 \left(\frac{3+3m\alpha-3\alpha}{m+2} \right)^2 e^{\left(\frac{6+6m\alpha-6\alpha}{m+2}-1\right)lt}} \quad (63)$$

$$8\pi\rho = X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3+3m\alpha-3\alpha}{m+2} \right)^2 e^{\left(\frac{6+6m\alpha-6\alpha}{m+2}-1\right)lt}} \quad (64)$$

The expression for density and pressure for dark fluid as

$$8\pi\rho_D = X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3+3m\alpha-3\alpha}{m+2} \right)^2 e^{\left(\frac{6+6m\alpha-6\alpha}{m+2}-1\right)lt}} - 8\pi\rho_0 (de^{lt})^{-3(1+w_m)} \quad (65)$$

$$8\pi\bar{p}_D = X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 w_m (de^{lt})^{-3(1+w_m)} \quad (66)$$

The equation of state (EoS) parameter for dark fluid is

$$w_D = \frac{\left\{ X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 w_m (de^{lt})^{-3(1+w_m)} \right\}}{\left\{ X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 (de^{lt})^{-3(1+w_m)} \right\}} \quad (67)$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{ X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 w_m (de^{lt})^{-3(1+w_m)} - 24\pi\xi \right\}}{\left\{ X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 (de^{lt})^{-3(1+w_m)} \right\}} \quad (68)$$

The expression for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{\rho_0}{3l^2} (de^{lt})^{-3(1+w_m)} \quad (69)$$

$$\Omega_D = \frac{1}{24\pi^2} \left\{ X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\rho_0 (de^{lt})^{-3(1+w_m)} \right\} \quad (70)$$

Where, X_2 and X_3 be the constants.

$$\rho_m = \lambda_1 e^{-3l(1+w_m-\sigma)t} \quad (71)$$

3.2. ii) For interacting two fluid model

Proceeding just as in Sec. 3, Case-I (ii), we again obtain the expression for the density and pressure of barotropic fluid

Where, $\lambda_1 = \rho_{00} d^{-3l(1+w_m-\sigma)}$.

The expression for density and pressure for dark fluid as

$$8\pi\rho_D = X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 (de^{lt})^{-3(1+w_m-\sigma)} \quad (72)$$

$$8\pi\bar{\rho}_D = X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 w_m e^{-3l(1+w_m-\sigma)t} \quad (73)$$

The equation of state (EoS) parameter for dark fluid is

$$w_D = \frac{\left\{ X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 w_m e^{-3l(1+w_m-\sigma)t} \right\}}{\left\{ X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 (de^{lt})^{-3(1+w_m-\sigma)} \right\}} \quad (74)$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{ X_3 + \frac{(m^2 + m + 1)}{3\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 w_m e^{-3l(1+w_m-\sigma)t} - 3l\xi \right\}}{\left\{ X_2 - \frac{(m^2 + m + 1)}{\left(\frac{3 + 3m\alpha - 3\alpha}{m + 2}\right)^2 e^{\left(\frac{6 + 6m\alpha - 6\alpha}{m + 2} - 1\right)lt}} - 8\pi\lambda_1 (de^{lt})^{-3(1+w_m-\sigma)} \right\}} \quad (75)$$

The kinematical quantities i.e., the average scale factor and spatial volume, expansion scalar and, anisotropic parameter are:

1. Average scale factor $(a) = de^{lt}$.
2. Spatial volume $(V) = (de^{lt})^3$.
3. Expansion scalar $(\theta) = 3l$.
4. Anisotropic parameter $(A_m) = X_5 = \text{constant}$.

The model has no initial singularity. The spatial volume, scale factors are constant for $t = 0$ and the other cosmological parameters are independent of time t . Thus the universe starts evolving with a constant volume and expands with exponential rate.

As t increases, the scale factors and the spatial volume increase exponentially as $t \rightarrow \infty$ while anisotropic parameter is constant. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model.

4. Conclusion

In this paper, we have discussed two-fluid viscous dark energy models based on exact solutions of Einstein's field equations for the anisotropic general type of Bianchi type space-time filled with two fluids: (i) Barotropic (ii) Dark fluid in

interacting and non-interacting modes with variable EoS parameter

In the derived models, w is obtained as time varying, which is consistent with the observation of [7]. The law of generalized linearly varying deceleration parameter defined by Eqn. (26) for Bianchi type-III, Bianchi type-V, Bianchi type-VI₀, and Bianchi type-VI_h space-time gives two types of cosmologies. The first form for ($n \neq 0$) gives the solution for positive value of deceleration parameter for ($n > 1$) and also negative value of deceleration parameter for ($0 \leq n < 1$) indicating the power law expansion of the universe whereas The second form for ($n = 0$) gives the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. In both the cases, it has also been observed that the ratio θ/σ becomes constant at $t \rightarrow \infty$. Thus the model approaches anisotropic space-time and the matter is dynamically negligible near the origin. This agrees with a result already given by Collins [49]. Thus our Bianchi type model represents realistic matter dominated universe.

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