

Self-gravitating Plane Symmetric Domain Walls in Modified Theory of Gravitation

R. Venkateswarlu^{1,*} and K. Sreenivas²

¹*GITAM School of International Business, GITAM University, Visakhapatnam, India*

²*Department of Mathematics, BVSR Engineering College, Chimakurthy, India*

Non-static plane symmetric domain walls are studied in Barber's second self-creation cosmology. The solutions of the field equations are obtained with the help of non-static plane symmetric space-time. It is shown that the self-gravitating (or stiff) thick domain walls do survive while the thick domain walls do not exist in case of $p_1 = 3\rho$. Some properties of this self-gravitating model are studied at the end.

1. Introduction

Many alternative theories and modifications to Einstein's theory of relativity have been proposed from time to time to unify gravity and matter fields in various forms. The scalar-tensor theories proposed by Brans and Dicke [1], Nordvedt [2] and Saez and Ballester [3] are noteworthy among several alternatives. Barber [4] has proposed two continuous self-creation theories by modifying the general relativity and Brans and Dicke (BD) theory. The Barber's first theory is considered to be an alternative to BD theory but Brans [5] has pointed out that the first theory severely violates the equivalence principle. The second theory is a modification of general relativity to a variable G-theory and the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. It is postulated that this scalar couples to the trace of the energy momentum tensor. Hence the field equations, in Barber's second theory, are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi} T_{ij} \quad (1)$$

and

$$\square \phi = \frac{8\pi}{3} \eta T \quad (2)$$

In this theory, the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. For a

detailed discussion of the self-creation theories of gravitation one may refer to the work of Barber [4].

Cosmic strings and domain walls are the topologically stable objects, which might have been formed during a phase transition in the early universe (Kibble [6]). According to Vilenkin [7,8], the domain walls are formed when the universe undergoes a series of phase transitions with discrete symmetry being spontaneously broken. After symmetry breaking different regions of the universe can be settled into different parts of the vacuum with domain walls forming boundaries between their regions. In particular, the appearance of domain wall is associated with spontaneously broken discrete symmetry i.e., the vacuum manifold consists of several disconnected components. For example, symmetry breaking is accomplished by a real scalar field. It is believed that the formation of galaxies is due to domain walls produced during the phase transition after the time of recombination of matter and radiation.

Therefore, some researchers have focused on solutions to Einstein field equations without cosmological constant describing the gravitational field of a planar wall i.e., a surface layer with plane symmetry. A vacuum domain wall in the thin-wall approximation is a planar wall in which the tension is equal to the surface energy density. After the time dependent solutions for domain walls considered by Vilenkin [7,8] and Isper and Sikivie [9], attempts have been made to focus on trying to find the expansion in thick domain walls by Widrow [10], Goetz [11], and Mukherjee [12].

In literature, it is found that considerable amount of work has been done on cosmic strings and domain walls in general relativity and in alternative theories of gravitation. Rahman [13-15],

*rangavajhala_v@yahoo.co.in

Reddy [16-18], Pradhan et al [19], Adhav et al. [20,21], Reddy and Naidu [22], and Reddy et al. [23], are some of the authors who have investigated various aspects of domain walls and cosmic strings in alternative theories of gravitation. Recently Venkateswarlu et al. [24] have studied non-static plane symmetric and Bianchi type-I domain walls in the presence of zero mass scalar fields. The Barber's second self-creation theory has been extensively discussed by Soleng [25,26], Reddy [27], Mohanty et al. [28,29], Panigrahi and Sahu [30], Venkateswarlu and Pavan kumar [31], and Venkateswarlu et al. [32]. It was found from the literature that the non-static plane symmetric space-time in the presence of thick domain walls has not been considered in Barber's second self-creation theory. Thus, in this paper, we consider thick domain walls for non-static plane symmetric space-time in Barber's second self-creation theory of gravitation.

2. Field Equations and the Metric

We consider the non-static plane symmetric metric as

$$ds^2 = e^{2h} (dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2) \quad (3)$$

Where, h and S are functions of t only.

A thick domain wall can be viewed as a soliton like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy-momentum tensor describing a scalar field ψ with self interactions contained in a potential $V(\psi)$ given by

$$\psi_i \psi_j - g_{ij} \left[\frac{1}{2} \psi_k \psi^k - V(\psi) \right] \quad (4)$$

The second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho (g_{ij} + \omega_i \omega_j) + p_1 \omega_i \omega_j \quad (5)$$

together with $\omega_i \omega^i = -1$.

Here ρ is the energy density of the domain walls, p_1 is the pressure in the direction normal to the plane of the domain walls and ω_i is the unit space-like vector in the same direction, i.e., along the x-axis.

We may use a correspondence to general relativity and define equivalent pressure and density (cf. Soleng [25]) as

$$p_{eq} = \frac{p_1}{\phi}, \quad \rho_{eq} = \frac{\rho}{\phi} \quad (6)$$

Using co-moving coordinate system, the non-vanishing component of T_{ij} can be obtained as

$$T_1^1 = -p_1, \quad T_3^3 = T_2^2 = T_4^4 = \rho \quad (7)$$

The field equations (1) and (2) with the help of Eqns. (3)-(7) can be written as

$$3\dot{h}^2 + \frac{2\dot{S}\dot{h}}{S} = -\frac{8\pi}{\phi} p_1 e^{2h} \quad (8)$$

$$\frac{\ddot{S}}{S} + 2\ddot{h} + \dot{h}^2 + \frac{2\dot{S}\dot{h}}{S} = \frac{8\pi}{\phi} \rho e^{2h} \quad (9)$$

$$2\ddot{h} + \dot{h}^2 = \frac{8\pi}{\phi} \rho e^{2h} \quad (10)$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{S}}{S} + 2\dot{h} \right) = \frac{8\pi}{3} \lambda (3\rho - p_1) e^{2h} \quad (11)$$

Where, a over head dot denotes ordinary derivative with respect to t .

3. Solution to the Field Equations

The set of field equations (8) to (11) are four equations in five unknowns. Thus, the deterministic solution of the field equations is obtained in the following two cases: (i) $p_1 = \rho$, which is analogues to stiff matter in general relativity and (ii) $p_1 = 3\rho$, which is again analogues to radiating model in general relativity.

Case 1: Let us consider the case when

$$p_1 = \rho \quad (12)$$

Now subtracting Eqns. (9) and (10) and integrating, we get

$$\dot{S} e^{2h} = c_1 \quad (13)$$

Where, c_1 is an arbitrary constant of integration.

From Eqns. (8)-(10) and using Eqn. (12), we obtain

$$\frac{\ddot{s}}{\dot{s}} - \frac{2\ddot{s}}{\dot{s}} + \frac{\dot{s}}{s} = 0 \quad (14)$$

Which, admits the solution of the form

$$s = ((1-a)(bt+c))^{\frac{1}{(1-a)}} \quad (15)$$

Where, a , b and c are arbitrary constants.

From Eqn. (13), we have

$$e^{2h} = h_0 (bt+c)^{\frac{-a}{(1-a)}} \quad (16)$$

Where, h_0 is a constant.

With the help of (15), (16) and (12), Eqn. (11) reduces to

$$(bt+c)^2 \ddot{\phi} + b(bt+c) \dot{\phi} - k_1^2 b^2 \phi = 0 \quad (17)$$

Where, $k_1^2 = \frac{\lambda a(4-3a)}{6(1-a)^2}$.

The solution of Eqn. (17) is given by

$$\phi = \phi_1 (bt+c)^{m_1} + \phi_2 (bt+c)^{m_2} \quad (18)$$

Where, $m_1 = \frac{\sqrt{\lambda a(4-3a)}}{\sqrt{6}(1-a)}$, $m_2 = \frac{\sqrt{\lambda a(4-3a)}}{\sqrt{6}(a-1)}$ and $0 < |\lambda| < 10^{-1}$.

The energy density and pressure are obtained as

$$\rho_{eq} = p_{eq} = \frac{ab^2(4-3a)}{32\pi(1-a)^2(bt+c)^{\frac{2-3a}{1-a}}} \quad (19)$$

And other kinematical parameters are

Proper volume: $V = r(1-a)^{\frac{1}{(1-a)}} h_0 (bt+c)^{\frac{2a-1}{a-1}}$

Scalar expansion: $\theta = \frac{(3a-2)b}{h_0(a-1)(bt+c)^{\frac{3a-2}{2(a-1)}}} \quad (20)$

Shear Scalar: $\sigma = \frac{b\sqrt{63a^2+60a+16}}{6h_0(bt+c)^{\frac{3a-2}{2(a-1)}}}$

It is clear that, from Eqn. (18), the two modes of Barber's scalar field ϕ behave differently.

When, $0 < a < 1$, the first mode is growing while the second mode is decaying as $t \rightarrow \infty$. This situation is quite similar the one obtained by Panigrahi and Sahu [29]. The Barber's scalar field becomes constant when $\lambda \rightarrow 0$, which shows that the theory lead to Einstein's theory in every respect.

The conditions $\rho_{eq} > 0$ and $p_{eq} > 0$ are satisfied only if $0 < a < 4/3$. When $a = 4/3$, the scalar field ϕ becomes a constant and $\rho_{eq} (= p_{eq})$ will vanish identically. Thus the solution reduces to non-static plane symmetric vacuum solution in general relativity. When $a = 1/4$, spatial volume increases (decrease) with the increase (decrease) in t . This indicates that the model is corresponding to both closed and open model.

The energy density of the domain walls $\rho \rightarrow 0$ as $t \rightarrow 0$ and $\rho \rightarrow \infty$ as $t \rightarrow \infty$, which shows the existence of Big-bang singularity in the model. Since $\frac{\sigma}{\theta} = \text{constant}$, the model does not approach isotropy for any t .

Case 2: Here we consider the equation of state as

$$p_1 = 3\rho \quad (21)$$

The above equation is taken again as an analog to disorder radiation in general relativity.

The solution of the field equations (8)-(10), together with (21), is obtained as

$$\begin{aligned} s &= t^{-3} \\ e^{2h} &= t^4 \\ \phi &= \phi_0 \log t \end{aligned} \quad (22)$$

Where, ϕ_0 is an arbitrary constant.

Use of Eqn. (22) in Eqns. (8)-(11), we obtain

$$\frac{\rho}{\phi} = \frac{p_1}{\phi} = 0$$

This shows that the non-static plane symmetric domain walls do not exist in Barber's second self-creation theory.

4. Conclusions

In this paper, we have studied the non-static plane symmetric thick domain walls in Barber's second self-creation theory of gravitation. It is observed that the self-gravitating thick domain walls do survive in this theory where as the same do not

survive in case of disordered radiation (i.e., $p_1 = 3\rho$). It is also noted that the self-gravitating model possess an initial singularity for a particular value of the constant a . It is found that the scalar field becomes a constant when $\lambda \rightarrow 0$. As it is well known that at the early stage of evolution of universe topologically stable objects such as domain walls and cosmic strings play a fundamental role in the formation of universe, the present study will contribute for better understanding of the Universe.

References

- [1] C. H. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
- [2] K. Nordvedt Jr., Astrophys. J. **161**, 1069 (1970).
- [3] D. Saez and V. J. Ballester, Phys. Lett. A **1113**, 467 (1985).
- [4] G. A. Barber, Gen. Rel. Grav. **14**, 117 (1982).
- [5] C. Brans, Gen. Rel. Grav. **19**, 949 (1987).
- [6] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
- [7] A. Vilenkin, Phys. Rep. **121**, 263 (1985).
- [8] A. Vilenkin, Phys. Rev. D **23**, 852 (1981).
- [9] J. Ipser and P. Sikivie, Phys. Rev. D **30**, 712 (1984).
- [10] L. M. Windrow, Phys. Rev. D **39**, 3571 (1989).
- [11] G. Goets, J. Math. Phys. **31**, 2683 (1990).
- [12] M. Mukherjee, Class. Quant. Grav. **10**, 131 (1993).
- [13] F. Rahaman, S. Chakraborty, M. Hossain and S. Das, Bulg. J. Phys. **28**, 217 (2001).
- [14] F. Rahaman, S. Chakraborty, M. Hossain, N. Begum and J. Bera, Indian J. Phys. B **76**, 747 (2002).
- [15] F. Rahaman, S. Chakraborty, M. Hossain, N. Begum and M. Kalam, Pramana **60**, 1153 (2003).
- [16] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, J. Dyn. Syst. Geom. Theories **3**, 1 (2005).
- [17] D. R. K. Reddy, R. L. Naidu, V. U. M. Rao, Astrophys. Space Sci. **300**, 381 (2005).
- [18] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, Astrophys. Space Sci. **306**, 185 (2006).
- [19] A. Pradhan and Singh. Atocmshi, Astrophys. Space. Sci. **288**, 315 (2003).
- [20] K. S. Adhav, A. S. Nimkar and M. V. Dawande, Astrophys. Space Sci. **310**, 231 (2007).
- [21] K. S. Adhav, V. G. Mete, A. M. Pund and R. B. Raut, J. Vectorial Rel. **5(4)**, 26 (2010).
- [22] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, Int. J. Theor. Phys. **46**, 1443 (2007).
- [23] D. R. K. Reddy, R. L. Naidu and A. S. Rao, Int. J. Theor. Phys. **47**, 1016 (2008).
- [24] R. Venkateswarlu, K. Pavan Kumar, Int. J. Theor. Phys. **49**, 1894 (2010).
- [25] H. H. Soleng, Astrophys. Space Sci. **137**, 373 (1987).
- [26] H. H. Soleng, Astrophys. Space Sci. **139**, 13 (1987).
- [27] D. R. K. Reddy and R. L. Naidu, Int. J. Theor. Phys. **47**, 2339 (2008).
- [28] G. Mohanty, B. Mishra and A. K. Biswal, Czech. J. Phys. **52** 1289 (2002).
- [29] R. C. Sahu and G. Mohanty, Astrophys. Space Sci. **306** 179 (2006).
- [30] U. K. Panigrahi and R. C. Sahu, Czech. J. Phys. **54** 543 (2004).
- [31] R. Venkateswarlu and K. P. Kumar, Astrophys. Space Sci. **301**, 73 (2006).
- [32] R. Venkateswarlu, V. U. M. Rao and K. P. Kumar, Int. J. Theor. Phys. **47**, 640 (2008).

Received: 7 August, 2014

Accepted: 31 January, 2015