

Five Dimensional Plane Symmetric Universe in Creation Field Cosmology

U. K. Panigrahi¹, R. Patra² and Mita Sharma^{3,*}

¹*Academy of Technocrats, Berhampur, Odisha, India*

²*PG Department of Mathematics, Berhampur University, Berhampur, Odisha, India*

³*Department of Mathematics and Humanities, College of Engineering and Technology, Bhubaneswar, Odisha, India*

We have studied the Hoyle-Narlikar C-field cosmology with five-dimensional plane symmetric metric space-time. Using the method of Narlikar and Padmanabham (1985), various solutions have been studied when the creation field C is a function of time t only. The geometrical and physical aspects of this model are also studied.

1. Introduction

The study of higher dimensional physics is important because of several prominent results obtained in the development of the superstring theory. In the latest study of superstrings and super gravity theories, Weinberg [2] studied the unification of fundamental forces with gravity, which reveals that the space-time should be different from four. Since the concept of higher dimensions is not unphysical, the string theories are discussed in both 10 dimensions and 26 dimensions of space-time. Because of this, many researchers are inspired to study higher dimensions to explore the hidden knowledge of the universe. Chodos and Detweller [3], Lorentz-Petzold [4], Ibanez and Verdaguer [5], Gleiser and Diaz [6], Banerjee and Bhuli [7], Reddy and Venkateswara [8], Khadekar and Gaikwad [9], Adhav et al. [10] have studied multi-dimensional cosmological models in general relativity and in other alternative theories of gravitation.

The three important observations in astronomy namely the phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) were supposed to be successfully explained by big-bang cosmology based on Einstein's field equations. However, Smoot et al. [11] revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly meet our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue

to contradict the theoretical explanations given from big bang type of models. Also, CMBR discovery did not prove it to be an outcome of big bang theory. In fact, Narlikar et al. [12] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle [13], Bondi and Gold [14] proposed the steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. Moreover, they have shown that the statistical properties of the large scale features of the universe do not change. Further, the constancy of the mass density has been accounted for by the continuous creation of matter in contrast to the one time infinite and explosive creation of matter at $t=0$ as in the earlier standard model. But, the principle of conservation of matter was violated in this formalism. To overcome this difficulty Hoyle and Narlikar [15] adopted a field theoretic approach by introducing a mass-less and charge-less scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C -field theory introduced by Hoyle and Narlikar, there is no big-bang type of singularity as in the steady state theory of Bondi and Gold [14]. A solution of Einstein's field equations admitting radiation with negative energy mass-less scalar creation fields C was obtained by Narlikar and Padmanabhan [1]. The study of Hoyle and Narlikar theory [15-17] in the space-time of dimensions more than four was carried out by Chatterjee and Banerjee [18]. The solutions of Einstein's field equations in the presence of creation field have been obtained for Binachi type-V universe in four dimensions by Singh and Chaubey [19].

*mitasharma08@gmail.com

In the present work, we have studied the Hoyle and Narlikar C-field cosmology in plane symmetric space time. We have assumed that $C(x, t) = C(t)$, that is, the creation field C is a function of time 't' only.

2. Hoyle and Narlikar C-field Cosmology

Introducing a mass-less scalar field called the creation field, or the C-field, Einstein's field equations are modified. Hoyle and Narlikar field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi (^mT_{ij} + {}^cT_{ij}) \quad (1)$$

Where, ${}^mT_{ij}$ is matter tensor of Einstein theory and ${}^cT_{ij}$ is matter tensor due to the C-field, which is given by

$${}^cT_{ij} = -f \left(C_i C_j - \frac{1}{2}g_{ij}C^k C_k \right) \quad (2)$$

Where, $f > 0$, and, $C_i = \frac{\partial C}{\partial x^i}$.

Because of the negative value of T^{00} ($T^{00} < 0$), the C-field has negative energy density producing repulsive gravitational field, which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$$-{}^cT_{ij}^{ij} = {}^mT_{ij}^{ij} = f C^i C^j_{;j} \quad (3)$$

i.e., matter creation through non zero left-hand side is possible while conserving the overall energy and momentum.

The above equation is similar to

$$mg_{ij} \frac{dx^i}{ds} - C_j = 0 \quad (4)$$

This implies that the 4-momentum of the created particle is compensated for by the 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy. Further, the C-field satisfy the source equation: $f C^i_{;j} = J^i_{;j}$ and $J^i = \rho \frac{dx^i}{ds} = \rho v^i$. Where, ρ is the homogeneous mass density.

3. Metric and Field Equations

We consider the five-dimensional plane metric

$$ds^2 = dt^2 - A^2(dx^2 + dy^2 + dz^2) + B^2 dm^2 \quad (5)$$

Where, A and B are function of t only.

We assume $C(x, t) = C(t)$ and that

$${}^mT_j^i = \text{diag}(\rho, -p, -p, -p, -p) \quad (6)$$

We have also assumed that the velocity of light and the gravitational constant are equal to one unit.

The field equations for metric (Eqn. (5)) lead to

$$3 \frac{\dot{A}^2}{A^2} + 3 \frac{\dot{A}\dot{B}}{AB} = 8\pi \left(\rho - \frac{1}{2}f\dot{C}^2 \right) \quad (7)$$

$$2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2 \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = 8\pi \left(-p + \frac{1}{2}f\dot{C}^2 \right) \quad (8)$$

$$3 \frac{\ddot{A}}{A} + 3 \frac{\dot{A}^2}{A^2} = 8\pi \left(-p + \frac{1}{2}f\dot{C}^2 \right) \quad (9)$$

$$\dot{\rho} + \left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \right) (\rho + p) = f\dot{C} \left[\ddot{C} + \left(\frac{3\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{C} \right] \quad (10)$$

Where, (.) denotes the derivative with respect to 't' and $\dot{C} = \frac{dC}{dt}$.

We can rewrite Eqn. (10) as

$$\frac{d}{dv}(V\rho) + p = f\dot{C}(V) \frac{d}{dv}[VC(V)] \quad (11)$$

Where

$$V = A^3 B \quad (12)$$

In order to obtain a unique solution, one has to specify the rate of creation of matter energy (at the expense of the negative energy of the C-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing C-field energy density. That is, the rate of creation of matter energy density per unit proper volume is given by

$$\frac{d}{dv}(V\rho) + p = \alpha^2 \dot{C}^2 \equiv \alpha^2 g^2(V) \quad (13)$$

Where, α is a constant and we have defined $\dot{C}(V) \equiv g(V)$.

From Eqns. (11) and (13), we get

$$g(V) \frac{d}{dv}(gV) = \frac{\alpha^2}{f} g^2(V) \quad (14)$$

On integration it gives

$$g(V) = A_1 V^{\frac{\alpha^2}{f}-1} \quad (15)$$

Where, A_1 is an arbitrary constant of integration. We take the equation of state of matter as $p = \gamma\rho$. Then from Eqns. (13) and (15), we have

$$\frac{d}{dV}(V\rho) + \gamma\rho = \frac{\alpha^2}{A_1^2} V^2 \left(\frac{\alpha^2}{f} - 1\right) \quad (16)$$

From Eqn. (16)

$$\rho = \frac{\alpha^2 A_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^2 \left(\frac{\alpha^2}{f} - 1\right) \quad (17)$$

$$p = \gamma \frac{\alpha^2 A_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^2 \left(\frac{\alpha^2}{f} - 1\right) \quad (18)$$

Now subtracting Eqn. (8) from Eqn. (9), we get

$$\frac{d}{dt} \left[\frac{A}{A} - \frac{B}{B} \right] + \left[\frac{A}{A} - \frac{B}{B} \right] \left[3 \frac{A}{A} + \frac{B}{B} \right] = 0 \quad (19)$$

Then from Eqns. (12) and (19)

$$\frac{d}{dt} \left[\frac{A}{A} - \frac{B}{B} \right] + \left[\frac{A}{A} - \frac{B}{B} \right] \frac{\dot{V}}{V} = 0 \quad (20)$$

Integrating the above equation, we get

$$\frac{A}{B} = k_2 \exp \left(k_1 \int \frac{1}{V} dt \right) \quad (21)$$

Where, k_1 and k_2 are constants of integrations.

In view of the fact that $V = A^3 B$, we write A and B in the explicit form

$$A = D_1 V^{\frac{1}{4}} \exp \left(X_1 \int \frac{1}{V} dt \right) \quad (22)$$

$$B = D_2 V^{\frac{1}{4}} \exp \left(X_2 \int \frac{1}{V} dt \right) \quad (23)$$

Where, $D_i (i = 1, 2)$ and $X_i (i = 1, 2)$ satisfy the relation $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

Using Eqns. (7), (8) and (9) we get

$$6 \frac{\dot{A}^2}{A^2} + 6 \frac{\dot{A}\dot{B}}{AB} + 3 \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} = \frac{16}{3} \pi (2 - \gamma) \rho \quad (24)$$

Now using Eqn. (12) we get

$$\frac{\ddot{V}}{V} = \frac{16}{3} \pi (2 - \gamma) \rho \quad (25)$$

From Eqns. (17) and (25), we get

$$\frac{\ddot{V}}{V} = \frac{16}{3} \pi (2 - \gamma) \frac{\alpha^2 A_1^2}{\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^2 \left(\frac{\alpha^2}{f} - 1\right) \quad (26)$$

This implies

$$V = \left\{ A_1 (f - \alpha^2) \left[\frac{16\pi(2-\gamma)}{3(2\alpha^2 - f - \gamma f)} \right]^{1/2} \right\}^{f/f - \alpha^2} t^{f/f - \alpha^2} \quad (27)$$

From Eqns. (15) and (27), we get

$$g = \frac{1}{f - \alpha^2} \left[\frac{16\pi(2-\gamma)}{3(2\alpha^2 - f - \gamma f)} \right]^{-1/2} \frac{1}{t} \quad (28)$$

Also from equation, $\dot{C}(V) = g(V)$ we get

$$C = \frac{1}{f - \alpha^2} \left[\frac{16\pi(2-\gamma)}{3(2\alpha^2 - f - \gamma f)} \right]^{-1/2} \log t \quad (29)$$

From Eqns. (17) and (27), the homogeneous mass density becomes

$$\rho = \frac{3\alpha^2 f}{16\pi(2-\gamma)(f - \alpha^2)^2} \frac{1}{t^2} \quad (30)$$

Hence from Eqn. (18)

$$p = \frac{3\alpha^2 \gamma f}{16\pi(2-\gamma)(f - \alpha^2)^2} \frac{1}{t^2} \quad (31)$$

From Eqns. (30) and (31), it is observed that

- 1) When time, $t \rightarrow \infty$, we get the density and pressure tending to zero i.e., the model reduces to vacuum.
- 2) When, $f = \alpha^2$, there is singularity in the density and pressure.
- 3) There is also singularity in the density and pressure for $\gamma = 2$.

Using Eqns. (22), (23) and (27), we can obtain

$$A(t) = D_1 k^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_1}{k} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (32)$$

$$B(t) = D_2 k^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_2}{k} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (33)$$

Now the scalar expansion

$$\theta = 4H \quad (34)$$

$$\theta = \frac{4}{3} \frac{f}{(f - \alpha^2)} t^{-1} \quad (35)$$

The mean anisotropy parameter A is given by

$$A = \frac{1}{4} \sum_{i=1}^4 \left(\frac{\Delta H_i}{H} \right)^2 \quad (36)$$

$$A = 27 \frac{X_1^2 (f - \alpha^2)^2}{k^2 f^2} t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)} \quad (37)$$

The shear scalar σ^2 is

$$\sigma^2 = \frac{1}{2} [\sum_{i=1}^4 H_i^2 - 4H^2] = \frac{4}{2} AH^2 \quad (38)$$

$$\sigma^2 = 6 \frac{X_1^2}{k^2} t^{\frac{2f}{\alpha^2 - f}} \quad (39)$$

The declaration parameter q is given by

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 \quad (40)$$

$$q = 2 - \frac{\alpha^2}{f} \quad (41)$$

Where, $k = \left\{ A_1 (f - \alpha^2) \left[\frac{16\pi(2-\gamma)}{3(2\alpha^2 - f - \gamma f)} \right]^{1/2} \right\}^{f/f - \alpha^2}$.

If $f > \alpha^2$ then for large t , the model tends to isotropic case.

4. Particular Cases

Case I: $\gamma = 0$ (Dust)

In this case, we obtain the values of various parameters

$$g = \frac{1}{f - \alpha^2} \left[\frac{32\pi}{3(2\alpha^2 - f)} \right]^{-1/2} \frac{1}{t} \quad (42)$$

$$C = \frac{1}{f - \alpha^2} \left[\frac{32\pi}{3(2\alpha^2 - f)} \right]^{-1/2} \log t \quad (43)$$

$$\rho = \frac{3\alpha^2 f}{32\pi(f - \alpha^2)^2} \frac{1}{t^2} \quad (44)$$

$$p = 0 \quad (45)$$

$$A(t) = D_1 k_1^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_1}{k_1} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (46)$$

$$B(t) = D_2 k_1^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_2}{k_1} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (47)$$

Now, the scalar expansion is

$$\theta = \frac{4}{3} \frac{f}{(f - \alpha^2)} t^{-1} \quad (48)$$

The mean anisotropy parameter A is given by

$$A = 27 \frac{X_1^2 (f - \alpha^2)^2}{k_1^2 f^2} t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)}$$

The shear scalar σ^2 is given by

$$\sigma^2 = 6 \frac{X_1^2}{k_1^2} t^{\frac{2f}{\alpha^2 - f}} \quad (49)$$

The declaration parameter q is given by

$$q = 2 - \frac{\alpha^2}{f} \quad (50)$$

Where, $k_1 = \left\{ A_1 (f - \alpha^2) \left[\frac{32\pi}{3(2\alpha^2 - f)} \right]^{1/2} \right\}^{f/f - \alpha^2}$.

If $f > \alpha^2$ then for large t , the model tends to isotropic case.

Case II: $\gamma = \frac{1}{3}$ (Disordered Radiation Universe)

In this case, we obtain the values of various parameters

$$\rho = \frac{9\alpha^2 f}{80\pi(f - \alpha^2)^2} \frac{1}{t^2} \quad (51)$$

$$p = \frac{3\alpha^2 f}{80\pi(f - \alpha^2)^2} \frac{1}{t^2} \quad (52)$$

$$A(t) = D_1 k_2^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_1}{k_2} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (53)$$

$$B(t) = D_2 k_2^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_2}{k_2} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (54)$$

Now, the scalar expansion

$$\theta = \frac{4}{3} \frac{f}{(f - \alpha^2)} t^{-1} \quad (55)$$

The mean anisotropy parameter A is given by

$$A = 27 \frac{X_1^2 (f - \alpha^2)^2}{k_2^2 f^2} t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)} \quad (56)$$

The shear scalar σ^2 is

$$\sigma^2 = 6 \frac{X_1^2}{k_2^2} t^{\frac{2f}{\alpha^2 - f}} \quad (57)$$

The declaration parameter q is given by

$$q = 2 - \frac{\alpha^2}{f} \quad (58)$$

Where, $k_2 = \left\{ A_1(f - \alpha^2) \left[\frac{80\pi}{3(6\alpha^2 - 4f)} \right]^{1/2} \right\}^{f/f - \alpha^2}$.

If $f > \alpha^2$ then for large t , the model tends to isotropic case.

Case III: $\gamma = 1$ (Zeldovich fluid or stiff fluid)

In this case, we obtain the values of various parameters

$$\rho = \frac{3\alpha^2 f}{16\pi(f - \alpha^2)^2} \frac{1}{t^2} \quad (59)$$

$$p = \frac{3\alpha^2 f}{16\pi(f - \alpha^2)^2} \frac{1}{t^2} \quad (60)$$

$$A(t) = D_1 k_3^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_1}{k_3} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (61)$$

$$B(t) = D_2 k_3^{1/3} t^{\frac{f}{3(f - \alpha^2)}} \exp \left\{ \frac{X_2}{k_3} \left(1 - \frac{f}{\alpha^2} \right) t^{\frac{\alpha^2}{\alpha^2 - f}} \right\} \quad (62)$$

Now the scalar expansion

$$\theta = \frac{4}{3} \frac{f}{(f - \alpha^2)} t^{-1} \quad (63)$$

The mean anisotropy parameter A is given by

$$A = 27 \frac{X_1^2}{k_3^2} \frac{(f - \alpha^2)^2}{f^2} t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)} \quad (64)$$

The shear scalar σ^2 is

$$\sigma^2 = 6 \frac{X_1^2}{k_3^2} t^{\frac{2f}{\alpha^2 - f}} \quad (65)$$

The declaration parameter q is given by

$$q = 2 - \frac{\alpha^2}{f} \quad (66)$$

Where, $k_3 = \left\{ A_1(f - \alpha^2) \left[\frac{8\pi}{3(\alpha^2 - f)} \right]^{1/2} \right\}^{f/f - \alpha^2}$.

If $f > \alpha^2$ then for large t , the model tends to isotropic case.

5. Discussion

For each case of $2f < \alpha^2$, we get negative deceleration parameter indicating that the universe is accelerating. This observation is consistent with the present day observation. The universe becomes isotropic for large value of t . For a small of t , the

ratio $\frac{\sigma}{\theta}$ tends to zero. Therefore, the model approaches to isotropy at late times. Also, we have observed that the matter density is inversely proportional to square of time t . We can interpret our result as: the matter is supposed to move along the geodesic normal to the surface $t = \text{constant}$. As the matter moves further apart, it is assumed that more mass is continuously created to maintain the matter density. However, the matter density tends to zero when time will be infinitely large.

6. Conclusion

In this work, the Hoyle-Narlikar C-field Cosmology has been studied in the five-dimensional plane symmetric metric. Various solutions have been studied in the case when the creation field C is a function of time only. The geometrical and physical aspects have been studied in some particular cases.

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