

Contribution of Non-thermal Electrons to Ion-acoustic Soliton Formation in Warm Positive and Negative Ions Plasma

Sankar Chattopadhyay*

*Centre for Theoretical Plasma Research, 346, R.K. Pally (Balaka), P.O. & P.S. Sonarpur,
Kolkata – 150, West Bengal, India*

*Department of Mathematics, Daria J.L.N. Vidyalaya, P.O. Thakuraniberia,
P.S. Canning, West Bengal, India*

Ion-acoustic solitary waves have been investigated theoretically in presence of positive and negative ions with non-thermal electrons by pseudo-potential method. The effect of concentration and temperature of both positive and negative ions on non-thermal electrons are discussed in the formation of solitary wave structure in which both compressive and rarefactive solitary waves are found.

1. Introduction

In plasma physics, ion-acoustic solitary waves (both compressive and rarefactive) have been studied by various authors [1-5] for the last two decades under different physical situations. The presence of non-thermal electrons in plasma gives rise to many interesting characteristics in non-linear propagation of waves including excitations of ion-acoustic solitary waves in plasma. Cairns et al. [6] took the ions to be cold and neglected the effect of the streaming velocity on electrons. They did not consider the temperature of ions and static magnetic field. Rarefactive and compressive solitary waves are found in presence of positive ions, negative ions and warm electrons in unbounded plasma. Cairns et al. [6] found compressive solitons in a plasma having positive ions only. Recently, Cairns et al. [6] have shown that solitary waves exist in the plasma consisting of non-thermal electrons and an excess of energetic particles and ions. This investigation was motivated by a recent observation of solitary wave structures with density depletion made by Freja satellite [7]. Subsequently, Mamun [8-9] considered the effect of non-thermal electrons for the study of ion-acoustic solitary waves. In a series of papers, Bandopadhyay and Das [10-14] investigated the stability of solitary waves in un-magnetized and magnetized plasma consisting of non-thermal electrons together with positive ions and obtained very interesting results. Bhattacharaya et al. [15] studied theoretically the effect of non-thermal

electrons on cold negative and positive ion, and on ion-acoustic solitary waves in a bounded plasma using pseudo-potential technique, and obtained both compressive and rarefactive solitons for any value of the non-thermal electron parameter β . They analyzed the structure of the solitary waves as a function of non-thermal electron parameter β , which measures the deviation from the thermalized state. However they did not consider the temperature and drift velocities of the ions. Paul et al. [17] studied analytically the plasma consisting of non-thermally distributed electrons and warm positive ions with streaming motion. This type of distribution arises in space plasmas such as solar wind and cometary plasmas from the force field present there. They obtained exact analytical solutions with some necessary and sufficient conditions for the existence of ion-acoustic solitons with special attention to the critical values of non-thermal electron parameter, ion temperature and velocity of the solitary wave. But, in this paper, we discuss the unbounded plasma with non-thermal electrons and warm negative as well as positive ions with drifts. The profiles of ion-acoustic solitary waves (first and second order) are shown for the plasma (He^+ , O^-) with different negative ion concentrations, ionic temperatures of both ions and non-thermal electron parameter.

The present paper is organized in the following way. In Sec. 2, the basic equations for plasma having warm positive ions, warm negative ions and non-thermal electrons in an unbounded system are given. Sagdeev potential is calculated in this section with valid expression of n_i and n_j . Sec. 3 contains the condition for solitary wave solution for

*sankar_fjln@yahoo.com

non-thermal electron plasma from which critical value of β is obtained. In this section, first and second order solitary wave solutions, amplitudes and widths are also calculated. Dispersion relation for the non-thermal plasma is calculated here. The most important and interesting phenomenon “stability” is discussed in detail and the nature of solitary waves is analyzed in this section very carefully. Discussions are kept in Sec. 4 and concluding remarks are given in Sec. 5.

2. Formulation

In our present study, we consider collisionless unmagnetized plasma consisting of warm positive and negative ions with streaming motion in the presence of non-thermal electrons. We also assume that the velocities of ions and electrons are both non-relativistic. The normalized basic equations in unidirectional propagation for such types of plasmas [16] are:

For positive ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} (n_i u_i) = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma_i}{n_i} \frac{\partial p_i}{\partial x} = - \frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3 p_i \frac{\partial u_i}{\partial x} = 0 \quad (3)$$

For negative ions

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x} (n_j u_j) = 0 \quad (4)$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \frac{\sigma_j}{n_j} \frac{\partial p_j}{\partial x} = - \frac{\partial \phi}{\partial x} \quad (5)$$

$$\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + 3 p_j \frac{\partial u_j}{\partial x} = 0 \quad (6)$$

Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z n_j \quad (7)$$

The non-thermal electron distribution function can be taken as [6]

$$f_e(v) = \frac{n_0}{(1+3\alpha)\sqrt{2\pi v_e^2}} \left[1 + \alpha \left(\frac{v^2}{v_e^2} - 2\phi \right)^2 \right] \cdot \exp \left[-\frac{1}{2} \left(\frac{v^2}{v_e^2} - 2\phi \right) \right] \quad (8)$$

Where, $V_e = \sqrt{\frac{K_B T_e}{m_e}}$, m_e is the mass of electron, K_B is the Boltzmann constant and T_e is the temperature of electron.

In this case, the subscript i, j denote positive and negative ions. n_i, u_i, p_i, σ_i and n_j, u_j, p_j, σ_j are, respectively, the concentration, velocity, pressure, and temperature of positive and negative ions. Again n_e is the concentration of non-thermal electrons connected with the parameter β , which measures the deviation from thermalized state. Also, α determines the presence of fast particles in the model, ϕ denotes the electrostatic potential, $Q (= m_j/m_i)$ is the ratio of the masses of negative to positive ions and Z is the charge.

In the above equations, the velocities are normalized by $\sqrt{\frac{K_B T_e}{m_i}}$, the densities by the equilibrium ion density n_0 , all the length by the Debye length $\sqrt{\frac{K_B T_e}{4\pi n_0 e^2}}$, and the potential by $\frac{K_B T_e}{e}$. Where, K_B is the Boltzmann constant, T_e is the absolute temperature of electron and m_i is the mass of positive ion and e is the charge of electron. The electron density is also normalized by the equilibrium ion number density n_0 so that we get

$$n_e = (1 - \beta\phi + \beta\phi^2) e^\phi \quad (9)$$

Where,

$$\beta = \frac{4\alpha}{1+3\alpha} \quad (10)$$

[with $\alpha \geq 0$ and $0 \leq \beta < \frac{4}{3}$].

For solitary wave solution from Eqns. (1) to (7), we use the Galilean transformation

$$\eta = x - V t \quad (11)$$

Where, V is the velocity of the solitary waves.

We also assume the boundary conditions

$$n_i \rightarrow n_{i0}, n_j \rightarrow n_{j0}, u_i \rightarrow u_{i0}, u_j \rightarrow u_{j0},$$

$$p_i \rightarrow 1, p_j \rightarrow 1, \phi \rightarrow 0, \text{ at } |x| \rightarrow \infty \quad (12)$$

The charge neutrality condition of the plasma is $n_{i0} = 1 + Z n_{j0}$.

By using Eqn. (11) and boundary conditions in Eqn. (12), we get from Eqns. (1) to (3) as

$$p_i = \left(\frac{v - u_{i0}}{v - u_i} \right)^3 \quad (13)$$

$$n_i^2 = \frac{n_{io}^3}{12\sigma_i} \left[\sqrt{\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}} - \sqrt{\left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}} \right]^2 \quad (14)$$

From (14), we observe that n_i^2 is non-negative if and only if $\sqrt{\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}}$ and $\sqrt{\left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}}$ are both real, and this will be possible if $\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}$ and $\left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}$ are both real and non-negative. As a consequence of this, $\phi \leq \frac{1}{2}(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2$ and $\phi \leq \frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2$ and these two conditions hold simultaneously if and only if

$$\Phi \leq \text{Min.} \left\{ \frac{1}{2}(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2, \frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 \right\} \quad (14a)$$

Also, it is easily seen that $\frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 \leq \frac{1}{2}(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2$.

Thus Eqn. (14a) gives,

$$\phi \leq \frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2$$

The relation in Eqn. (14) gives theoretically valid expression of n_i^2 if $\phi \leq \frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2$. It is not possible to get the correct value of n_i from Eqn. (14) for small values of σ_i . After removing σ_i from the denominator of Eqn. (14) we get

$$n_i = \frac{2n_{io}(V - u_{io})}{\sqrt{\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}} + \sqrt{\left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}}} \quad (14b)$$

Eqn. (14b) gives both theoretically and also numerically correct expression of n_i even for

$$\sigma_i = 0 \text{ when } \phi \leq \frac{1}{2}(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2.$$

Similarly using Eqn. (11) and the boundary condition in Eqn. (12) we get from Eqns. (4) to (6) as

$$p_j = \left(\frac{V - u_{jo}}{V - u_j} \right)^3 \quad (15)$$

$$n_j^2 = \frac{Qn_{jo}^3}{12\sigma_j} \left[\sqrt{\left\{ (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}} - \sqrt{\left\{ (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}} \right]^2 \quad (16)$$

By the similar argument as before the theoretically and numerically correct expression of n_j is

$$n_j = \frac{2n_{jo}(V - u_{jo})}{\sqrt{\left\{ (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}} + \sqrt{\left\{ (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}}} \quad (16a)$$

Where, $\phi \leq -\frac{Q}{2Z} \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^2$ even when $\sigma_j = 0$.

Again by using Eqns. (11), (14), (16) and (9) we get finally from Eqn. (7) as

$$\begin{aligned} \frac{d^2\phi}{d\eta^2} &= (1 - \beta\phi + \beta\phi^2)e^\phi - \frac{n_{io}^{\frac{3}{2}}}{\sqrt{12}\sigma_i} \\ &\left[\sqrt{\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}} - \sqrt{\left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}} \right]^2 \\ &+ Z \frac{Qn_{jo}^3}{12\sigma_j} \left[\sqrt{\left\{ (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}} - \sqrt{\left\{ (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}} \right]^2 \end{aligned} \quad (17)$$

Eqn. (17) can be written in the form

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi} \quad (18)$$

Where, $\psi(\phi)$ is known as the sagdeev potential and is given by

$$\begin{aligned} \psi(\phi) &= (1 + 3\beta) - \left[(1 + 3\beta) - 3\beta\phi + \beta\phi^2 \right] e^\phi - \\ &\frac{n_{io}^{\frac{3}{2}}}{6\sqrt{3}\sigma_i} \left[\left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}^{\frac{3}{2}} - \left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi \right\}^{\frac{3}{2}} \right] \end{aligned}$$

$$\begin{aligned} & \sqrt{\frac{3\sigma_i}{n_{io}}}^2 - 2\phi\}^{\frac{3}{2}} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^3 + (V - u_{io} - \\ & \sqrt{\frac{3\sigma_i}{n_{io}}})^3 \Big] - \frac{Q^{\frac{3}{2}}n_{jo}^{\frac{3}{2}}}{6\sqrt{3}\sigma_j} \left[\left\{ (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}^{\frac{3}{2}} - \right. \\ & \left. \left\{ (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi}{Q} \right\}^{\frac{3}{2}} - (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^3 + (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^3 \right] \end{aligned} \quad (19)$$

The Sagdeev potential $\Psi(\phi)$ in Eqn. (19) will be reduced to that potential which Cairns et al. obtained for $n_{jo} = 0$, $n_{io} = 1$ and $u_{io} = 0$. When $n_{jo} = 0$, we get that form of $\Psi(\phi)$ that Paul et al. [17] got. The function $\Psi(\phi)$ will be valid for such value of ϕ where ϕ satisfies the inequality

$$-\frac{Q}{2Z} \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^2 < \phi < \frac{1}{2} (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 \quad (20)$$

3. (A) Existence of Solitary Wave Solution

For solitary wave solution, the Sagdeev potential $\Psi(\phi)$ must satisfy the following conditions:

- i) $\Psi(\phi) = 0 = \frac{\partial \Psi}{\partial \phi}$ for all V at $\phi = 0$.
- ii) $\Psi(\phi) = 0$ for some $\phi = \phi_m$, ϕ_m is some maximum value of ϕ .
- iii) $\Psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$

$$\begin{aligned} a_0 &= Z^2 \left[Q(V - u_{jo})^2 + Z(V - u_{io})^2 \right] \\ a_1 &= Z \left[2Q(V - u_{jo})^2 - 3Z\sigma_j + Z(V - u_{io})^2 - 3Z\sigma_i - Q(V - u_{io})^2(V - u_{jo})^2 \right. \\ & \quad \left. + \beta Q(V - u_{io})^2(V - u_{jo})^2 \right] \\ a_2 &= Q(V - u_{jo})^2 - 6Z\sigma_j - Q(V - u_{io})^2(V - u_{jo})^2 + 3Z\sigma_j(V - u_{io})^2 + 3Q\sigma_i(V - u_{jo})^2 \\ & \quad + \beta Q(V - u_{io})^2(V - u_{jo})^2 - 3\beta Z\sigma_j(V - u_{io})^2 - 3\beta Q\sigma_i(V - u_{jo})^2 \\ a_3 &= 3\sigma_j[(V - u_{io})^2 - 1 - 3\sigma_i - \beta(V - u_{io})^2 + 3\beta\sigma_i] \end{aligned}$$

This is a more general cubic equation of critical negative ion concentration (n_{jc}) for non-thermal plasma. The above cubic equation in n_{jc} reduces to isothermal case when $\beta = 0$. From the above cubic equation in n_{jc} after simplifying some steps we get finally the solution for n_{jc} as

In addition to that for localized soliton solutions, we require the condition

$$\frac{\partial^2 \Psi}{\partial \phi^2} < 0 \text{ at } \phi = 0$$

Which, implies the inequality

$$\frac{n_{io}}{(V - u_{io})^2 - \frac{3\sigma_i}{n_{io}}} + \frac{Z^2 n_{jo}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} < 1 - \beta \quad (21)$$

Inequality in Eqn. (21) gives the condition for the existence of a potential well. In the absence of non-thermal parameter β ($\beta = 0$), Eqn. (21) reduces to Refs. [18, 21] and for cold ion plasma ($\sigma_i = \sigma_j = 0$) inequality in Eqn. (21) reduces to Ref. [19]. For $n_{jo} = 0$, inequality in Eqn. (21) follows Ref. [17]. Again for critical negative ion concentration (n_{jc}), we get from the above inequality in Eqn. (21) as

$$\frac{1 + Zn_{jc}}{(V - u_{io})^2 - \frac{3\sigma_i}{1 + Zn_{jc}}} + \frac{Z^2 n_{jc}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jc}}} = 1 - \beta \quad (21a)$$

After simplifying some steps, we get the cubic equation in n_{jc} as

$$a_0 n_{jc}^3 + a_1 n_{jc}^2 + a_2 n_{jc} + a_3 = 0 \quad (21b)$$

Where,

$$\begin{aligned} n_{jc} &= \left[\frac{1}{2} \left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0} \right) \right. \\ & \quad \left. + \frac{1}{2} \sqrt{\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0} \right)^2 - \frac{4}{729} \left(\frac{a_1^2}{a_0^2} - \frac{3a_2}{a_0} \right)^3} \right]^{\frac{1}{3}} \end{aligned}$$

$$+ \left[\frac{1}{2} \left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0} \right) - \frac{1}{2} \sqrt{\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0} \right)^2 - \frac{4}{729} \left(\frac{a_1^2}{a_0^2} - \frac{3a_2}{a_0} \right)^3} \right]^{\frac{1}{3}} - \frac{a_1}{3a_0}$$

This will give three values of the critical negative ion concentration (n_{jc}) and all these values will be real if the inequality

$$\left(\frac{a_1 a_2}{3a_0^2} - \frac{2a_1^3}{27a_0^3} - \frac{a_3}{a_0} \right)^2 > \frac{4}{729} \left(\frac{a_1^2}{a_0^2} - \frac{3a_2}{a_0} \right)^3$$

holds.

Again, from the inequality (21), the critical values of soliton velocities V_c (i.e., phase velocities) are obtained from the equation given below as

$$\frac{n_{i0}}{(V_c - u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{Z^2 n_{j0}}{Q(V_c - u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}} = 1 - \beta \quad (21c)$$

Eqn. (21c) can be written in the following as

$$\begin{aligned} & Qn_{i0}n_{j0}(1-\beta)V_c^4 - 2Qn_{i0}n_{j0}(1-\beta)(u_{i0} + u_{j0}) \\ & V_c^3 + [(1-\beta)(Qn_{i0}n_{j0}u_{j0}^2 + Qn_{i0}n_{j0}u_{i0}^2 + \\ & 4Qn_{i0}n_{j0}u_{i0}u_{j0} - 3Qn_{j0}\sigma_i - 3n_{i0}\sigma_j) - Qn_{i0}^2n_{j0} - \\ & Z^2n_{i0}n_{j0}^2] V_c^2 + 2[(1-\beta)(3n_{i0}u_{i0}\sigma_j + \\ & 3Qn_{j0}u_{j0}\sigma_i - Qn_{i0}n_{j0}u_{i0}u_{j0}^2 - Qn_{i0}n_{j0}u_{i0}^2u_{j0}) + \\ & Qn_{i0}^2n_{j0}u_{j0} + Z^2n_{i0}n_{j0}^2u_{i0}]V_c + \\ & [(1-\beta)(Qn_{i0}n_{j0}u_{i0}^2u_{j0}^2 - 3n_{i0}u_{i0}^2\sigma_j - \\ & 3Qn_{j0}\sigma_i u_{j0}^2 + 9\sigma_i\sigma_j) - Qn_{i0}^2n_{j0}u_{j0}^2 + 3n_{i0}^2\sigma_i - \\ & Z^2n_{i0}n_{j0}^2u_{i0}^2 + 3Z^2n_{j0}\sigma_i] = 0 \end{aligned}$$

We are now discussing and analyzing the critical phase velocities V_c from Eqn. (21c) under three different situations.

(i) Critical phase velocities for equal ion drifts

For $u_{i0} = u_{j0} = u$ (i.e., motion with equal ion drifts) and $\sigma_i \rightarrow 0$, $\sigma_j \rightarrow 0$ (cold ion), we get from Eqn. (21c)

$$\frac{n_{i0}}{(V_c - u)^2} + \frac{Z^2 n_{j0}}{Q(V_c - u)^2} = 1 - \beta$$

From this we get $V_c = u \pm \frac{1}{\sqrt{1-\beta}} \sqrt{n_{i0} + \frac{Z^2 n_{j0}}{Q}}$ when $\beta < 1$.

The above relation gives two different phase velocities of the motion (slow and fast). This is

more general form than Ref. [21] and it will be reduced to Ref. [21] for $\beta = 0$ (i.e., isothermal plasma).

(ii) Critical phase velocities in the absence of negative ion

For $u_{j0} \rightarrow 0$, $n_{j0} \rightarrow 0$, $n_{i0} \rightarrow 1$ and $\sigma_j \rightarrow 0$ (i.e., in absence of negative ions), we get from Eqn. (21c)

$$\frac{1}{(V_c - u_{i0})^2 - 3\sigma_i} = 1 - \beta$$

Thus $V_c = u_{i0} \pm \sqrt{3\sigma_i + \frac{1}{1-\beta}}$ which is the critical soliton's velocity or phase velocities in presence of positive ion drift only with non-thermal electrons. It is the general form of Ref. [21] and is turned into Ref. [21] when $\sigma_i \rightarrow 0$ and $\beta \rightarrow 0$ (i.e., cold isothermal plasma). The above equation for V_c supports work in [17].

(iii) Critical phase velocities without ion drifts

For $u_{i0} \rightarrow 0$ and $u_{j0} \rightarrow 0$ (i.e., without ion drifts) we get from Eqn. (21c)

$$\begin{aligned} & V_c^4 - \left[3 \left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}} \right) + \frac{1}{1-\beta} \left(n_{i0} + \frac{Z^2 n_{j0}}{Q} \right) \right] V_c^2 + \frac{3}{Q} \\ & \left[\frac{3\sigma_i\sigma_j}{n_{i0}n_{j0}} + \frac{1}{1-\beta} \left(\frac{\sigma_j n_{i0}}{n_{j0}} + \frac{Z^2 \sigma_i n_{j0}}{n_{i0}} \right) \right] = 0 \end{aligned}$$

This is general form than Refs. [16, 21] and it supports Ref.[21] for $\sigma_i \rightarrow 0$, $\sigma_j \rightarrow 0$ and $\beta \rightarrow 0$. Two cases now arise.

Case (a): For warm non-thermal plasma, the four values of critical phase velocities (V_c) are given as

$$V_c = \frac{1}{2} \left[\sqrt{\lambda_1 + 2\sqrt{\lambda_2}} + \sqrt{\lambda_1 - 2\sqrt{\lambda_2}} \right]$$

$$V_c = -\frac{1}{2} \left[\sqrt{\lambda_1 + 2\sqrt{\lambda_2}} + \sqrt{\lambda_1 - 2\sqrt{\lambda_2}} \right]$$

$$V_c = \frac{1}{2} \left[\sqrt{\lambda_1 + 2\sqrt{\lambda_2}} - \sqrt{\lambda_1 - 2\sqrt{\lambda_2}} \right]$$

$$V_c = -\frac{1}{2} \left[\sqrt{\lambda_1 + 2\sqrt{\lambda_2}} - \sqrt{\lambda_1 - 2\sqrt{\lambda_2}} \right]$$

Where,

$$\lambda_1 = \left[3 \left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}} \right) + \frac{1}{1-\beta} \left(n_{i0} + \frac{Z^2 n_{j0}}{Q} \right) \right]$$

$$\lambda_2 = \frac{3}{Q} \left[\frac{3\sigma_i \sigma_j}{n_{io} n_{jo}} + \frac{1}{1-\beta} \left(\frac{\sigma_j n_{io}}{n_{jo}} + \frac{Z^2 \sigma_i n_{jo}}{n_{io}} \right) \right]$$

Case (b): For cold non-thermal plasma when $\sigma_i \rightarrow 0$ and $\sigma_j \rightarrow 0$ then we get finally

$$V_c = \pm \frac{1}{\sqrt{1-\beta}} \sqrt{n_{io} + \frac{Z^2 n_{jo}}{Q}}$$

In this case, the actual value of the critical soliton velocity (i.e., linear velocity), is the positive value of V_c in presence of positive and negative ion with non-thermal electron and it supports Ref. [16] for $\beta \rightarrow 0$. The other value of V_c (i.e., negative value) is absurd in this case. In all the cases, the most important restriction on β is $\beta < 1$.

Now from Eqn. (21a), the critical value of the non-thermal parameter β_c ($\beta = \beta_c$) is

$$\beta_c = 1 - \frac{n_{io}}{(V - u_{io})^2} - \frac{Z^2 n_{jo}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} \quad (22)$$

This β_c is more general than the one given in Ref. [17] and it will be reduced to that in Ref. [17] for $n_{jo} = 0$ and $n_{io} = 1$.

Expanding $\Psi(\phi)$ in power series of ϕ , we get from Eqn. (18)

$$\frac{d^2 \phi}{d\eta^2} = S_1 \phi + S_2 \phi^2 + S_3 \phi^3 + S_4 \phi^4 + S_5 \phi^5 + \dots \quad (23)$$

Where,

$$\begin{aligned} S_1 &= (1 - \beta) - \frac{n_{io}}{(V - u_{io})^2} - \frac{Z^2 n_{jo}}{Q(V - u_{jo})^2 - \frac{3\sigma_j}{n_{jo}}} \\ S_2 &= -\frac{1}{2} \left[1 + \frac{n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}} \left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-3} - (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-3} \right\} - \right. \\ &\quad \left. \frac{Z^3 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{3}{2}}\sqrt{3\sigma_j}} \left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-3} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-3} \right\} \right] \\ S_3 &= \frac{1}{6} \left[(1 + 3\beta) + \frac{3n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}} \left\{ (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-5} - (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-5} \right\} - \right. \\ &\quad \left. \frac{3Z^4 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{5}{2}}\sqrt{3\sigma_j}} \left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-5} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-5} \right\} \right] \\ S_4 &= \frac{1}{24} \left[(1 + 8\beta) - \frac{15n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}} \left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-7} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-7} \right\} + \right. \\ &\quad \left. \frac{15Z^5 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{7}{2}}\sqrt{3\sigma_j}} \left\{ \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-7} - \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-7} \right\} \right] \\ S_5 &= \frac{1}{120} \left[(1 + 15\beta) - \frac{105n_{io}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}} \left\{ (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^{-9} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^{-9} \right\} + \right. \\ &\quad \left. \frac{105Z^6 n_{jo}^{\frac{3}{2}}}{2Q^{\frac{9}{2}}\sqrt{3\sigma_j}} \left\{ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-9} - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}} \right)^{-9} \right\} \right] \end{aligned} \quad (24)$$

Taking terms up to ϕ^2 from Eqn. (23), we get the first order K-dV soliton solution [20] as

$$\phi_1 = \frac{3S_1}{2S_2} \text{Sech}^2 \left(\sqrt{\frac{S_1}{4}} \eta \right) \quad (25)$$

Again for higher order non-linearity effect after taking terms up to ϕ^3 we get the higher order M-KdV solitary wave solution [20] from Eqn. (23) as

$$\phi_2 = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}} \left[2 \text{Cosh}^2 \left(\sqrt{\frac{S_1}{4}} \eta \right) - 1 \right] \quad (26)$$

For small amplitude solitary wave, the first (ϕ_{01}) and second (ϕ_{02}) order amplitudes of the solitary wave solution, respectively, are

$$\phi_{01} = \frac{3S_1}{2S_2} \quad (27)$$

and

$$\phi_{02} = \frac{6S_1}{2S_2 + \sqrt{4S_2^2 - 18S_1S_3}} \quad (28)$$

The first(δ_1) and second(δ_2) order width of that solitary waves, respectively, are

$$\delta_1 = \frac{2}{\sqrt{S_1}} \quad (29)$$

and

$$\delta_2 = \delta_1 \cosh^{-1} \left\{ \left[\frac{1.381 S_2}{\sqrt{4S_2^2 - 18S_1S_3}} + 1.6905 \right]^{\frac{1}{2}} \right\} \quad (30)$$

(B) Dispersion Relation

For the solitary wave solution, the dispersion relation is very essential. The dispersion relation is obtained from $\Psi(\phi) = 0$ at $\phi = \phi_m$ where ϕ_m is some maximum value of ϕ . Thus $\Psi(\phi_m) = 0$ gives

$$\begin{aligned} (1 + 3\beta) - [(1 + 3\beta) - 3\beta\phi_m + \beta\phi_m^2] e^{\phi_m} - \frac{n_{io}^2}{6\sqrt{3}\sigma_i} \left[\{(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi_m\}^{\frac{3}{2}} - \{(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^2 - 2\phi_m\}^{\frac{3}{2}} - (V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}})^3 + (V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}})^3 \right] - \frac{Q^2 n_{jo}^2}{6\sqrt{3}\sigma_j} \left[\{(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi_m}{Q}\}^{\frac{3}{2}} - \{(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^2 + \frac{2Z\phi_m}{Q}\}^{\frac{3}{2}} - (V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^3 + (V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}})^3 \right] = 0 \end{aligned} \quad (31)$$

(C) Stability of Solitary Wave Solution

From Eqn. (18), we get

$$\frac{d^2\phi}{d\eta^2} + \psi'(\phi) = 0 \quad (32)$$

Integrating Eqn. (32) and using the boundary condition, we finally get

$$\frac{1}{2} \left(\frac{d\phi}{d\eta} \right)^2 + \Psi(\phi) = 0 \quad (33)$$

From the energy integral (Eqn. (33)), the first term can be regarded as the kinetic energy (K.E.) of a particle of unit mass at position ϕ and time η with velocity $\left(\frac{d\phi}{d\eta} \right)$, whereas the second term $\Psi(\phi)$ is the potential energy (P.E.) of the same particle at that instant. As the kinetic energy is always non-negative, so is P.E. $\Psi(\phi) \leq 0$ for the entire motion. It is found from Eqn. (33) that the maximum value of P.E. $\Psi(\phi)$ is zero. Also, from Eqn. (32), the force acting on the particle at position ϕ is $-\psi'(\phi)$. We now assume $\Psi(\phi) = 0 = \psi'(\phi)$ at $\phi = 0$ and as a consequence of this, one may say that the velocity $\left(\frac{d\phi}{d\eta} \right)$ as well as the force $\frac{d^2\phi}{d\eta^2}$ acting on the particle at $\phi = 0$ are simultaneously equal to zero. If $\phi = 0$ can be considered an unstable position of equilibrium, the particle is then slightly displaced from its unstable position of equilibrium and continues its motion until its velocity $\left(\frac{d\phi}{d\eta} \right)$ is equal to zero, whereas the force acting on the particle of unit mass is $-\psi'(\phi_m)$ at $\phi = \phi_m$ for some $\phi_m \neq 0$.

Now for $\phi_m > 0$, the force acting on the particle at $\phi = \phi_m$ is directed towards the point $\phi = 0$ when $-\psi'(\phi_m) < 0$ i.e., if $\psi'(\phi_m) > 0$, the particle reflects back again to $\phi = 0$. Also for $\phi_m < 0$, the force acting on the particle at $\phi = \phi_m$ is directed towards the point $\phi = 0$ if $-\psi'(\phi_m) > 0$ i.e., if $\psi'(\phi_m) < 0$, the particle in this case reflects back again at $\phi = 0$. Moreover, if $\Psi(\phi) = 0 = \psi'(\phi)$ at $\phi = \phi_m$ then the velocity $\left(\frac{d\phi}{d\eta} \right)$ and the force $\frac{d^2\phi}{d\eta^2}$ are simultaneously equal to zero at $\phi = \phi_m$ and as a result of this, the particle cannot be reflected back again at $\phi = 0$. Actually, if $\psi'(\phi_m) > 0$ for $\phi_m > 0$ or $\psi'(\phi_m) < 0$ for $\phi_m < 0$ the particle takes an infinitely long time to move away from the unstable position of equilibrium continuing its motion until ϕ takes the value ϕ_m and again it takes an infinitely long time to come back to its unstable position of equilibrium. Thus for the existence of a solitary wave solution of the energy integral (Eqn. (33)), we must have the following two conditions:

- (i) Position of unstable equilibrium of particle: $\phi = 0$ where P.E. $\Psi(\phi)$ attains its maximum value i.e., $\frac{\partial^2 \Psi}{\partial \phi^2} < 0$ along with $\Psi(\phi) = 0$.
- (ii) Condition for oscillation of the particle:

$\Psi(\phi_m) = 0$, $\Psi'(\phi_m) > 0$ for some $\phi_m > 0$ (positive potential solitary wave) or $\Psi'(\phi_m) < 0$ for some $\phi_m < 0$ (negative potential solitary wave) within the interval $\min. (0, \phi_m) < \phi < \max. (0, \phi_m)$.

(iii) Condition for negative value of $\Psi(\phi)$: $\Psi(\phi) < 0$ for all $0 < \phi < \phi_m$ or $\phi_m < \phi < 0$, which is the condition to define the energy integral (Eqn. (33)) within the interval $\min. (0, \phi_m) < \phi < \max. (0, \phi_m)$.

Again, if P.E. $\Psi(\phi)$ of the particle attains its minimum value at $\phi = 0$ i.e., $\frac{\partial^2 \Psi}{\partial \phi^2} > 0$, then it is impossible to make any oscillation of the particle even when it is slightly displaced from its position of stable equilibrium ($\phi = 0$) and as a consequence of this solitary waves are formed for $\frac{\partial^2 \Psi}{\partial \phi^2} > 0$. In other words, for unstable equilibrium of the particle at $\phi = 0$ i.e., $\frac{\partial^2 \Psi}{\partial \phi^2} < 0$, the function $\Psi(\phi)$ must be convex within a neighbourhood (n.b.h.) of $\phi = 0$ and in this case both type of solitary waves (negative or positive potential) may exist if other conditions are fulfilled. We now assume that $\frac{\partial^2 \Psi}{\partial \phi^2} = 0$ and $\frac{\partial^3 \Psi}{\partial \phi^3} = 0$ at $\phi = 0$, then if $\frac{\partial^4 \Psi}{\partial \phi^4} < 0$ at $\phi = 0$, the potential energy, $\Psi(\phi)$, of the particle attains its max. value at $\phi = 0$ and consequently $\phi = 0$ is the position of unstable equilibrium. On the other hand, if $\frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$, $\frac{\partial^3 \Psi(0)}{\partial \phi^3} = 0$ and $\frac{\partial^4 \Psi(0)}{\partial \phi^4} > 0$, P.E. $\Psi(\phi)$ of the particle attains its min. value at $\phi = 0$ and so $\phi = 0$ is position of stable equilibrium of the particle and in this case there is no question for the existence of solitary of solitary wave solution of the energy integral (Eqn.(33)).

It is evident from the fact that if $\Psi(0) = \frac{\partial \Psi(0)}{\partial \phi} = \frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$ and $\frac{\partial^3 \Psi(0)}{\partial \phi^3} < 0$ then we can say that $\Psi(\phi)$ is a concave function in $(-\phi_\epsilon, 0)$ and also convex function in $(0, \phi_\epsilon)$, where ϕ_ϵ is a sufficiently small strictly positive quantity. Thus if a particle be slightly displaced from the place $\phi = 0$ towards the positive potential side it falls within the interval $(0, \phi_\epsilon)$ and due to convexity it moves away from $\phi = 0$ and continues its motion until its velocity is equal to zero i.e., until ϕ takes $\phi = \phi_m > 0$ where $\Psi(\phi_m) = 0$ and in this case if $\frac{\partial \Psi(\phi_m)}{\partial \phi} > 0$, one may obtain a positive potential solitary wave (PPSW) solution of the energy integral (Eqn. (33)). On the other hand, if the same particle is slightly displaced from $\phi = 0$ towards the negative potential

side, it falls within the interval $(-\phi_\epsilon, 0)$ and due to concavity it moves towards $\phi = 0$, and consequently no solitary wave solution is found in the negative potential side. We can conclude in the same way that if $\Psi(0) = \frac{\partial \Psi(0)}{\partial \phi} = \frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$ and $\frac{\partial^3 \Psi(0)}{\partial \phi^3} > 0$ then one may obtain a negative potential solitary wave (NPSW) solution of the energy integral (Eqn. (33)), whereas there does not exist any positive potential solitary wave (PPSW). In this situation, it is very important to note that we cannot expect the coexistence of both negative and positive potential solitary waves at $\frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$. Hence, the necessary conditions for the existence of PPSW are $\Psi(0) = \frac{\partial \Psi(0)}{\partial \phi} = \frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$ and $\frac{\partial^3 \Psi(0)}{\partial \phi^3} < 0$ whereas for NPSW are $\Psi(0) = \frac{\partial \Psi(0)}{\partial \phi} = \frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$ and $\frac{\partial^3 \Psi(0)}{\partial \phi^3} > 0$ at $\frac{\partial^2 \Psi(0)}{\partial \phi^2} = 0$.

4. Discussion

In this paper, by using the Sagdeev's pseudo-potential approach, we investigate the structure of first (ϕ_1) and second (ϕ_2) order solitary waves, first (δ_1) as well as the second (δ_2) order widths of the waves that are shown in Figs. 1–5.

The effect of n_{j0} on first (ϕ_1) and second (ϕ_2) order solitary wave solutions are studied carefully and are shown graphically in Fig. 1. It is understood from this figure that the values of first order (ϕ_1) solitary waves are decreasing and rarefactive in nature while second (ϕ_2) order solitary waves are compressive with spiky form at lowest n_{j0} for $(\text{He}^+, \text{O}^-)$ plasma when negative ion concentrations (n_{j0}) are increasing. The maximum value of ϕ_1 or ϕ_2 is obtained for smallest value of negative ion concentration (n_{j0}). Again as negative ion concentration (n_{j0}) increases the first (ϕ_{01}) and second (ϕ_{02}) order amplitudes increase against the non-thermal electron parameter β .

Fig. 2 shows the structure of first (ϕ_1) and second (ϕ_2) order solitary waves with the variation of different ionic temperatures (σ_i, σ_j) of positive (i) and negative (j) ions. When the temperature of both the ions ($\sigma_i \neq \sigma_j$) increases, the first order amplitude (ϕ_{01}) decreases with rarefactive nature for the plasma $(\text{He}^+, \text{O}^-)$ and the second (ϕ_{02}) order amplitude increases due to increase of the temperature of ions (σ_i, σ_j). Again, when the temperature of positive and negative ions are equal ($\sigma_i = \sigma_j = \frac{1}{30}$), the first order K-dV amplitude (ϕ_{01}) is maximum and is rarefactive in nature but the

second order solitary waves are compressive in nature for the said plasma.

Fig. 3 represents the structure of first (ϕ_1) and second (ϕ_2) order to an increase of waves with the variation of the non-thermal electron parameter (β) for (He^+, O^-) plasma. It is evident from this figure that the values of ϕ_1 are decreasing upto a certain value of η beyond, then ϕ_1 increases when β increases, but the interesting result occurs when the non-thermal electron parameter (β) takes the value $\beta = 0$. The non-thermal case then reduces to isothermal case (for $\beta = 0$) and in this situation the first order K-dV amplitude is maximum with respect to the other values of the non-thermal parameter (β). The second (ϕ_{02}) order amplitude increases when non-thermal electron parameter (β) increases from 0 to 0.2 except at $n_{j0}=0.5$. For (He^+, O^-) plasma compressive solitons exist in second order solution.

The first (δ_1) and second (δ_2) order widths verses negative ion concentration (n_{j0}) with the variation of the temperature of positive and negative ions (σ_i, σ_j) are shown in Fig. 4. The first order width (δ_1) increases when temperature of positive and negative ions (σ_i, σ_j) increases and for equal ion temperature ($\sigma_i = \sigma_j = \frac{1}{30}$) the first (δ_1) and second (δ_2) order width increases due to increase of negative ion concentration for a particular $\beta=0.2$. For increasing value of σ_i, σ_j , the second (δ_2) order width decreases at first upto a certain value of n_{j0} and then increases normally for increasing values of n_{j0} for (He^+, O^-) plasma.

Fig. 5 shows the first (δ_1) and second (δ_2) order widths against the negative ion concentration (n_{j0}) with the variation of the non-thermal electron parameter (β) for the plasma (He^+, O^-). When n_{j0} increases, the first (δ_1) order width increases gradually for increasing values of β , but δ_1 gets smaller values for $\beta = 0$, which shows an isothermal situation. It is also shown that second (δ_2) order width increases when β increases. Also it is important to note that the first (δ_1) order width is always greater than the second (δ_2) order width for some values of n_{j0} .

5. Concluding Remarks

In this paper, we have investigated theoretically the propagation of ion-acoustic solitary waves (compressive and rarefactive) with the analysis of the first and second order widths and amplitudes in an unbounded plasma consisting of positive ions, negative ions and non-thermal electrons. The influence of negative ions, temperature of both

positive and negative ions and non-thermal electron parameter (β) have been discussed and represented by the respective Figs. 1–5. The structures of both compressive and rarefactive solitary waves (ϕ_1 and ϕ_2) are shown in Figs. 1–3 for different values of negative ion concentration (n_{j0}), temperature of positive (σ_i) and negative (σ_j) ions and non-thermal electron parameter (β) for the plasma (He^+, O^-). We observe from those figures that for the above plasma, the first (ϕ_1) order solitary wave is rarefactive, but the second (ϕ_2) order solitary wave is compressive in nature. It is also important to note that the amplitude of second (ϕ_{02}) order compressive soliton is higher than that of first (ϕ_{01}) order rarefactive soliton. The second (ϕ_2) order compressive soliton has a peculiar spiky form with respect to the first order (ϕ_1) rarefactive soliton. From Figs. 4–5, it is seen that the first (δ_1) and second (δ_2) order widths vary directly with β and n_{j0} . On the other hand, the first order (δ_1) width varies directly with the temperature of ions and the second (δ_2) order width decreases at first, but after that δ_2 increases. The existence of both compressive and rarefactive solitons is observed from Freja satellite [7] in the auroral zone and also by Cairns et al. [6] with non-thermally distributed electrons and cold ions. Our motivation is to establish the generalized form of the Sagdeev potential including positive and negative ion drifts and their different ionic temperatures with the first and second order solitary wave solutions. In many space and laboratory plasmas, non-thermal electron distribution is observed where electron tail is produced by wave damping. We shall develop in future a plan for a third order solitary wave solution near the critical density of the negative ion with non-thermal electron together with positive and negative ions and electron drifts.

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Figure Captions

Fig.1: Structures of first (ϕ_1) and second (ϕ_2) order solitary wave solutions versus η with the variation of negative ion concentration (n_{jo}) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$, $Q = 4$ and $\beta = 0.2$.

Fig.2: Structures of first (ϕ_1) and second (ϕ_2) order solitary wave solutions versus η with the variation of the temperature of positive (σ_i) and negative (σ_j) ions for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $n_{jo} = 0.1$ and $\beta = 0.2$.

Fig.3: Structures of first (ϕ_1) and second (ϕ_2) order solitary wave solutions versus η with the variation of the non-thermal electron parameter (β) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and $n_{jo} = 0.1$.

Fig.4: First (δ_1) and Second (δ_2) order width verses negative ion concentration (n_{jo}) with a variation of the temperature of positive (σ_i) and negative (σ_j) ions for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$ and $\beta = 0.2$.

Fig.5: First (δ_1) and Second (δ_2) order width against negative ion concentration (n_{jo}) with the variation of the non-thermal electron parameter (β) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$.

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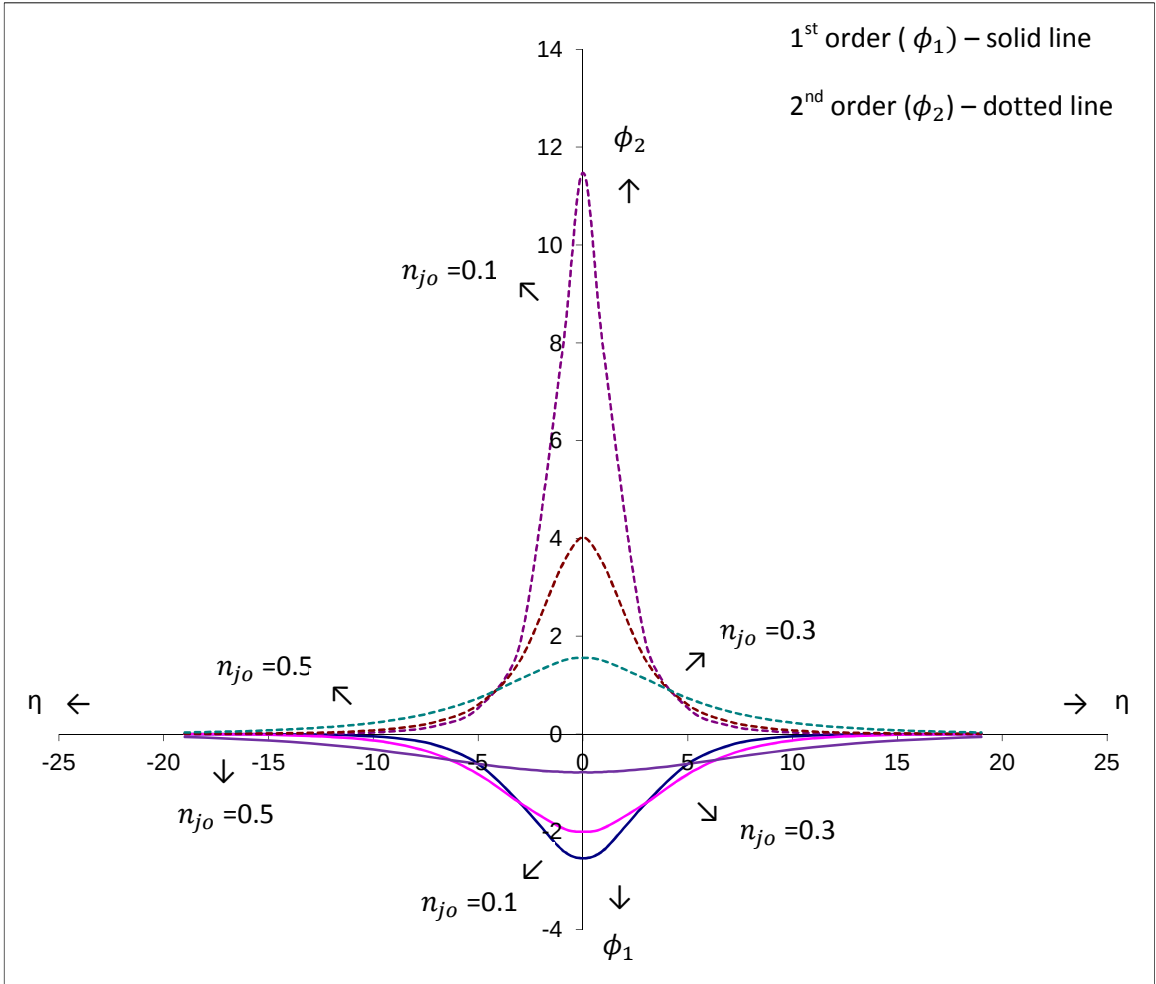


Fig.1

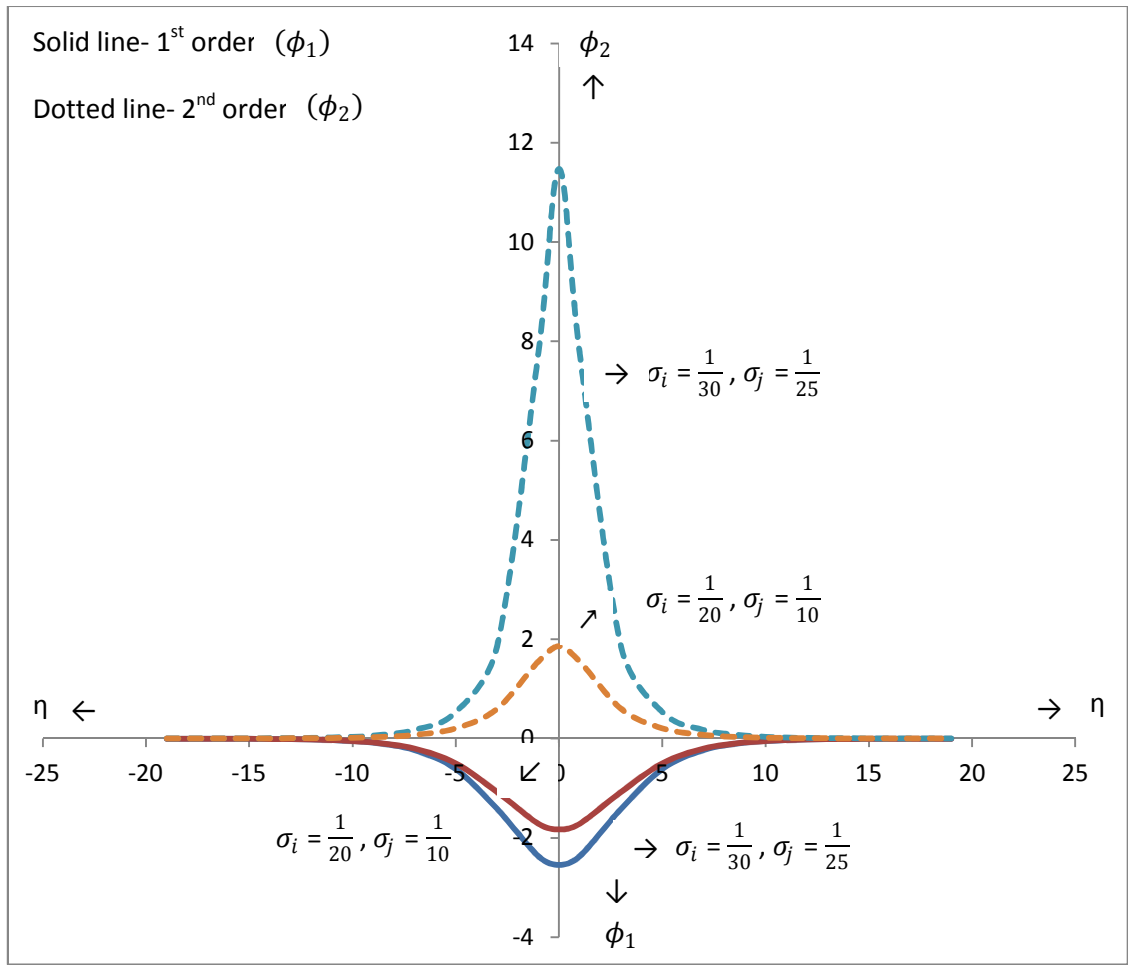


Fig.2

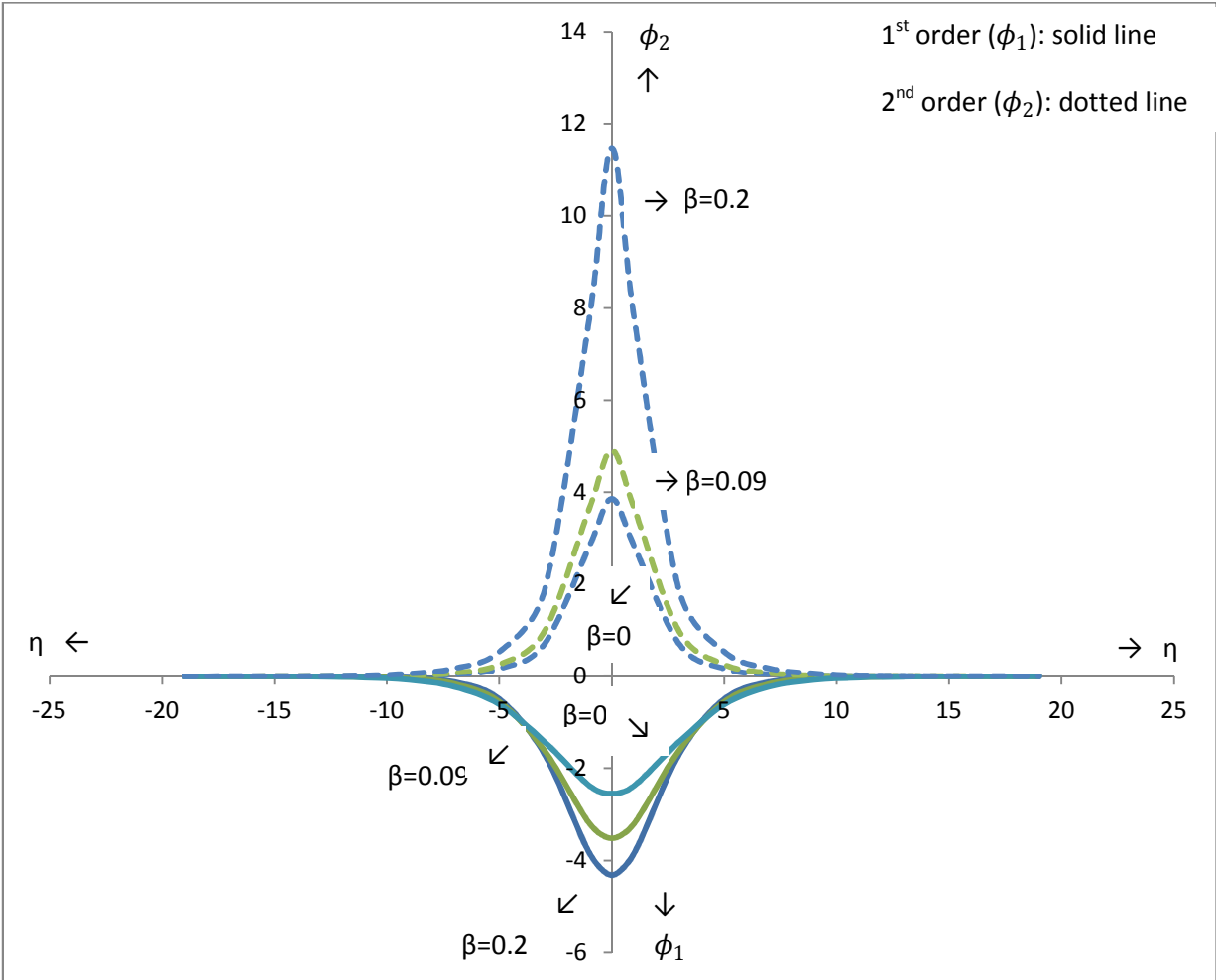


Fig.3

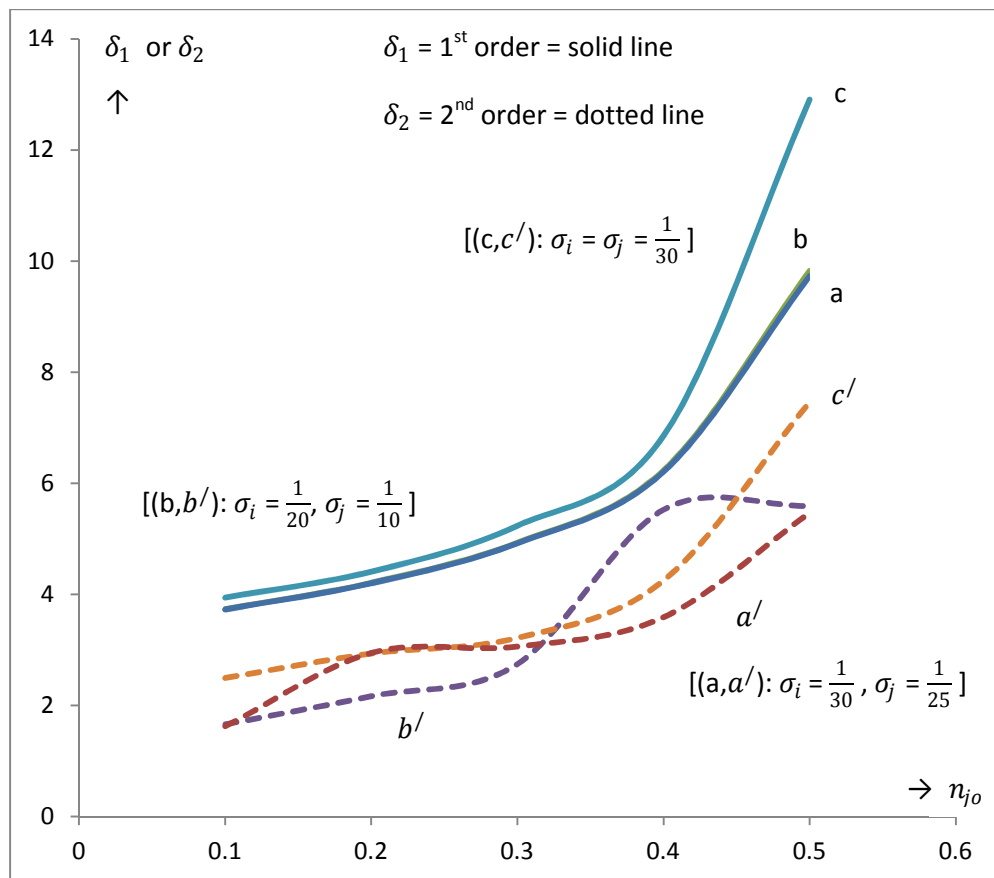


Fig.4

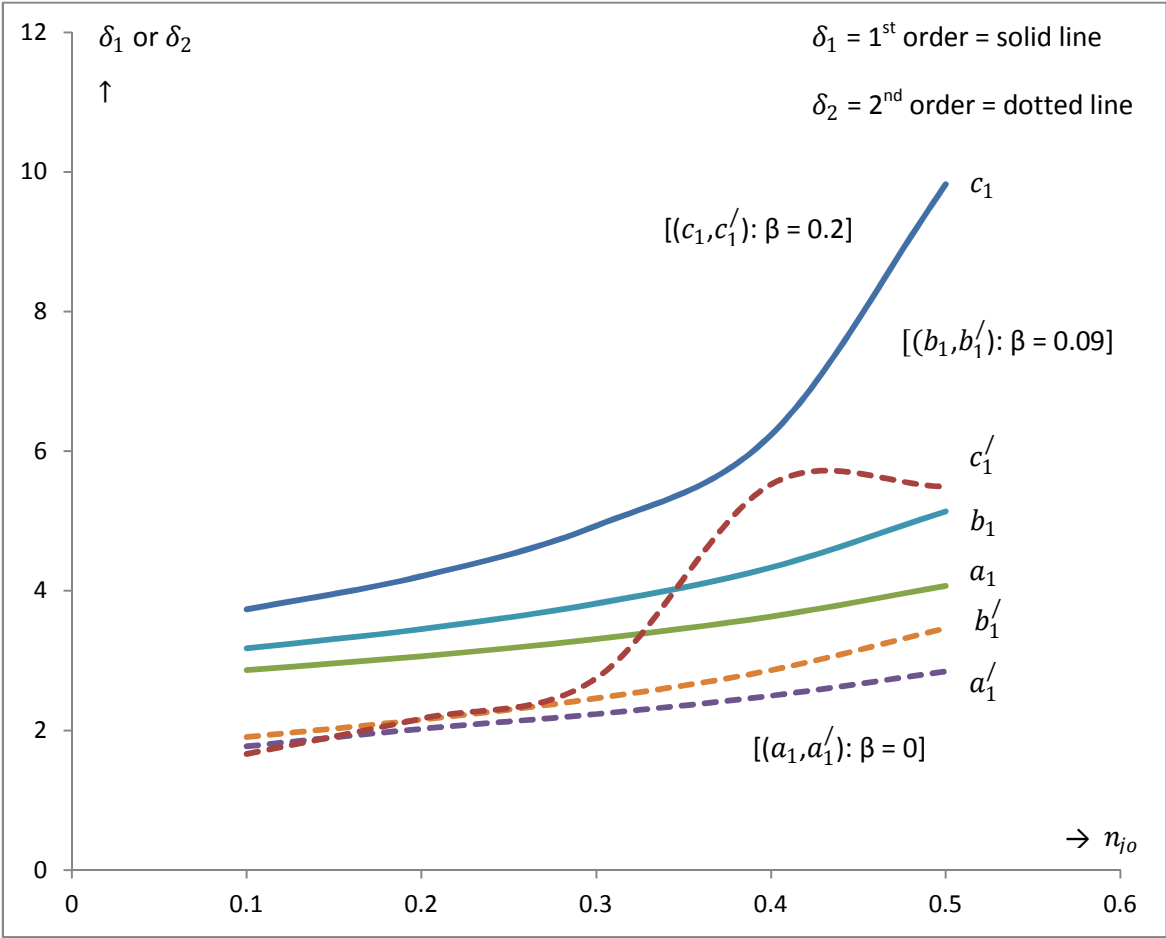


Fig.5