

Anisotropic Cosmological Models in $f(R)$ Gravity with Quark and Strange Quark Matter and Special Form of Deceleration Parameter

K. S. Adhav*, G. B. Tayade and S. L. Munde

Department of Mathematics, Sant Gadge Baba Amravati University, Amravati, India

In this paper, we have studied the Bianchi type-I and V cosmological models with quark and strange quark matter in $f(R)$ theory of gravity. The general solutions of the field equations of Bianchi type-I and type-V space-times have been obtained under the assumption of special form of deceleration parameter. The physical and geometrical aspects of the models are also discussed.

1. Introduction

The $f(R)$ theory of gravity has been helpful in describing the evolution of the universe. The $f(R)$ gravity is getting a lot of attention since it can describe early acceleration and late acceleration and also reproduce Einstein's General Relativity and Gravitation Theory [1]. Most of the work has been done in $f(R)$ gravity with different matter sources. Carroll et al. [2] explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. Several $f(R)$ gravity models are reviewed by Capozziello and Faraoni [3].

It is well known that quark-gluon plasma existed during one of the phase transitions of the universe in the early time when cosmic temperature was $T \sim 200$ MeV. Typically, strange quark matter is modeled with an (EoS) equation of state $p = \frac{(\rho - 4B_c)}{3}$ based on the phenomenological bag model of quark matter in which quark confinement is described by an energy term proportional to the volume. In this equation, B_c is the difference between the energy density of the perturbative and non-perturbative QCD vacuum, known as the bag constant and ρ and p are the energy density and thermodynamic pressure of the quark matter respectively. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (bag constant). Also, in the framework of this model, the quark matter is composed of massless u and d quarks, massive s quarks and electrons. In the

simplified version of the bag model, it is assumed that quarks are massless and non-interacting.

Therefore, we have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density), the total energy density is $\rho = \rho_q + B_c$ and the total pressure is $p = p_q - B_c$. There are many studies on quark matter in general relativity. Mak and Harko [4] have studied charged strange quark matter in the spherically symmetric space-time admitting conformal motion. The study of strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion has been done by Yavuz et al. [5]. Adhav et al. [6,7] have discussed string cloud and domain walls with quark matter in the n -dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Katore [8] studied cosmological model with strange quark matter attached to cosmic string for FRW space-time in general relativity. Adhav [9] discussed the Kantowski-Sachs string cosmological model in $f(R)$ theory of gravity. Recently, Sahoo and Mishra [10] studied the axially symmetric space-time with strange quark matter attached to string cloud in bi-metric theory.

Yilmaz et al. [11] have discussed quark and strange quark matter in $f(R)$ gravity for Bianchi I and V space time models assuming that Hubble parameter is proportional to volume. Wherein, they have concluded that quark matter may behave like phantom-type dark energy for $\epsilon < -1$ and quark matter may be source of early dark energy, which causes early acceleration of the universe due to negative pressure. Furthermore, $f(R)$ solutions obtained represent early eras of the universe since

*ati_ksadhav@yahoo.co.in

$f(R)$ solutions for quark matter coincide with $f(R)$ equations for inflation.

In this paper, we have also studied the Bianchi type-I and V cosmological models with quark and strange quark matter in $f(R)$ theory of gravity. The general solutions of the field equations of Bianchi type-I and V space-times have been obtained under the assumption of special form of deceleration parameter. The physical and geometrical aspects of the models are also discussed.

2. $f(R)$ Gravity Field Equations and Deceleration Parameter

The $f(R)$ theory of gravity is the generalization of general relativity. The action for $f(R)$ theory of gravity is given by

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R) + L_m \right) d^4x \quad (1)$$

Here $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. It should be noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein-Hilbert action.

Now, by varying the action (Eqn. (1)) with respect to the metric ($g_{\mu\nu}$), we get the corresponding field equations as

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = \kappa T_{\mu\nu} \quad (2)$$

Where, $F(R) = \frac{df(R)}{dR}$, $\square \equiv \nabla^\mu \nabla_\mu$, ∇_μ is the covariant derivative, $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m and $\kappa (= 8\pi G/c^4 = 1)$ is the coupling constant in gravitational units. These are the fourth order partial differential equations in the metric tensor $g_{\mu\nu}$. The fourth order is due to the last two terms on the left-hand side of the equation. If we take $f(R) \equiv R$, these equations reduce to the field equations of general relativity.

Now, contracting the field Eqns. (2), we get

$$F(R)R - 2f(R) + 3\square F(R) = T \quad (3)$$

From Eqn. (3), we get

$$f(R) = \frac{1}{2} [F(R)R + 3\square F(R) - T] \quad (4)$$

Using Eqn. (4) in Eqn. (2), the $f(R)$ gravity field equations can be written as

$$R_{\mu\nu} - \frac{1}{F(R)} [\nabla_\mu \nabla_\nu F(R) + T_{\mu\nu}] = \frac{1}{4F(R)} [F(R)R - \square F(R) - T] g_{\mu\nu} \quad (5)$$

The energy-momentum tensor for quark matter is given as

$$T^{(Quark)}_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$$

or

$$T^{(Quark)\mu}_{\nu} = \text{diag}(\rho, -p, -p, -p) \quad (6)$$

Where, $\rho = \rho_q + B_c$, $p = p_q - B_c$ and $u^\mu = u^\mu_\mu = \delta^\mu_0$ is the four velocity in the co-moving coordinates. Since quark matter behaves nearly as a perfect fluid [11,12-15] we will use the following equation of state for quark matter

$$p = \varepsilon \rho, \quad 0 \leq \varepsilon \leq 1 \quad (7)$$

Also the linear equation of state for strange quark matter is [16,17]

$$p = \varepsilon(\rho - \rho_0) \quad (8)$$

Where, ρ_0 is the energy density at zero pressure and ε is a constant.

When $\varepsilon = 1/3$ and $\rho_0 = 4B_c$ the above linear equation of state is reduced to the following equation of state for strange quark matter in the bag model [11,15]

$$p = \frac{(\rho - 4B_c)}{3} \quad (9)$$

Where, B_c is the Bag constant.

In order to solve the Einstein's field equations in $f(R)$ gravity, we will use a special form of deceleration parameter which defined by Singha and Debnath [18] for FRW metric as

$$q = -\frac{\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^\alpha} \quad (10)$$

Where, $\alpha > 0$ is a constant and a is mean scale factor of the universe.

After solving Eqn. (10) one can obtain the mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = k(1 + a^{-\alpha}) \quad (11)$$

Where, k is a constant of integration.

On integrating Eqn. (11), we obtain the volume scale factor as

$$V = (e^{\alpha t} - 1)^{3/\alpha} \quad (12)$$

The physical parameters, namely the volume scale factor (V), mean Hubble's parameter (H), expansion scalar (θ), shear scalar (σ^2), mean anisotropy parameter (Δ) and deceleration parameter (q) are given by

$$V = ABC \quad (13)$$

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (14)$$

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (15)$$

$$\sigma^2 = \frac{\theta^2}{3} - \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \right) \quad (16)$$

$$\Delta = 6 \left(\frac{\sigma}{\theta} \right)^2 \quad (17)$$

and

$$q = 2 - 3 \frac{V\ddot{V}}{\dot{V}^2} \quad (18)$$

3. Bianchi Type-I Model

The line element for the Bianchi type-I space time is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \quad (19)$$

Where, A , B and C are functions of the cosmic time t only.

The corresponding Ricci scalar for Bianchi type-I space time is given by

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \right] \quad (20)$$

Where, overhead dot ($\dot{}$) represents derivative with respect to time t .

The $f(R)$ gravity field equations (5) for the metric (Eqn. (19)) with the help of Eqn. (6) are given by

$$-\frac{4F}{3} \left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{S}{F} \right) = \rho + p \quad (21)$$

$$4F \left(\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{F}}{AF} - \frac{S}{F} \right) = \rho + p \quad (22)$$

$$4F \left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{F}}{BF} - \frac{S}{F} \right) = \rho + p \quad (23)$$

$$4F \left(\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{F}}{CF} - \frac{S}{F} \right) = \rho + p \quad (24)$$

Where, S is

$$S = \frac{F}{2} \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{1}{2} \left(\frac{\ddot{F}}{F} + \frac{\ddot{\theta}}{\theta} \right) \right] \quad (25)$$

Using equations (21)-(24), we obtain the metric functions explicitly as

$$A = l_1 V^{1/3} \exp \left(X_1 \int \frac{dt}{VF} \right) \quad (26)$$

$$B = l_2 V^{1/3} \exp \left(X_2 \int \frac{dt}{VF} \right) \quad (27)$$

$$C = l_3 V^{1/3} \exp \left(X_3 \int \frac{dt}{VF} \right) \quad (28)$$

Where, $l_1, l_2, l_3, X_1, X_2, X_3$ are constants of integration and satisfy the relation

$$l_1 l_2 l_3 = 1 \text{ and } X_1 + X_2 + X_3 = 0 \quad (29)$$

Using Eqns. (26)-(28) in Eqns. (4), (16) and (20), we get

$$f(R) = RF - 2\rho + 2\frac{\dot{V}}{V}F\left(\frac{\dot{V}}{3V} + \frac{\dot{F}}{F}\right) - \frac{2X^2}{FV^2} \quad (30)$$

$$R = 2\left[\frac{1}{3}\left(\frac{\dot{V}}{V}\right)^2 - \frac{\ddot{V}}{V}\right] - \frac{2X^2}{F^2V^2} \quad (31)$$

and

$$\sigma^2 = \frac{X^2}{V^2F^2} \quad (32)$$

Where, $2X^2 = X_1^2 + X_2^2 + X_3^2$.

We will examine the behaviors of the quark matter, which existed in the first seconds of the Universe. So, we can choose for the solution that the anisotropy parameter of the early Universe equals to constant, since, Bianchi models are anisotropic. Also, Collins et al. [19] have shown that $\frac{\sigma}{\theta} \cong 0.3$ according to red-shift studies. This assumption is similar to the assumption of mean anisotropy parameter used for the study of anisotropic cosmological models in the papers of Harko and Mak [20] and [21] (and references therein). In our study, this means that the anisotropy parameter Δ or $\frac{\sigma}{\theta}$ of the universe equal to constant β . Harko and Mak [20,21] have studied extensively anisotropy problem for Bianchi type I and V universe models in Brane cosmology.

From Eqns. (15) and (32), we have

$$\frac{\sigma}{\theta} = \frac{X}{F\dot{V}} = \beta \quad (33)$$

From Eqn. (33), we get

$$F = \frac{X}{\beta\dot{V}} \quad (34)$$

Using Eqns. (12) and (34) in Eqns. (26)-(28), we obtain the scale factor as

$$A = l_1 (e^{\alpha t} - 1)^{\frac{X+3X_1\beta}{\alpha X}} \quad (35)$$

$$B = l_2 (e^{\alpha t} - 1)^{\frac{X+3X_2\beta}{\alpha X}} \quad (36)$$

$$C = l_3 (e^{\alpha t} - 1)^{\frac{X+3X_3\beta}{\alpha X}} \quad (37)$$

Using Eqns. (12) in Eqns. (15)-(18), (31) and (34), we get

$$F = \frac{X}{3\beta} \frac{(e^{\alpha t} - 1)^{\frac{\alpha-3}{\alpha}}}{e^{\alpha t}} \quad (38)$$

$$R = \frac{ke^{2\alpha t}}{(e^{\alpha t} - 1)^2} - 6\alpha \frac{e^{\alpha t}}{(e^{\alpha t} - 1)} \quad (39)$$

$$\theta = 3[1 + (e^{\alpha t} - 1)^{-1}] \quad (40)$$

$$\sigma^2 = \frac{9\beta^2 e^{2\alpha t}}{(e^{\alpha t} - 1)^2} \quad (41)$$

$$\Delta = 6\beta^2 \quad (42)$$

$$q = -1 + \frac{\alpha}{e^{\alpha t}} \quad (43)$$

Adding Eqn. (22) and three times (21), we get the following equation as

$$(\rho + p) = -F\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{S}{F} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{F}}{AF}\right) \quad (44)$$

Using Eqns. (35)-(37) and (38) in the above equation, we get the following equation as

$$\rho + p = \frac{X}{3\beta} \frac{(e^{\alpha t} - 1)^{\frac{1-3}{\alpha}}}{e^{\alpha t}} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha t}}{(e^{\alpha t} - 1)^2} + (\alpha - 6) \frac{\alpha e^{\alpha t}}{(e^{\alpha t} - 1)} - \alpha^2 \right] \quad (45)$$

3.1. Quark matter for Bianchi type-I model

Using Eqns. (2) in Eqns. (45), we obtain the energy density and pressure for quark matter as

$$\rho_q = \frac{X}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^\alpha - 1)^2} + (\alpha - 6) \frac{\alpha e^\alpha}{(e^\alpha - 1)} - \alpha^2 \right] \quad (46)$$

$$p_q = \frac{X \varepsilon}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^\alpha - 1)^2} + (\alpha - 6) \frac{\alpha e^\alpha}{(e^\alpha - 1)} - \alpha^2 \right] \quad (47)$$

Using Eqns. (12), (38), (39) and (46) in Eqn. (30), the $f(R)$ function for quark matter is found to be

$$f(R) = \frac{2X}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[\frac{(3\beta^2 - 4\alpha + 6)e^{2\alpha}}{(e^\alpha - 1)^2} + (6 - \alpha) \frac{\alpha e^\alpha}{(e^\alpha - 1)} + \alpha^2 \right] - \frac{4\alpha X}{\beta} (e^\alpha - 1)^{-\frac{3}{\alpha}} + \left[\frac{X}{3\beta} + \frac{2X(\alpha - 2)}{\beta} - 6X\beta \right] \frac{e^\alpha}{(e^\alpha - 1)^{1+\frac{3}{\alpha}}} \quad (48)$$

3.2. Strange quark matter for Bianchi type-I model

Using Eqns. (8) in Eqn. (45), we obtain the energy density and pressure for strange quark matter as

$$\rho_{sq} = \frac{X}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^\alpha - 1)^2} + (\alpha - 6) \frac{\alpha e^\alpha}{(e^\alpha - 1)} - \alpha^2 \right] + \frac{\varepsilon \rho_0}{(1+\varepsilon)} \quad (49)$$

$$p_{sq} = \frac{X \varepsilon}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^\alpha - 1)^2} + (\alpha - 6) \frac{\alpha e^\alpha}{(e^\alpha - 1)} - \alpha^2 \right] - \frac{\varepsilon \rho_0}{(1+\varepsilon)} \quad (50)$$

Using Eqns. (12), (38), (39) and (49) in Eqns. (30), the $f(R)$ function for strange quark matter is found to be

$$f(R) = \frac{2X}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[\frac{(3\beta^2 - 4\alpha + 6)e^{2\alpha}}{(e^\alpha - 1)^2} + (6 - \alpha) \frac{\alpha e^\alpha}{(e^\alpha - 1)} + \alpha^2 \right] - \frac{4\alpha X}{\beta} (e^\alpha - 1)^{-\frac{3}{\alpha}} + \left[\frac{X}{3\beta} + \frac{2X(\alpha - 2)}{\beta} - 6X\beta \right] \frac{e^\alpha}{(e^\alpha - 1)^{1+\frac{3}{\alpha}}} - \frac{2\varepsilon \rho_0}{(1+\varepsilon)} \quad (51)$$

4. Bianchi type-V Model

The line element for the Bianchi type-V space time is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2mx} dy^2 - C^2 e^{-2mx} dz^2 \quad (52)$$

Where, A , B and C are functions of the cosmic time t only and $m \neq 0$ is an arbitrary constant.

The corresponding Ricci scalar for Bianchi type-V space time is given by

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} \right] \quad (53)$$

Where, overhead dot (·) represents derivative with respect to time t .

The corresponding $f(R)$ gravity field Eqn. (5) for the metric (Eqn. (41)) with the help of Eqn. (6) are given by

$$-\frac{4F}{3} \left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{S}{F} \right) = \rho + p \quad (54)$$

$$4F \left(\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{F}}{AF} - \frac{2m^2}{A^2} - \frac{S}{F} \right) = \rho + p \quad (55)$$

$$S = \frac{F}{2} \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} + \frac{1}{2} \left(\frac{\ddot{F}}{F} + \frac{\ddot{\theta}}{F} \right) \right] \quad (59)$$

Using Eqns. (54)-(58), we obtain the metric functions explicitly as

$$A = V^{1/3} \quad (60)$$

$$B = dV^{1/3} \exp \left(X \int \frac{dt}{VF} \right) \quad (61)$$

$$C = \frac{1}{d} V^{1/3} \exp \left(-X \int \frac{dt}{VF} \right) \quad (62)$$

Where, d and X are constants of integration.

Using Eqns. (60)-(62) in Eqn. (4) and (53), we get

$$f(R) = -2\rho + RF + 2\frac{\dot{V}}{V}F \left(\frac{\dot{V}}{3V} + \frac{\dot{F}}{F} \right) - \frac{2X^2}{FV^2} - \frac{6m^2F}{V^{2/3}} \quad (63)$$

$$R = 2 \left[\frac{1}{3} \left(\frac{\dot{V}}{V} \right)^2 - \frac{\ddot{V}}{V} \right] - \frac{2X^2}{F^2V^2} + \frac{6m^2}{V^{2/3}} \quad (64)$$

Using Eqns. (12) and (34) in Eqns. (60)-(62), we obtain the scale factor as

$$A = (e^{2t} - 1)^{\frac{1}{\alpha}} \quad (65)$$

$$4F \left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{F}}{BF} - \frac{2m^2}{A^2} - \frac{S}{F} \right) = \rho + p \quad (56)$$

$$4F \left(\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{F}}{CF} - \frac{2m^2}{A^2} - \frac{S}{F} \right) = \rho + p \quad (57)$$

$$m \left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0 \quad (58)$$

Where, S is

$$B = d(e^{\alpha} - 1)^{\frac{1+3\beta}{\alpha}} \quad (66)$$

$$C = \frac{1}{d}(e^{\alpha} - 1)^{\frac{1-3\beta}{\alpha}} \quad (67)$$

Using Eqns. (12) in Eqns. (15)-(18), (34) and (64), we get

$$F = \frac{X}{3\beta} \frac{(e^{\alpha} - 1)^{\frac{\alpha-3}{\alpha}}}{e^{\alpha}} \quad (68)$$

$$R = 6(e^{\alpha} - 1)^{\frac{-2}{\alpha}} m^2 + \frac{ke^{2\alpha}}{(e^{\alpha} - 1)^2} - 6\alpha \frac{e^{\alpha}}{(e^{\alpha} - 1)} \quad (69)$$

$$\theta = 3[1 + (e^{\alpha} - 1)^{-1}] \quad (70)$$

$$\sigma^2 = \frac{9\beta^2 e^{2\alpha}}{(e^{\alpha} - 1)^2} \quad (71)$$

$$\Delta = 6\beta^2 \quad (72)$$

and

$$q = -1 + \frac{\alpha}{e^{\alpha}} \quad (73)$$

Adding Eqn. (22) and three times (21), we get the following equation as

$$(\rho + p) = -F \left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{F}}{F} + \frac{S}{F} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{F}}{AF} \right) \quad (74)$$

Using Eqns. (65)-(67) and (68) in the above equation, we get

$$\rho + p = \frac{X}{3\beta} \frac{(e^{\alpha} - 1)^{1-\frac{3}{\alpha}}}{e^{\alpha}} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^{\alpha} - 1)^2} + (\alpha - 6) \frac{\alpha e^{\alpha}}{(e^{\alpha} - 1)} - \alpha^2 - \frac{2m^2}{(e^{\alpha} - 1)^{\frac{2}{\alpha}}} \right] \quad (75)$$

4.1. Quark matter for Bianchi type-V model

Using Eqn. (7) in Eqn. (75), we obtain the energy density and pressure of the quark matter as

$$\rho_q = \frac{X}{3(1+\varepsilon)\beta} \frac{(e^{\alpha} - 1)^{1-\frac{3}{\alpha}}}{e^{\alpha}} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^{\alpha} - 1)^2} + (\alpha - 6) \frac{\alpha e^{\alpha}}{(e^{\alpha} - 1)} - \alpha^2 - \frac{2m^2}{(e^{\alpha} - 1)^{\frac{2}{\alpha}}} \right] \quad (76)$$

$$p_q = \frac{X\varepsilon}{3(1+\varepsilon)\beta} \frac{(e^{-\alpha} - 1)^{1-\frac{3}{\alpha}}}{e^{\alpha}} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^{\alpha} - 1)^2} + (\alpha - 6) \frac{\alpha e^{\alpha}}{(e^{\alpha} - 1)} - \alpha^2 - \frac{2m^2}{(e^{\alpha} - 1)^{\frac{2}{\alpha}}} \right] \quad (77)$$

Using Eqns. (12), (68), (69) and (76) in Eqn. (63), the $f(R)$ function for quark matter is found to be

$$f(R) = \frac{2X}{3(1+\varepsilon)\beta} \frac{(e^{\alpha} - 1)^{1-\frac{3}{\alpha}}}{e^{\alpha}} \left[\frac{k_1 e^{2\alpha}}{(e^{\alpha} - 1)^2} + (6 - \alpha) \frac{\alpha e^{\alpha}}{(e^{\alpha} - 1)} + \alpha^2 + \frac{2m^2}{(e^{\alpha} - 1)^{\frac{2}{\alpha}}} \right] - \frac{4\alpha X}{\beta} (e^{\alpha} - 1)^{-\frac{3}{\alpha}} + \left[\frac{kX}{3\beta} + \frac{2X(\alpha - 2)}{\beta} - 6X\beta \right] \frac{e^{\alpha}}{(e^{\alpha} - 1)^{1+\frac{3}{\alpha}}} \quad (78)$$

4.2. Strange quark matter for Bianchi type-V model

Using Eqn. (8) in Eqn. (75), we obtain the energy density and pressure of the quark matter as

$$\rho_{sq} = \frac{X}{3(1+\varepsilon)\beta} \frac{(e^{\alpha} - 1)^{1-\frac{3}{\alpha}}}{e^{\alpha}} \left[(4\alpha - 3\beta^2 - 6) \frac{e^{2\alpha}}{(e^{\alpha} - 1)^2} + (\alpha - 6) \frac{\alpha e^{\alpha}}{(e^{\alpha} - 1)} - \alpha^2 - \frac{2m^2}{(e^{\alpha} - 1)^{\frac{2}{\alpha}}} \right] + \frac{\varepsilon \rho_0}{(1+\varepsilon)} \quad (79)$$

$$p_{sq} = \frac{X \varepsilon}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[\frac{(4\alpha - 3\beta^2 - 6) e^{2\alpha}}{(e^\alpha - 1)^2} + (\alpha - 6) \frac{\alpha e^\alpha}{(e^\alpha - 1)} - \alpha^2 - \frac{2m^2}{(e^\alpha - 1)^{\frac{2}{\alpha}}} \right] - \frac{\varepsilon \rho_0}{(1+\varepsilon)} \quad (80)$$

Using Eqns. (12), (68), (69) and (79) in Eqn. (63), the $f(R)$ function for quark matter is found to be

$$f(R) = \frac{2X}{3(1+\varepsilon)\beta} \frac{(e^\alpha - 1)^{1-\frac{3}{\alpha}}}{e^\alpha} \left[\frac{k_1 e^{2\alpha}}{(e^\alpha - 1)^2} + (6 - \alpha) \frac{\alpha e^\alpha}{(e^\alpha - 1)} + \alpha^2 + \frac{2m^2}{(e^\alpha - 1)^{\frac{2}{\alpha}}} \right] - \frac{4\alpha X}{\beta} (e^\alpha - 1)^{-\frac{3}{\alpha}} + \left[\frac{kX}{3\beta} + \frac{2X(\alpha - 2)}{\beta} - 6X\beta \right] \frac{e^\alpha}{(e^\alpha - 1)^{1+\frac{3}{\alpha}}} - \frac{2\varepsilon \rho_0}{(1+\varepsilon)} \quad (81)$$

5. Discussion and Conclusion

In this paper, we have studied Bianchi type-I and V cosmological model with quark and strange quark matter in $f(R)$ gravity. The solutions of the field equations are obtained under the assumption of special form of deceleration parameter for both the models.

(i) It is observed that (in both models) the volume of the universe is zero when $t \rightarrow 0$ and as time increases volume V increases exponentially.

(ii) When, $t \rightarrow 0$, $\rho \rightarrow \infty$ and when $t \rightarrow \infty$, $\rho \rightarrow 0$, which indicates that the universe starts with initial (Big-Bang) type of singularity (for both the models).

(iii) For both the models, the anisotropy of expansion is maintained throughout the evolution of the universe.

(iv) The deceleration parameter q varies from $+1$ to -1 . It shows that the universe accelerates after an epoch of deceleration. The deceleration parameter q is in the range, $-1 \leq q \leq 0.5$, which matches with the observations made by Riess et al. [22] and Perlmutter et al. [23] and the present day universe is undergoing accelerated expansion.

This may be the transition of quark matter to dark matter during the decelerating phase of the universe at very early time. For $\varepsilon < -1$ the quark matter may behave like dark energy.

Acknowledgements

The authors are thankful to UGC, New Delhi for financial assistance through Major Research Project.

References

- [1] A. Elizalde et al., Phys. Rev. D **83**, 086006 (2011).
- [2] S. M. Carroll et al., Phys. Rev. D **70**, 043528, (2004).
- [3] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, 2010).
- [4] M. K. Mak and T. Harko, Int. J. Mod. Phys. D **13**, 149 (2004).
- [5] I. Yavuz et al., Int. J. Mod. Phys. D **14**, 1365, (2005).
- [6] K. S. Adhav et al., Int. J. Theor. Phys. **47**, 2002 (2008).
- [7] K. S. Adhav et al., Astrophys. Space Sci. **319**, 81 (2009).
- [8] S. D. Katore, Int. J. Theor. Phys. **51**, 83 (2012).
- [9] K. S. Adhav, Can. J. Phys. **90**, 119 (2012).
- [10] P. K. Sahoo and B. Mishra, IJPAM, **82**(1), 87 (2013).
- [11] I. Yilmaz et al., Gen. Rel. Grav. **44**, 2313 (2012).
- [12] J. Adams et al., (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005).
- [13] K. Adcox et al., (PHENIX Collaboration), Nucl. Phys. A **757**, 184 (2005).
- [14] B. B. Back et al., (PHOBOS Collaboration), Nucl. Phys. A **757**, 28 (2005).
- [15] C. Aktas et al., Gen. Rel. Grav. **43**(6), 1577 (2011).
- [16] R. Sharma et al., IJMPD **15**(3), 405 (2006).
- [17] R. Sharma et al., MNRAS, **375**(4), 1265 (2007).
- [18] A. K. Singha and U. Debnath, Int. J. Theor. Phys. **48**(2), 351 (2009).

- [19] C. B. Collins, E. N. Glass and D. A. Wilkinson, *Gen. Rel. Grav.* **12**(10), 805 (1980).
- [20] T. Harko and M. K. Mak, (2002) (gr-qc/0212075v1).
- [21] T. Harko and M. K. Mak, (2004) (gr-qc/0401069v1).
- [22] A. G. Riess et al., *Astron. J.* **116**, 1009 (1998).
- [23] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999).

Received: 29 August, 2014

Accepted: 31 January, 2015