

Holographic Dark Energy Model with Variable Deceleration Parameter, Quintessence and Tachyon Field in Bianchi Type III Space-Time

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In this paper, we study the correspondence between the quintessence and tachyon scalar field models with the holographic dark energy in anisotropic Bianchi type III space-time. This correspondence allows reconstructing the quintessence potential and the dynamics of scalar field model, which describes the accelerated expansion of the Universe. The exact solutions to the corresponding Einstein's field equations are obtained. Some physical properties are briefly discussed.

1. Introduction

Recent astrophysical data show that the current universe is expanding at an accelerating rate [1-4]. The existence of dark energy is the most accepted hypothesis consistent with recent observations [5,6]. Despite this favorable observational data, the nature of dark energy is still a challenging problem in modern cosmology. In physical cosmology, the simplest candidate for dark energy is the cosmological constant [7] satisfying the equation of state $\omega = -1$. There are several other possible forms of dark energy among them the next simple model is the quintessence [8-10] with the equation of state $\omega > -1$. Other scalar field models proposed to explain the nature of dark energy are related to k-essence models, which have a non-standard form of kinetic energy [11,12], for example, string theory fundamental scalars known as tachyon [13], dilation [14] and phantom dark energy [15]. Recently, an interesting investigation is made to determine the nature of dark energy within the frame work of quantum gravity, which is called holographic dark energy [16-20]. This principle is enlightened by Hooft [17] in his investigations of the quantum property of black holes and later by Susskind [20] in string theory. According to the holographic principle, the entropy of a system scales not with its volume, but with its surface area [21]. In the context of dark energy problem, though the holographic principle proposes a relation between the holographic DE density ρ_Λ and the Hubble parameter H given as $\rho_\Lambda = H^2$, it does not contribute to the present accelerated

expansion of the universe. Granda and Olivers [22] proposed a holographic density of the form $\rho_\Lambda = 3M_p^{-2}(\alpha H^2 + \beta \dot{H})$, where $M_p^{-2} = 8\pi G$, H is the Hubble parameter and α, β are constants, which must satisfy the restrictions imposed by the current observational data. This new model of dark energy shows the accelerated expansion of the universe and thus consistent with the recent observations. They studied [23] the correspondence between quintessence, tachyon, k-essence, and dilation DE models with this holographic dark energy model in the flat FRW universe. There are other cosmological versions of holographic principle in the literature [24-28]. Recently, many authors [29-30] have investigated the holographic dark energy with quintessence. Karami and Fehri [31] has investigated the correspondence between the quintessence, tachyon, K-essence, and dilation scalar field models with the new holographic dark energy model in non-flat FRW universe. Bianchi type universes are important as they seem to be homogeneous and anisotropic. Bali and Chandnani [32], Yadav and Yadav [33], Singh et al. [34] and Bali et al. [35] have investigated Bianchi type-III models in different aspects.

The purpose of present work is to investigate the nature of the holographic dark energy in Bianchi type III space-time and establish a correspondence between the holographic dark energy with quintessence and tachyon field. In Sec. 2, we obtain the EoS parameter for the holographic DE model in Bianchi type-III space time. In Sec. 3, we have solved the Einstein's field equations. The physical properties of the model are discussed in Sec. 4. The correspondence between the holographic dark energy with quintessence and

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tachyon scalar field are shown in Sec. 5. The paper ends with a conclusion in Sec. 6.

2. The Metric and Field Equations

The Bianchi type-III space-time is considered in the form

$$ds^2 = dt^2 - A^2 dx^2 - e^{-2\alpha x} B^2 dy^2 - C^2 dz^2 \quad (1)$$

Where, α is a constant and A, B and C are functions of cosmic time t .

The Einstein's field equation is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -(T_{ij} + \bar{T}_{ij}) \quad (2)$$

Where, R_{ij} is the Ricci tensor, R is the Ricci scalar.

The energy momentum tensor for matter and the holographic dark energy are defined [29] as

$$T_{ij} = \rho_m u_i u_j \quad \text{and} \quad \bar{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j - g_{ij} p_\Lambda. \quad (3)$$

Where, ρ_m, ρ_Λ are the energy densities of matter and holographic dark energy and p_Λ is the pressure of the holographic dark energy.

The field equations for Bianchi type-III metric are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p_\Lambda \quad (4)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -p_\Lambda \quad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p_\Lambda \quad (6)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = \rho_m + \rho_\Lambda \quad (7)$$

$$\alpha \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0 \quad (8)$$

Where an overhead dot denotes differentiation with respect to time t . Here, we have used the geometrized unit in which $8\pi G = 1$ and $c = 1$.

The physical quantities that are important in cosmology are proper volume V , average scale

factor a , expansion scalar θ , shear scalar σ and Hubble parameter H . For the metric (Eqn. (1)), they have the following form

$$V = a^3 = ABCe^{\alpha x} \quad (9)$$

$$\theta = u^\mu{}_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (10)$$

$$\sigma^2 = \frac{1}{2} \sigma_{\mu\nu} \sigma^{\mu\nu} = \frac{1}{2} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \theta^2 \quad (11)$$

$$H = \frac{\dot{a}}{a} \quad (12)$$

An observational quantity in cosmology is the deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} \quad (13)$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to a standard decelerating model, whereas negative sign indicates inflation.

The average anisotropy parameter (Δ) is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (14)$$

Where, H_i ($i=1,2,3$) are the directional Hubble parameters.

The holographic dark energy density is given by

$$\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H}) \quad (15)$$

That is, $\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H})$ with $M_p^{-2} = 8\pi G = 1$ [23].

The continuity equation can be obtained as

$$\dot{\rho}_m + \dot{\rho}_\Lambda + 3H(\rho_m + \rho_\Lambda + p_\Lambda) = 0 \quad (16)$$

The continuity equation of the matter is

$$\dot{\rho}_m + 3H\rho_m = 0 \quad (17)$$

The continuity equation of the holographic dark energy is

$$\dot{\rho}_{\Lambda} + 3H(\rho_{\Lambda} + p_{\Lambda}) = 0 \quad (18)$$

The barotropic equation of state is

$$p_{\Lambda} = \omega_{\Lambda} \rho_{\Lambda} \quad (19)$$

Using Eqns. (15) and (19) in (18) we get

$$\omega_{\Lambda} = -1 - \frac{2\alpha H\dot{H} + \beta\ddot{H}}{3H(\alpha H^2 + \beta\dot{H})} \quad (20)$$

3. Cosmological Solution

From Eqn. (8), we obtain

$$A = kB \quad (21)$$

Where, k is a constant of integration. Using Eqn. (21), Eqns. (4)-(8) reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p_{\Lambda} \quad (22)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} = -p_{\Lambda} \quad (23)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = \rho_m + \rho_{\Lambda} \quad (24)$$

Now we have three linearly independent Eqns. (22)-(24) and five unknown parameters $A, C, p_{\Lambda}, \rho_{\Lambda}$, and ρ_m . Two additional constraints relating these parameters are required to obtain explicit solutions of the system.

(i) We assume that the expansion (θ) in the model is proportional to the shear (σ). This leads to

$$B = C^n \quad (25)$$

Where, n is a positive constant.

(ii) Next we assume the value of directional scale factor C as increasing function of time as

$$C = (t^m e^t) \quad (26)$$

Where, m is a positive constant.

Using (26) in Eqns. (21) and (25) we get

$$A = k(t^m e^t)^n \quad (27)$$

and

$$B = (t^m e^t)^n \quad (28)$$

4. Some Physical Properties of the Model

The physical and kinematical parameters of the model have the following expressions

$$V = a^3 = ABC = k(t^m e^t)^{2n+1} \quad (29)$$

$$\theta = (2n+1)\left(\frac{m}{t} + 1\right) \quad (30)$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left(\frac{m}{t} + 1\right)^2 \quad (31)$$

$$H = \frac{(2n+1)}{3}\left(\frac{m}{t} + 1\right) \quad (32)$$

$$\Delta = 2\left(\frac{n-1}{2n+1}\right)^2 \quad (33)$$

Since Δ is constant, the space-time is anisotropic.

$$q = -1 + \frac{3m}{(2n+1)(m+t)^2} \quad (34)$$

From Fig. 1, we have observed that the deceleration parameter is negative. Thus our model shows the present day accelerating phase of the universe.

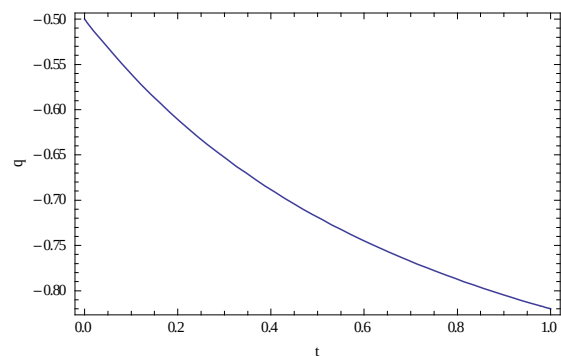


Fig.1: Variation of DP (q) with cosmic time t for $m=1.5$ and $n=1.5$.

The matter density parameter (Ω_m) and holographic dark energy density parameter (Ω_Λ) are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \quad \text{and} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} \quad (35)$$

Using Eqns. (26), (27) and (28) in Eqn. (7) we get

$$\Omega_m + \Omega_\Lambda = \frac{3}{(2n+1)^2} \left[(n^2 + 2n) - \alpha^2 k^{-2} \left(\frac{m}{t} + 1 \right)^{-2} (t^m e^t)^{-2n} \right] \quad (36)$$

Eqn. (36) shows that the sum of the energy density parameters approaches to $\frac{3}{(2n+1)^2} (n^2 + 2n)$ at late time, so at late times the universe becomes flat.

Using Eqn. (32) in Eqn. (17) we get

$$\rho_m = k_2 t^{-m(2n+1)} e^{-(2n+1)t} \quad (37)$$

Again using Eqn. (32) in Eqn. (15), we get

$$\rho_\Lambda = \alpha \frac{(2n+1)^2}{3} \left(\frac{m}{t} + 1 \right)^2 - \frac{\beta(2n+1)m}{t^2} \quad (38)$$

The behavior of matter energy density ρ_m and the dark energy density ρ_Λ are shown in Figs. 2 and 3, respectively. From these figures we can conclude that the matter energy density and dark energy density tend to a non-zero constant at later times.

Using Eqn. (32) in Eqn. (20) we get

$$\omega_\Lambda = -1 + \frac{2m}{(2n+1)(m+t)} \left[\frac{\alpha(m+t)(2n+1) - 3\beta}{\alpha(2n+1)(m+t)^2 - 3\beta m} \right] \quad (39)$$

Since the recent observational data indicates that the universe is accelerating and the value of deceleration parameter lies in the range $-1 < q < 0$. Therefore, this model describes an expanding, shearing and accelerating universe with a big bang at $t = 0$ for $n > 0$. It is also observed from Eqn. (39) and Fig. 4 that $-1 < \omega_\Lambda < -1/3$.

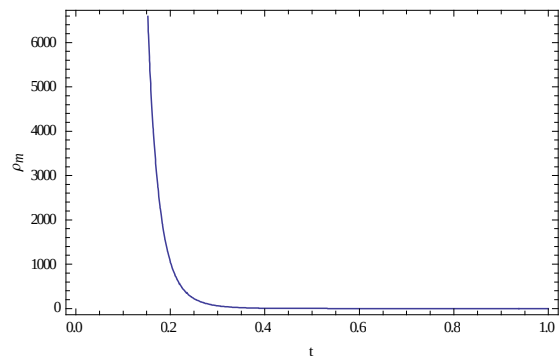


Fig.2: Variation of matter energy density ρ_m with cosmic time t for $m = 1.5$, $n = 1.5$, $k_2 = .15$.

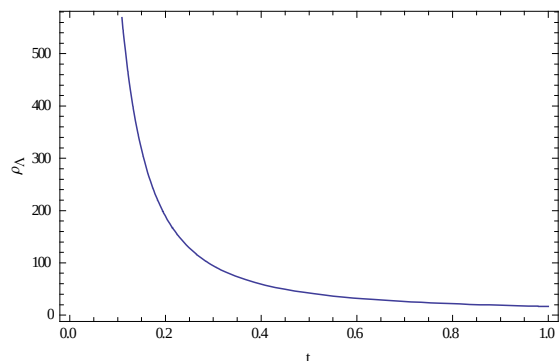


Fig.3: Variation of dark energy density ρ_Λ with cosmic time t for $m = 1.5$, $n = 1.5$, $\alpha = .5$, $\beta = .025$.

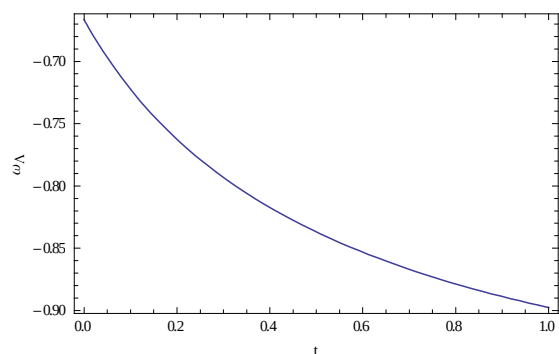


Fig.4: Variation of ω_Λ with cosmic time t for $m = 1.5$, $n = 1.5$, $\alpha = .5$, $\beta = .025$.

5. The Correspondence

5.1. Correspondence between the holographic and quintessence scalar field models

To establish the correspondence between the holographic dark energy with quintessence dark energy models, we compare the EoS and the dark energy density for the corresponding model of dark energy.

The action for the quintessence scalar ϕ is given by

$$s = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi) \right] \quad (40)$$

The energy density and pressure for the quintessence scalar field are represented by

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (41)$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (42)$$

Where, $V(\phi)$ is the quintessence potential.

The equation of state for scalar field is given by

$$\phi - \phi_0 = \sqrt{\frac{2m}{3}} \int \sqrt{[\alpha(2n+1) - 3\beta(m+t)^{-1}]} t^{-1} dt \quad (46)$$

$$V(\phi) = \frac{(2n+1)}{3} t^{-2} [\alpha(2n+1)(m+t)^2 - 3\beta m] - \frac{m}{3} t^{-2} [\alpha(2n+1) - 3\beta(m+t)^{-1}] \quad (47)$$

Therefore, one can establish a correspondence between the holographic dark energy and quintessence scalar field.

5.2. Holographic tachyon model

As said earlier that among various scalar field models proposed to explain the nature of dark energy, tachyon field is one that is an unstable field, and has become important in string theory through its role in the Dirac-Born-Infeld (DBI) action, which is used to describe the D-brane action [36-42]. The effective Lagrangian density of tachyon matter is [36-42]

$$\mathcal{L} = -V(\phi) \sqrt{1 + \partial_\mu \partial^\mu \phi} \quad (48)$$

$$\omega_\phi = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \quad (43)$$

For the accelerated expansion of the universe, the equation of state parameter for quintessence must be less than $-\frac{1}{3}$.

From Eqns. (45) and (49) we have

$$\begin{aligned} & -1 + \frac{2m}{(2n+1)(m+t)} \left[\frac{\alpha(m+t)(2n+1) - 3\beta}{\alpha(2n+1)(m+t)^2 - 3\beta m} \right] \\ &= \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \end{aligned} \quad (44)$$

Also, comparing Eqns. (14) and (41) we get

$$\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H}) = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (45)$$

From Eqns. (43) and (44), the kinetic energy term and the quintessence potential in power law form can be obtained as follows

The energy density and pressure for the tachyon field as [36]

$$\rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (49)$$

$$p_T = -V(\phi) \sqrt{1 - \dot{\phi}^2} \quad (50)$$

Where, $V(\phi)$ is the tachyon potential. The EoS parameter for the tachyon scalar field is obtained as

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1 \quad (51)$$

From Eqns. (39) and (51) we have

$$-1 + \frac{2m}{(2n+1)(m+t)} \left[\frac{\alpha(m+t)(2n+1) - 3\beta}{\alpha(2n+1)(m+t)^2 - 3\beta m} \right] = \dot{\phi}^2 - 1 \quad (52)$$

$$\rho_{\Lambda} = 3(\alpha H^2 + \beta \dot{H}) = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (53)$$

From Eqns. (51) and (52) we get

Comparing Eqns. (15) and (49) we get

$$\phi - \phi_0 = \int \left[\frac{2m}{(2n+1)(m+t)} \left\{ \frac{\alpha(m+t)(2n+1) - 3\beta}{\alpha(2n+1)(m+t)^2 - 3\beta m} \right\} \right]^{1/2} dt \quad (54)$$

$$V(\phi) = \frac{(2n+1)^{1/2}}{3t^2(m+t)^{1/2}} \left[\alpha(2n+1)^2(m+t)^3 - (3\beta + 2\alpha)m(2n+1)(m+t) + 6\beta m \right]^{1/2} \left[\alpha(2n+1)(m+t)^2 - 3\beta m \right]^{1/2} \quad (55)$$

6. Conclusion

We consider in this paper a homogeneous Bianchi type-III universe filled with matter and holographic dark energy. The exact solutions of Einstein field equations have been obtained. In our investigation, we have found that the deceleration parameter shows the present day accelerating phase of the universe. Also our model represents the early era of the universe, because the model remains anisotropic. The physical and geometrical properties of the model are consistent with the present day observations. We have also established a correspondence between the holographic DE with quintessence and tachyon field models. We reconstruct the potentials and dynamics of these scalar field models for this accelerating model of the universe.

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