Bianchi Type-V Inflationary Model in the Scale-Covariant Theory of Gravity

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In this paper, we investigate an inflationary cosmological model of Bianchi type V filled with a perfect fluid within the framework of scale-covariant theory of gravity. Exact solutions of the field equations are obtained by using an exponential form of the average scale factor, derived by applying a special law of variation of Hubble parameter. The interesting feature of the model is that it has no finite singularity. The anisotropic cosmological model is accelerating with uniform exponential expansion and tends to isotropy for large time. The physical and kinematical behaviors of the model are discussed.

1. Introduction

Inflation, the stage of accelerated expansion of the universe, nowadays is attracting the attention of many cosmologists. Guth [1] first proposed inflationary model in the context of grand unified theory, which has been accepted soon as a model of early universe. Einstein's general relativity originally constructed as a theory of gravitation. Owing to the scaling behavior exhibited in high-energy particle scattering experiments, there has been considerable interest in manifestly scale-invariant theory. Such theories are considered valid only in the limit of high energies or vanishing rest masses. It is held that in elementary particle theories, rest masses are considered constants and that scale invariance is generally valid only when the constant rest-mass condition is relaxed. By associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space time distances, Canuto et al. [2] formulated a scale-covariant theory of gravitation, corresponding to each dynamical system of units is a gauge condition that determines the otherwise arbitrary gauge function. For gravitational units, they have chosen the gauge condition so that the standard Einstein's equations are recovered. Assuming atomic units, derivable form atomic dynamics, to be distinct from gravitational units, they imposed different gauge condition. It is suggested that Dirac's large number hypothesis be used for the determination of this condition so that the gravitational phenomenon can be described in atomic units. This theory is a viable alternative to general relativity which allows a natural interpretation of the possible variation of the gravitational constant G [3,4]. Beesham [5] discussed power asymptotic singularities in the scale-covariant theory of gravitation with special attention to the Friedmann model and Kasner model, and has generalized the corresponding relativistic results. Reddy et al. [6] have presented an LRS Bianchi type-I cosmological model with a negative constant deceleration parameter. Reddy et al. [7] considered Kaluza-Klein space-time in the presence of a perfect fluid and have obtained a cosmological model in five dimensions with negative deceleration parameter by applying a special law of variation of hubble parameter. Shri Ram et al. [8] have presented spatially homogeneous Bianchi type V cosmological model with power law expansion in scale-covariant theory of gravitation.

In this paper, we obtain a spatially homogeneous Bianchi type V space-time in the presence of a perfect fluid with exponential form of the average scale factor within the framework of scale-covariant theory of gravity. We present the metric and field equations and derive the exponential form of the average scale factor by applying a special law of variation of Hubble's parameter.Then we obtain exact solutions of the field equations. The detailed study of physical and kinematical properties of accelerated expanding cosmological model has been carried out.

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2. Metric and Field Equations

Canuto et al. [2] formulated the scale-covariant theory of gravity by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances. In this theory, Einstein's field equations are valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by the conformal transformation

$$\overline{g}_{\mu\nu} = \phi^2 g_{\mu\nu} \tag{1}$$

Where the gauge function ϕ is a function of coordinates. A bar denotes gravitational units and unbarred denotes atomic units. Using Eqn. (1), Canuto et al. [2] transformed the usual Einstein's field equations into

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + f_{\mu\nu}(\phi) = -8\pi G(\phi)T_{\mu\nu} + \Lambda(\phi)g_{\mu\nu} \tag{2}$$

Where

$$\phi^2 f_{\mu\nu} = 2\phi\phi_{;\mu\nu} - 4\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}(\phi\phi^{;\mu}_{;\mu} - \phi^{,\mu}\phi_{,\nu}) \tag{3}$$

Here semicolon denotes covariant differentiation and comma denotes ordinary differentiation. For a perfect fluid distribution, the energy momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - pg_{\mu\nu} \tag{4}$$

Where, ρ is the energy-density of matter, p the isotropic pressure v^μ is the four velocity vector satisfying $v^\mu v_\mu = 1$.

The general metric for a spatially homogeneous and anisotropic Bianchi type V metric is of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2] \tag{5}$$

Where, A, B and C are scale functions and m is a constant.

In comoving coordinates, the field Eqns. (2)-(4), for the metric in Eqn. (5), explicitly give the following set of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} - 2\frac{\dot{A}}{A}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} - 2\frac{\dot{B}}{B}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} - 2\frac{\dot{C}}{C}\frac{\dot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} = -8\pi Gp \tag{8}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \frac{\ddot{\phi}}{\phi} + 3\frac{\dot{\phi}^2}{\phi^2} = 8\pi G\rho \tag{9}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{10}$$

The energy conservation equation, which is a consequence of the field equations, is given by

$$\dot{\rho} + (\rho + p)v^\mu_{;\mu} = -\rho\left(\frac{\dot{G}}{G} + \frac{\dot{\phi}}{\phi}\right) - 3p\frac{\dot{\phi}}{\phi} \tag{11}$$

For the metric in Eqn. (5), this equation reduces to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \rho\left(\frac{\dot{G}}{G} + \frac{\dot{\phi}}{\phi}\right) + 3p\frac{\dot{\phi}}{\phi} = 0 \tag{12}$$

An overdot denotes ordinary differentiation with respect to cosmic time t.

We now define certain physical and kinematical parameters. For the metric (5), the expansion scalar (θ), the shear scaler (σ) are given by

$$\theta = v^\mu_{;\mu} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{13}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6} \tag{14}$$

The generalized mean Hubble's parameter H is defined by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{15}$$

Where, $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's parameters in the directions of x, y and z respectively. The volume scalar V and the average factor a are defined as

$$V = a^3 = ABC \tag{16}$$

From Eqns. (13), (15) and (16), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{17}$$

An important observational quantity is the deceleration parameter (DP) q, which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{18}$$

The sign of q indicates whether the model inflates or not. The positive sign corresponds to standard decelerating model whereas the negative sign indicates inflation.

From Eqns. (6)-(9), we obtain the expressions for energy density and pressure in terms of kinematical parameters as

$$8\pi G\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} - \frac{\ddot{\phi}}{\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 + 3H\frac{\dot{\phi}}{\phi} \tag{19}$$

$$8\pi Gp = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} - \frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 - H\frac{\dot{\phi}}{\phi} \tag{20}$$

Integration of Eqn. (10) yields

$$A^2 = BC \tag{21}$$

by absorbing the integration constant in the function B or C.

3. Solution of Field Equations

We follow the technique used by Saha and Rikhvitsky [9] to solve the field Eqns. (6)-(9) in quadrature forms. Subtracting Eqn.(7) from (6), Eqn. (8) from (7) and Eqn. (8) from Eqn. (6), respectively, and integrating the results, we obtain

$$\frac{B}{A} = d_1 \exp\left(k_1 \int \frac{dt}{a^3\phi^2}\right) \tag{22}$$

$$\frac{C}{B} = d_2 \exp\left(k_2 \int \frac{dt}{a^3\phi^2}\right) \tag{23}$$

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{dt}{a^3\phi^2}\right) \tag{24}$$

Where, $d^{'s}$ and $k^{'s}$ are constants of integration. After straightforward calculations, the metric functions A, B and C can be obtained [8] as

$$A(t) = a \tag{25}$$

$$B(t) = ba \exp\left(X \int \frac{dt}{a^3\phi^2}\right) \tag{26}$$

$$C(t) = b^{-1}a \exp\left(-X \int \frac{dt}{a^3\phi^2}\right) \tag{27}$$

Where, b and X are again arbitrary constants.

From the above equations we can determine the scale functions A, B and C if the average scale factor a(t) and the gauge function ϕ are given functions of cosmic time t. For further simplification, we assume that the gauge ϕ varies inversely proportional to the average scale factor i.e.,

$$\phi = \frac{k}{a} \tag{28}$$

k being a constant. Then Eqns. (25), (26) and (27) reduce to

$$A(t) = a \tag{29}$$

$$B(t) = ba \exp\left(\frac{X}{k^2} \int \frac{dt}{a}\right) \tag{30}$$

$$C(t) = b^{-1}a \exp\left(\frac{-X}{k^2} \int \frac{dt}{a}\right) \tag{31}$$

Now, to derive an appropriate ansatz for the special law of variation of Hubble's parameter that yields a constant value of DP. Here we make an assumption that the Hubble parameter H is related to the average scale factor by the relation

$$H = la^{-n} \tag{32}$$

Where, $l > 0$ and $n \geq 0$ are constants. Such type of relation has already been considered by Berman [10], Berman and Gomide [11] for solving field equations in FRW models. It may be noted that through the current observations of SNe Ia and CMB favour accelerating models, but they do not altogether rule out the decelerating ones, which is consistent also with these observations.

From Eqns. (17) and (32), we obtain

$$\dot{a} = la^{-n+1} \tag{33}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1} \tag{34}$$

From Eqns.(18), (33) and (34), we find that

$$q = n - 1 \tag{35}$$

Thus, we see that q is constant. From Eqn. (33), we obtain

$$a = (nlt + c_1)^{\frac{1}{n}}, \qquad n \neq 0 \tag{36}$$

$$a = c_2 \exp(lt), \qquad n = 0 \tag{37}$$

Where, c_1 and c_2 are constants of integration. Without loss of any generality, we take $c_1 = 0$ and $c_2 = 1$. Shri Ram et al. [8] have presented a decelerated expanding cosmological model of Bianchi type V filled with perfect fluid in scale covariant theory of gravitation by using power law form of the average scale factor $a(t)$ given by Eqn. (36).

Eqn. (37) gives the exponential form of the average scale factor. Inserting Eqn. (37) into the integrals on right side of Eqns. (29)-(31) and then integrating, we obtain the solution for the scale factors as

$$A(t) = e^{lt} \tag{38}$$

$$B(t) = be^{lt} \exp\left(\frac{-X}{lk^2}e^{-lt}\right) \tag{39}$$

$$C(t) = b^{-1}e^{lt} \exp\left(\frac{X}{lk^2}e^{-lt}\right) \tag{40}$$

The solution for the gauge function ϕ is given by

$$\phi = ke^{-lt} \tag{41}$$

The expansion scalar θ and shear scalar σ has values given by

$$\theta = 3l \tag{42}$$

$$\sigma^2 = \frac{2X^2}{k^4}e^{-2lt} \tag{43}$$

The directional Hubble's parameters and average Hubble parameter are given by as follows

$$H_1 = l \tag{44}$$

$$H_2 = l + \frac{X}{k^2}e^{-lt} \tag{45}$$

$$H_3 = l - \frac{X}{k^2}e^{-lt} \tag{46}$$

$$H = l \tag{47}$$

From Eqns. (19)-(20), we can find the expression for energy density and pressure involving the time varying gravitational constant G. We recall that in most of the time varying G cosmologies, G is a decreasing function of time. The possibility of an increasing G has also been discussed by Levit [12]. Beesham [13] has discussed the passibility of a creation field with G proportional to a power function t. Sistero [14] presented exact solutions for zero curvature Robertson-Walker cosmological models with G proportional to a power function of the average scale factor a. Here we obtain a physically realistic model of the universe by assuming a time-varying G of the form

$$G = \lambda a^2 = \lambda e^{2lt} \tag{48}$$

Where λ is a constant of proportionality. Using Eqn.(48) into Eqns. (19)-(20), we obtain energy density and pressure given by

$$8\pi\rho = \frac{2l^2}{\lambda}e^{-2lt} - \frac{1}{\lambda}\left(\frac{2X^2}{k^4} + 3m^2\right)e^{-4lt} \tag{49}$$

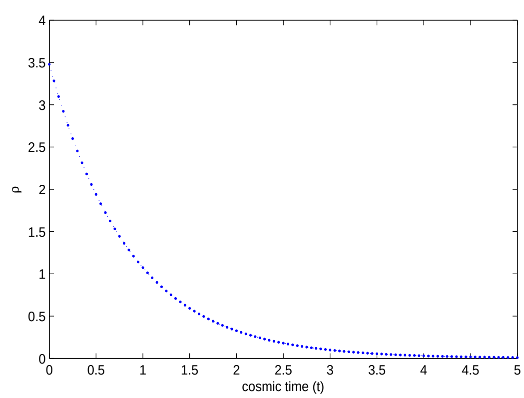

FIG. 1: Variation of energy density ρ with time t.

$$8\pi p = -\frac{2l^2}{\lambda}e^{-2lt} - \frac{1}{\lambda}\left(\frac{2X^2}{k^4} - m^2\right)e^{-4lt} \tag{50}$$

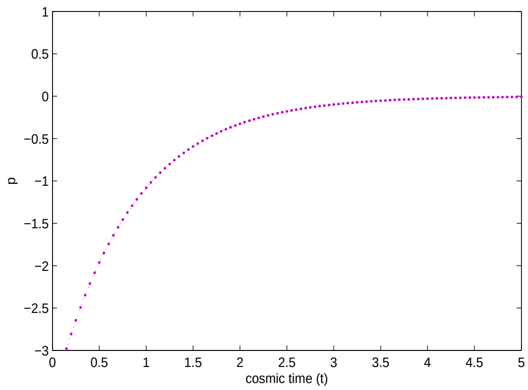

FIG. 2: Variation of pressure with time t.

As $q = -1$, the model presented by a set of inflationary solutions and which is in the phase of accelerated expansion. Thus, the model is consistent with the recent observations of Suopernovae Ia requiring the present universe to be accelerating [15,16]. The behaviors of energy density and pressure are shown in Figs. 1 and 2, respectively. We observe that energy density is positive and the pressure is negative for all finite time which corresponds to accelerated expansion of the universe.

4. Physical and Kinematical Behaviors of the Model

We now discuss the physical and kinematical behaviors of the cosmological model. We observe that spatial volume V and three scale factors are constants at $t = 0$. At this point, each of the following have constant values: the energy density, pressure, expansion scalar, shear scalar and gauge function. This means that model has no finite singularity at $t = 0$. The model is well behaved for $-\infty < t < \infty$. This shows that the universe is infinitely old and has exponential inflationary phase. The directional Hubble parameters are time dependent while the mean Hubble parameter is constant. The expansion scalar is constant throughout the time of evolution right from the beginning. The physical and kinematical quantities are all decreasing functions of time. As $t \to \infty$, the spatial volume is infinite. The energy density and pressure become zero. Thus, the model gives an empty space for large time. Since $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$ the model is isotropic for large time. Thus, the universe starts expanding from a big bang singularities in the infinite past and expands exponentially with constant rate of expansion and finally it approaches to isotropy for large time.

5. Conclusion

We have studied the spatially homogeneous and anisotropic Bianchi type V cosmological model with negative deceleration parameter in scale-covariant theory of gravitation in the presence of a perfect fluid. We have applied a special law of variation for Hubble's parameter to derive an exponential form of the average scale factor that yields the deceleration parameter equal to -1. Then using the exponential form of the average scale factor we have obtained the solutions of the scale factors. The corresponding cosmological models represents an exponentially expanding universe having singularity in the infinite past (i.e., $t \to -\infty$). The scale factor admit constant values at early times of the universe ($t \to 0$), afterwards scale factors starts increasing with the increase of time without showing any type of initial singularity and finally diverge to infinity as ($t \to \infty$). This shows that the universe expands exponentially approaching to infinite volume.

The expansion scalar for these scale factors exhibits the constant value which is $\theta = 3l$. This shows uniform exponential expansion for all time i.e., the universe expands homogeneously. Since $H = l$, the mean Hubble parameter is constant, whereas directional Hubble parameters are dynamical. The deceleration parameter $q = -1$ implies accelerating expansion of the universe as one can expect for exponential volumetric expansion. The matter pressure and energy density are monotonically decreasing function of time. The model is consistent with the present observations. Perlmutter et al. [15] and Riess et al. [16] have proved the decelerating parameter of the universe in the range $-1 \leq q \leq 0$, and the present day universe is undergoing accelerated expansion.

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