On a Recent Variant of Schrödinger's Cat Experiment

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Recently, a stimulating paper by Z. Yun appeared in which he proposes a variant of Schrödinger's cat experiment aiming to put into evidence that superpositions of macroscopically different quantum states do not occur within quantum mechanics and that no measurement is necessary to induce the reduction of the wave packet. Obviously such statements require a critical analysis of the position of the author concerning what happens when a microsystem interacts with a macroscopic one, and, in particular, concerning the measurement problem. The analysis of the above mentioned experiment will allow us to discuss in a quite simple but detailed way many aspects of the problem of the macro-objectification of quantum properties. We will show that Yun's experiment does not give rise to any problem if one takes the collapse theory position concerning quantum mechanics[2], while it gives rise precisely and exclusively to the well known problems when other interpretations are considered. Our analysis will require a reconsideration of the appropriate description of decay processes within quantum theory, a theme we have discussed in all details many years ago. Finally, since the author argues that the reduction process is, in some sense, incorporated in Feyman's path integral approach to quantum mechanics, we will briefly discuss this point.

1. Introduction

Schrödinger's cat history, and the problems it raises for the standard interpretation of quantum mechanics (and even for most of its alternatives) is very well known to all physicists. Recently Z. Yun has presented a paper [1]: *Path Integral approach on Schrödinger's cat* and has circulated another one: *Quantum process of an isolated system*, in which he argues, on the basis of a smart modification of the original proposal, that "Schrödinger's cat is not in a quantum superposition of alive and dead", that "the collapse of the wave function of an isolated system is possible without external observer" and that resorting to Feynman's path integral formulation of the theory "one can prove that standard quantum theory does not exhibit any difficulty in connection with measurement processes".

For people interested in the foundations of quantum mechanics it is obvious that such statements deserve a detailed critical investigation involving the different views on wave packet reduction which have been discussed for about 100 years and that we will briefly revisit here. We warn the reader that few results of this paper are new but that, analyzing Yun's proposal, we will have the opportunity of stressing once more the critical points of quantum mechanics and of putting into evidence some specific features of the various positions about them. In our intentions, the analysis should turn out to be useful and illuminating concerning some subtle aspects of the measurement problem which, in general, are not fully appreciated, particularly by the non experts in the field. The conclusion of our analysis will be that Yun's proposed experiment must be looked at by taking strictly into account the precise interpretation of the theory one adheres to and that, if one follows this line, the experiment itself does not raise any problem besides those (already familiar to everybody) which affect the various positions about quantum theory.

Moreover, from what follows it will emerge clearly that some recent proposals [2-5] of solving the measurement problem (within a Hilbert space context) modifying the evolution equation by adding to it a stochastic collapse mechanism are those that most naturally overcome the alleged difficulties connected with Yun's analysis.

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[2] As we will see, also the adoption of the Bohmian point of view solves Yun's dilemma. However, here we prefer to stick to a purely Hilbert space description of physical processes.

2. Yun's Proposal and Questions

Let us begin by summarizing the proposal of the author, which he qualifies as a "new" version of Schrödinger's cat experiment[3]. We attach here the figure that he has presented and which contains all essential elements of the game (Fig. 1).

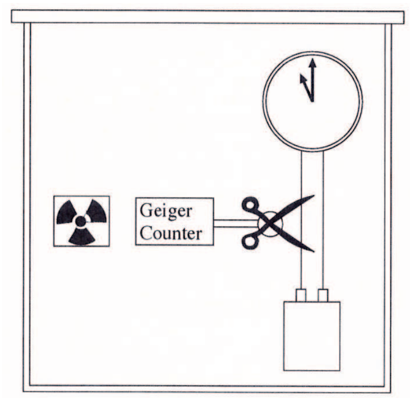

FIG. 1: Yun's experimental set-up.

As usual, one considers an unstable system with a certain life time which emits, when it decays, particles activating a Geiger counter. The counter is connected to a pair of scissors in such a way that, when the detector is activated, they snap the power line which keeps a clock running, so that it will cease to do so and its pointers will stop moving.

In the author's words: "Suppose the half life of a radioactive atom is one hour. Set the time of running clock to 11:00 and cover the lid of box. Let's uncover the lid at 12:00. According to the Copenhagen interpretation, before we make the observation at 12:00, the state of the clock is in quantum superposition of $|running>$ and $|stopped>$ states. In many world interpretation the universe of co-existing $|running>$ and $|stopped>$ states splits into the universes of one of the two states when we open the lid at 12:00". And now, according to Yun, a (supposedly) crucial problem emerges: "suppose when we make the observation at 12:00 we discover that the clock stopped at 11:30", an hypothesis which leads naturally to the question on which all his subsequent arguments are based:

> *In this case, do we still have to believe that the collapse of wave function (or splitting world in many world interpretation) occurred at 12:00, at the moment we open the lid and make an observation, as Copenhagen interpretation (or many world interpretation) insists? If the collapse of the wave function occurred at 12:00, then what should we call the physical event which occurred at 11:30?*

Note that the considered situation is slightly more subtle than the one usually connected with the measurement process. In fact, just to mention an example, if no clock is taken into account as in Schrödinger's case and one accepts that it is consciousness which induces the reduction, finding the cat dead at 12:00 does not allow to raise any question concerning the time at which it died. On the contrary, in the situation we are considering, we have a record explicitly referring to 11:30.

The author deals with this problem in Section II of his paper, and he makes the following (in our opinion to a large extent vague and inappropriate) claims that will be the main object of our analysis[4]:

- According to the Copenhagen interpretation (or many worlds interpretation) the wave function of an isolated system never collapses (or the word does not split) until the observer makes an observation, and, accordingly, the clock is in a superposition of running and stopped states and this state collapses to one of them when we observe it at 12:00. Then

[3] It is interesting to remark that H. Everett, already in 1957 had considered [6], in connection with Wigner's friend example i.e., with the assumption of reduction by consciousness, a formally rather similar, but conceptually different, situation in which a record of an outcome exists before a specific observer becomes aware of it. The idea goes as follows: an observer A performs a measurement, he gets one of various possible outcomes and he records his result in a notebook. A week after the registration a second observer B who knows the wavefunction of the measured system, of the measuring device and of the notebook looks at the notebook. Here a quite problematic situation emerges. For B, the various entries of the notebook have nonzero probabilities referring to results other than the recorded one (which might have also an extremely small probability of occurrence according to the theory). But A has written which was the outcome he had found one week before. As we have stated, the situation is similar but conceptually different from the one considered by Yun, since, in his opinion, standard quantum theory assumes that reduction takes undoubtedly place when any conscious observer (in our case A) becomes aware of the outcome.

[4] The statements that follow have been formulated by Yun, literally in the terms reported here.

do we always discover the clock either running or stopped at 12:00? This does not make sense, which yields a clear evidence that something physical event occurred at 11:30. That event must be the collapse of the wave function. So, a wave function of an isolated system can collapse without external observer. The collapse of the wave function does not require a conscious observer.

- Another possible scenario is that before we open the lid at 12:00, the state inside the box is a linear combination of an infinite number of quantum states corresponding to different timings of stopping clock:

$$\begin{aligned} |stopped\ at\ 11:01> &+ |stopped\ at\ 11:02> ... + |stopped\ at\ 11:30> + ... \\ &+ |stopped\ at\ 12:00> + |running> \end{aligned} \tag{1}$$

and when we open the lid at 12:00, the state collapses to one of them, for example, $|stopped\ at\ 11:30>$

- However, by using the path integral approach to quantum theory one can prove that there should be no quantum superposition of $|running>$ and $|stopped>$ clock states in this experimental setup.

3. On the Copenhagen Interpretation

The first necessary clarification concerns the author's position with respect to the Copenhagen interpretation of Quantum Mechanics, a point which has not been presented with the necessary care[5]. In fact, he considers an essential part of this interpretation that reduction takes place precisely and exclusively when a conscious observer has a definite perception about the outcome.

3.1. Bohr and Heisenberg

To make clear the point we have just made, we will start by stressing that, even though it has become a common (and inappropriate) attitude to include in the expression"the Copenhagen interpretation" various - even conceptually rather different - positions, there is no doubt that some basic distinctions are necessary. For example, concerning wave packet reduction, one should keep quite distinct the points of view of Bohr and Heisenberg, on the one side, and those of von Neumann [7], London and Bauer [8], and (the first) Wigner [9], on the other.

To stress this point we recall that Earman and Shimony [10], in their accurate analysis of the historical development of quantum mechanics, have reached the conclusion that the correct interpretation of most of the writings of Bohr and Heisenberg can be summarized in the claim that the final state of the pointer reading is definite, though unknown, when the final state of the microsystem plus apparatus is reached - but before registration upon the consciousness of the observer. It is extremely easy to present a long list of statements by the two above mentioned scientists pointing out that, within standard quantum mechanics and with reference to a measurement process, one does not need to call into the game a conscious observer.

In brief, the crucial step of the process, i.e., wave packet reduction, takes place when a microsystem in a superposition of different states interacts with a macrosystem in a ready state in such a way that the different states of the superposition lead to macroscopically different situations of the macrosystem itself (typically, the positions of its pointer). This is strictly connected with the repeated claims, in particular by Bohr, that the existence of (vaguely defined) classical systems and that their description in the usual classical language are logical prerequisites for the very formulation of quantum theory. Accordingly, the measuring process itself must be left unanalyzed.

I will characterize this position (which, as I will stress in what follows, I consider basically inconsistent and fundamentally imprecise) concerning the measurement problem as "The Copenhagen interpretation 1". To give more strength to my opinion that this is the appropriate interpretation of the so called "Copenhagen Interpretation" without any further characterization, I consider it useful to recall a clear cut sentence by M. Jammer [11], which has made clear what must go under this name:

[5] As we will see, an analogous remark applies to his position with respect to the Many Worlds Interpretation.

> *According to the Copenhagen interpretation in any measurement the state of the observed object is affected by the macroscopic measuring instruments whose existence and mode of operation, though necessary for the possibility of observing quantum mechanical processes, are not accounted for by the quantum theory itself but regarded as logically preceding the theory. It was further assumed that these macroscopic devices could be observed with arbitrary accuracy and that the very act of reading the pointer or the registration of the result had no effect on the outcome of the measurement.*

3.2. von Neumann and Wigner

The specification "The Copenaghen Interpretation" acquires a completely different status in the perspective of J. von Neumann [7], London and Bauer [8], and Wigner [9] (in his first times). For these scientists reduction of the wave packet takes place precisely and exclusively when a conscious observer becomes aware of the outcome of the measurement (I will characterize this point of view as "The Copenhagen interpretation 2"). Such a position presents a serious conceptual limitation deriving from its not making clear what the expression "conscious" actually means. Moreover, it implies that the universe evolved in a completely different way before the appearence of consciousness in the cosmos, as lucidly pointed out by Bell [12]:

> *What exactly qualifies some subsystems to play the role of measurers? Was the world wave function waiting to jump for thousands of millions of years until a single-cell living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer - with a Ph.D.?*

We consider also useful to call the attention of the reader on the fact that Wigner, after having championed the reduction by consciousness for a certain time, has subsequently changed radically his mind, as lucidly stressed by M. Esfeld [13] in his essay on *Wigner's view of Physical Reality* in which one reads:

> *An analogous consideration applies to Wigner's later change of mind. His idea contains a viable option: If one considers it to be inappropriate to take recourse to the mind or consciousness of an observer in the interpretation of quantum mechanics and if one regards state reductions as objective physical events, it is reasonable to envisage a modification of the Schrödinger dynamics. The aim then is to achieve a more general dynamics that encompasses state reductions. The most elaborate suggestion in this respect goes back to Ghirardi, Rimini and Weber.*

4. On the Many Worlds Intepretation

Just in the same way in which Yun has not adequately articulated his presentation of the Copenhagen Interpretation, he has not taken into account that the general expression "Many Worlds" embraces an extremely rich set of distinct positions. The only point which is common to all of them is the insistence on the fact that all changes in time are strictly ruled by the linear Schrödinger's equation. In particular, no reduction takes place, and, consequently, measurement processes do not play any role and there is no classical realm to be used to interpret the theory. Said differently, the guiding idea underlying this approach is that the vaguely defined suspension of Schrödinger's equation when a measurement takes place and the reduction of the wave packet occurs, can be avoided by assuming that it is just an illusion that a specific choice is made among the many macroscopic possibilities contained in the superposition of the entangled system-apparatus state.

Let us now be more specific on this interpretation. Its first formulation is due to Everett [6]; subsequently it has been championed by DeWitt [14] but in a remarkably modified way. In what follow we will invert the order of presentation of the two approaches, mainly since, concerning the problem of the proliferations of the worlds, Yun undoubtedly sticks strictly to the position of DeWitt and ignores the original proposal.

4.1. DeWitt

The basic idea of DeWitt is that the various terms in an entangled superposition should be interpreted as showing that the universe branches into a number of different worlds. To stress his specific attitude we cannot do better than to make reference to Davies [15], who quotes (without giving the reference) the following sentence by him:

> *Our universe must be viewed as constantly splitting into a stupendous number of branches ... Every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world into myriad copies of itself.*

This implies that, for example, if one applies this view to the final state ensuing from a measurement of a spin component of a spin 1/2 particle by an apparatus A (with obvious meaning of the symbols):

$$\frac{1}{\sqrt{2}}[|\uparrow> + |\downarrow>]|A_{ready}> \rightarrow \frac{1}{\sqrt{2}}[|\uparrow> |A_{\uparrow}> + |\downarrow> |A_{\downarrow}>] \quad (2)$$

one has an actual branching into two worlds during the course of the measurement interaction: in one of them the spin is pointing "up" and the apparatus has registered this fact by ending with its pointer pointing "up" and in the other the spin is "down" and the apparatus pointer points "down".

There are other important aspects which are ignored by Yun but which require to be discussed for an appropriate evaluation of his paper and statements. The first one has to do with the probability of occurrence, in a single world, of the various outcomes when the same experiment is repeated many times. This problem, in order to avoid misinterpretations, requires to be absolutely precise concerning the specific modalities of the branching.

Let us start by considering, as Yun seems to do, the case in which one assumes that the branching gives rise precisely to one universe for each possible outcome of the measurement. To depict the difficulties which emerge we will follow the lucid analysis recently performed by H. Putnam [16], even though the same problem had already been taken into account by Everett, by DeWitt and by Ballantine [17]. The argument goes as given in the sequel[6].

Suppose one is interested in the outcomes of measuring a dichotomic observable Ω of an individual quantum system and let us denote its eigenvalues as -1 and +1. For what concerns the state of the microsystem before the measurement let us assume that it is in a superposition $|\phi\rangle$ of the associated eigenstates, with coefficients α and β, whose moduli squared are different. Just to be specific, suppose that $|\alpha|^2 = 1/5$ and $|\beta|^2 = 4/5$.

Let us now suppose that, in a specific universe - typically ours, the same experiment, i.e., the measurement of Ω, is performed simultaneously many times on many identically prepared systems, i.e., all in the state $|\phi\rangle$. Due to the observation by the first observer, O_1, the universe splits in two universes, in one of which the outcome -1 and in the other the opposite one has been obtained. But we have to take into account also the second observer, O_2, who is playing the same game. Due to his action the universe bifurcates, so that we end up with 4 universes, in 1 of which both (replicas) of the observers have perceived the outcomes (-1,-1), in 2 of which they have obtained different outcomes, and, finally, in the last universe they have got the outcomes (+1,+1). Now the third observer O_3 enters the game. In this case we have 8 universes distributed, for what concerns the triple of possible outcomes, in the following way: in one universe we have three observers who have got the set (-1,-1,-1) of outcomes, in 3 universes they have got two outcomes +1 and one -1, in 3 two outcomes -1 and one outcome +1, and finally in the last universe the three identical outcomes (+1,+1,+1). Accordingly, the 8 replicas of the three observers end up in different universes in which the relative appearance of the outcomes +1 and -1 is different from the one implied by quantum mechanics. Actually the ratios of the outcomes (-1) and (+1) in the 8 universes, take the values (3/0, 2/3, 3/2 *and* 0/3), all of which differ appreciably from the ratio implied by quantum theory, i.e., $[(1/5)/(4/5) = 1/4]$. Obviously, with only 3 experiments the statistics is too low in order that one can attach to it any meaning. But this is not the whole story; actually, the situation becomes more and more embarrassing with the increasing of the number N of repetitions of the same experiment. For extremely large N, the number of universes in which one has a ratio of the two outcomes near to the quantum one becomes practically zero, with respect to the total number 2^N of universes.

So, the simultaneous repetition of identical experiments in our universe generates a set of universes and practically in all of them the probabil-

[6] It has to be stressed that this difficulty characterizes both the case in which the splitting of the world is induced by the micro-macro interaction or the one in which it is due to the act of the conscious perception by an observer.

ities of the two outcomes do not agree at all with the quantum ones, i.e., that about $N/5$ of the N observers get the outcome -1, and the remaining ones the opposite outcome.

To correct this absurd situation both Everett and DeWitt have assumed the existence of a "measure" associated with each vector of the Hilbert space which should reflect the probability that is usually associated with that branch of the wave function in the standard theory. Typically, with reference to the state

$$|\Psi >= \alpha|\psi_1 > +\beta|\psi_2 > \quad (3)$$

the measure associated to $|\psi_1 >$ is $|\alpha|^2$ and the one associated to $|\psi_2 >$ is $|\beta|^2$. However both Everett and DeWitt do not assume that the above "measures" are probabilities, but they attribute to them a purely abstract status of mathematical entities. It should be clear that when formulated in these terms the Many Worlds view suffers of many limitations which have been discussed, among others, by d'Espagnat [18] and by Deutsch [19]. This point has, in our opinion, a great relevance and makes clear that the Many Worlds interpretations both of DeWitt and of Everett (see below for a discussion of his views) meet enormous difficulties in accounting for the quantum probabilities.

There is another point which must be stressed. Yun assumes, and he is quite specific about this point, that what leads to the multiplication of the universes is the act of conscious perception by an observer. This reflects simply his opinion (or better the common naive view about the Many Worlds) and cannot be considered as the position of the proponents of the Interpretation.

In particular, DeWitt, to deal with this fundamental aspect, makes appeal to the complex and chaotic phase relations of the terms of the wavefunction in a superposition. From his emphasis on "complexity" one might deduce that the universe splits when the wave function reaches a certain degree of complication. At any rate, given for granted that his statements are rather vague and that he never puts forward a precise quantitative criterion concerning how the splitting of the universes is related to this feature of the "complexity" of the system under consideration, we believe that, if one tries to guess what DeWitt had in mind, one would be inclined to accept that he was more inclined to relate the multiplication of the universes to the the changes that the the state of a microsystem induces (as a consequence of their interactions) in a macroscopic one than to consciousness. Once more, Yun's position is basically at odds with DeWitt picture concerning Many Worlds. Accordingly, the problems that the hypothetical modified Schrödinger's cat example meets *de facto* with such an Interpretation do not differ significantly from those which arise when one takes the position we have denoted as the Copenhagen Interpretation 1.

At any rate, to cut any possible way out, in what follows we will briefly discuss Yun's position with respect to the Many Worlds Interpretation even accepting tentatively that his views about it are actually appropriate (which is not the case).

4.2. Everett

Everett's position is quite different from the one of DeWitt. He puts all the emphasis on the role of the subjective states of the observers, states which are strictly correlated with the various aspects of the universe. The theory, as Squires [20] has stressed, is better characterized as one dealing with many "viewpoints concerning the world" than with "Many Worlds". The problems arise from any attempt to be specific about the meaning and the implications which must be attached to the expression "viewpoints": why we only experience a single view?

The answer is quite natural (in a sense) when one makes appeal to the internal consistency of the situation occurring in connection with the so-called von Neumann's chain. Suppose we have a system in a superposition of different states, which becomes entangled with different states of the pointer of the apparatus, and, subsequently, with different perceptions by an observer and so on. The internal consistency we are referring derives from the fact that if one extends the "chain" to larger and larger systems (such as the observer's consciousness, the information that another guy gets by asking to the observer which result he has seen, and so on ...) each one of the richer and richer states of the superposition, do not give rise to any contradiction since the state in which the pointer points at "37" goes together with the statevector associated to "the observer reads 37", to his friend hearing him to claim that he has seen the pointer pointing at "37", and so on. The same holds, obviously for the terms of the superposition corresponding to other outcomes.

From this perspective, what really matters are the *correlations* between the various properties of the systems entering into the game. So, e.g., when one considers a superposition like the one of the r.h.s. of Eqn. (2), following Everett, one would state that, if the apparatus points at up, then necessarily, the electron has spin up, and similarly for the other possibility. The situation does not differs from the quantum one, exception made for the fact that, in the standard theory, the statement concerning the electron is conditioned on the fact that a spin measurement (with the associated reduction of the wave packet) has been performed, while here all levels (the micro, the macro, the conscious etc.) have the same standing, since no reduction takes ever place. The situation is to some extent supported and clarified by the fact that Everett has characterized his position by basing it on the idea of "a relative state". So, as just stressed with reference to our Eqn. (2), one would state that the state of the electron is $|\uparrow>$ relative to the state $|A_{\uparrow}>$ and so on. In particular, reference has been made by Everett himself (as well as by many other scientists interested in this approach), to *memory states*, which are intended to describe the processes by which we (or computers) store information. This should make plausible that we do not feel our mental states as splitting because, in every branch of the statevector, perfect correlations between our memory states and events that have occurred are present. In some sense, the only superquantum elements of the theory are these memory states.

The reader will have no difficulty in suspecting that the theory should meet serious difficulties in connection with the fact that the decomposition of an entangled state is in no way unique. For instance, in the case of two spin 1/2 particles one might consider two different ways of writing the singlet spin state:

$$|\Psi_{singlet}\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2] \equiv \frac{1}{\sqrt{2}}[|\leftarrow\rangle_1 \otimes |\rightarrow\rangle_2 - |\rightarrow\rangle_1 \otimes |\leftarrow\rangle_2] \quad (4)$$

where we have indicated as $|\leftarrow\rangle$ and $|\rightarrow\rangle$ the eigenstate of σ_x. With reference to the above equation one might then claim that the state of particle 2 is "downarrow" relative to the state "uparrow" of particle 1 and similarly for the other term, but equally legitimately he might claim that the state of particle 2 is "spin up along the x-axis" relative to the state "spin down along the x-axis" of particle 1 and so on.

This analysis leads to conclude that one needs to identify some special basis for the Hilbert space of one of the constituents, and all claims concerning the other constituents are relative to the states of the basis for it.

We do not intend to spend much time to discuss the situation which remains rather unclear at the fundamental level. However we believe that our analysis has been sufficiently exhaustive to show that it is quite inappropriate, as many do, to identify in some sense the positions of DeWitt and Everett by using for denoting them the same name of Many Worlds Interpretations. Moreover, it has to be stressed that also Everett has never made fully explicit that the different viewpoints have to be attached to the consciousness of an observer. Typically, his arguments have been often considered, as already remarked, to refer to a machine equipped with a rich memory, such as a computer. There follows that Yun's position besides contradicting, as we have remarked, the one of DeWitt (even though it shares with it the vague picture concerning the multiplication of the universes), it has almost nothing in common with Everett's view.

4.3. Subsequent developments of the Many Worlds view

Precisely for the difficulties of the two Many Worlds positions we have described in the previous sections, the program has been further analyzed and enriched. This is also due to the fact that the considered approach has raised a lot of interest particularly among cosmologists, for the reason that for them the fact that only Schrödinger's evolution is involved and no mention of observations is made fits very well with the fact that if one considers the wave function of the entire universe, the very meaning of an external observer becomes obscure. We will not go through a list of the many attempts based on DeWitt and/or Everett points of view, we will simply mention important investigations by Lockwood [21], Deutsch [19] and the systematic attempt which goes under the name of the Decoherent Histories [22-24] approach. Concerning the first two cases I would state that the authors have not succeeded in providing a fully consistent and satisfactory characterization of their proposals, and, concerning the last one which has raised recently a lot of interest, we believe that it is inconsistent and does not attain the objectives for which it has been worked out, as we have proven in [25].

We conclude this subsection by mentioning that a further elaboration of the ideas underlying the Many Worlds Interpretation has led Albert and Loewer [26] to the formulaton of what has become known as the "Many Minds Interpretation", in which, in place of a splitting of the universe or a multitude of viewpoints, it is assumed that our brains exhibit a sort of foliation, each sheet corresponding to the different perceptions associated to the various outcomes. This formulation is, in our opinion, much more in agreement with the position of Everett than with the one of DeWitt.

5. A Concise Summary of other Positions about Quantum Theory

Before going on we mention other alternative proposals and interpretations aimed to overcome the measurement problem which have been advanced and have raised a remarkable interest by people involved in foundational issues.

5.1. Collapse theories

The central idea of this approach consists in contemplating that the linear and deterministic evolution of the standard theory has not a universal validity. The basic Scrödinger's equation must be modified by the addition of nonlinear and stochastic terms, which account for the definite features of the reduction process.

As it is obvious, and as it has been stressed by many scientists, macro-objects are characterized by the fact that they correspond to perceptually different locations of (some) of their macroscopic parts (typically their "pointer"), so that it is quite natural to tackle "the preferred basis problem" by assuming that the modified dynamics will strive to make precise the positions of physical objects. Given this, we can be fully precise about the features of collapse models. For simplicity we will make reference to the original proposal of Ref. [2].

- *States.* A Hilbert space $\mathcal{H}$ is associated to any physical system and the state of the system at time t is represented by a normalized vector $|\psi_t>$ of $\mathcal{H}$.

- *Dynamics.* The evolution of the system is governed by Schrödinger's equation. In addition, at random times, with a Poissonian distribution with mean frequency λ (which, in accordance with a relevant remark by Pearle and Squires [27], is assumed to be proportional to the mass of the particle under consideration), each particle of any system is subjected to a spontaneous localization process of the form:

$$|\psi_t> \rightarrow \frac{L_n(\mathbf{x})|\psi_t>}{||L_n(\mathbf{x})|\psi_t>||}, \quad L_n(\mathbf{x}) = (\frac{\alpha}{\pi})^{3/4}\exp[-\frac{\alpha}{2}(\hat{\mathbf{x}}_n - \mathbf{x})^2] \tag{5}$$

  In this equation $\hat{\mathbf{x}}_n$ is the position operator of the $n-th$ particle of the system.

- *Collapse probability.* One assumes that the probability density that the collapse for the $n-th$ particle occurs at the space point $\mathbf{x}$ is given by:

$$p_n(\mathbf{x}) = ||L_n(\mathbf{x})|\psi_t>||^2 \tag{6}$$

- *Ontology.* Let $\psi_t(\mathbf{x}_1, ...\mathbf{x}_N)$ be the wave function in configuration space. Then:

$$m(\mathbf{x}, t) \equiv \sum_{n=1}^{N} m_n \int d^3\mathbf{x}_1...d^3\mathbf{x}_N \delta^{(3)}(\mathbf{x}_n - \mathbf{x})|\psi_t(\mathbf{x}_1, ...\mathbf{x}_N)|^2 \tag{7}$$

  is assumed [28] to describe the *density of mass* distribution of the system of N particles under consideration in three-dimensional space as a function of time.

- *The trigger mechanism.* The nicest feature of the model consists in the fact that the frequency of the localizations increases with the number of particles. In particular, it can be rigorously proved that any localization of any particle implies a localization of the centre of mass of the whole system. Accordingly, in the case of an almost rigid body, its position suffers a localization (within a spatial region having a linear extension of $10^{-5}cm$ - see below for the choice of the parameter α) accompanied by the collapse of the statevector to a well localized one, with a frequency amplified, with respect to the one of the individual constituents, by the number of constituents of the body.

- *Choosing the parameters of the theory.* The original choice of the values of the localization accuracy and of the mean frequency of the localizations *for a nucleon* has been:

$$\alpha = 10^{10} cm^{-2}, \ \ \lambda = 10^{-16} sec^{-1} \tag{8}$$

Note that with these choices a microscopic system suffers a localization about every 10^7 years, while a macroscopic system ($N \simeq 10^{23}$) one about every 10^{-7} sec. This is why the standard theory is left practically unchanged at the micro-level, while superpositions of macroscopically distinct states are suppressed in extremely short times.

Some important remarks:

1. The theory qualifyes itself as a consistent modification of quantum mechanics, which, on the basis of a unique, universal dynamics accounts both for the behaviour of microsystems, the reduction of the wave packet and the classical behavior of macroscopic objects.

2. The deviations from the quantum predictions implied by the modified dynamics depend essentially only on the product $\alpha \cdot \lambda$ with the only proviso that the localization accuracy must be much larger than the atomic dimensions in order that the modified dynamics leaves practically unaffected the internal motion.

3. Changing the above product of some orders of magnitude contradicts well established facts or it requires important changes like the introduction of an appropriate cutoff.

4. The theory has the status of a rival theory with respect to quantum mechanics and, accordingly, it can be subjected to crucial tests with respect to this theory. In particular, one can get from it indications concerning where to look for an hypothetical breaking of the superposition principle.

5. Recently a lot of attention has been paid to the possibility of devising crucial experimental tests of the theory against quantum mechanics. These tests cover a wide range of experimental situations. We will mention those making reference to its implications for superconductivity [2,29-31], the fact that the collapse processes tend to destroy quantum interference (the problem has been discussed in Refs. [32-34]). Other interesting effects follow from the fact that the collapse models imply spontaneous photon emission (see Refs. [35-37]), as well as from the fact that energy is not conserved (see Refs. [2,38]). For a general review of the problem of testing collapse models we refer the reader to [39,40].

5.2. Bohmian mechanics

This theory represents a (non purely Hilbert space) deterministic completion of quantum mechanics which requires, for the complete specification of the state of an individual physical system, the addition to the statevector $|\psi>$ of other variables which are assumed to be unaccessible (from which the name of hidden variables). These variables (let me call them λ) are appropriately described by a weight function $\rho(\lambda)$ depending on the statevector itself. The theory is built in such a way that the assignement, at the initial time $t = 0$, of the statevector and of the precise value of the hidden variables (which in Bohmian mechanics are identified with the positions - we will collectively indicate as q_j - of all particles of the system under consideration) uniquely determines the positions of all particles constituting the system at any subsequent time. However, due to the fact that we know only the distribution and not the precise values of such hidden variables at the initial time, to get the quantum probabilities, an average must be performed by resorting to the appropriate initial distribution $\rho(q_j, 0)$.

The extremely nice feature of the theory is that when the initial distribution agrees with the one implied by quantum mechanics (i.e. $\rho(q_j, 0) = |\psi(q_j, 0)|^2$) the same relation holds at any subsequent time, i.e., the theory reproduces exactly the probability density of the positions of the particles as predicted by the standard theory. The theory does not require a separate postulate concerning the reduction of the wave packet when macroscopic instruments are involved: measurement processes are fully governed by the deterministic equations of the theory and they do not require any specific intervention by an observer. I think that this theory represents the paradigmatic example of a perfectly consistent deterministic completion of quantum mechanics.

6. Decay Processes: How to Deal with Them

The process considered by Yun requires some specifications. We have presented a completely rigorous treatment of decay processes many years ago [41,42] and we have shown[7] that by dealing in the appropriate way with them one recovers the exponential decay law at all times, in spite of the fact that, as is well known, the quantum non decay probability is not (at very small and very large times) a pure exponential. It is useful to recall our procedure.

The key idea is that, actually, one is dealing with a system subjected to repeated and random measurement processes aimed to check whether the system is decayed or not. As already stated, this fact implies that the decay law is purely exponential at all times; the deviations from the exponential of the quantum non decay probability have as a consequence that the life time, e.g., in the case of a resonance of the Breit-Wigner type, is slightly different from the one implied by the imaginary part Γ of the energy appearing in such a formula. However the effect is totally negligible [46] for all physically meaningful cases [47].

Let us consider the situation at very small times after the preparation. One has a superposition, with a weight very near to 1, of the undecayed atom and a state (with an extremely small norm) in which the atom is decayed and the decay products propagate towards the counter without having reached it. We have depicted the two states of the superposition in Fig. 2. In brief, the actual state is

$$\begin{aligned} |\psi_{\Delta t}\rangle \; = \; & [\alpha(\Delta t)|\mathit{undecayed\ atom}\rangle \\ & +\beta(\Delta t)|\mathit{decayed\ atom}\rangle \otimes |\mathit{decay\ products}\rangle] \\ & \otimes|\mathit{untriggered\ counter}\rangle \otimes |\mathit{clock\ running}\rangle \end{aligned} \tag{9}$$

with $|\alpha(\Delta t)| >> |\beta(\Delta t)|$.

[7] Ideas similar to those which have guided us had been contemplated by Beskov and Nilsson [43] and by Zeh [44] who has considered the possibility that in the case of an unstable system the environment might enforce the exponential decay law. Subsequently, an important step in this direction has been made by Ekstein and Siegert [45] who proved that, if repeated measurements take place, the decay law turns out to be exponential and not power like for $t \to \infty$. The exhaustive and rigorous treatment of the process at all times has been presented, for the first time, in Refs. [41,42].

At this moment, the propagation of the decay fragments being usually remarkably fast, the localized part of the wave function associated to the decay products crosses the counter and correspondingly one is almost immediately led to a superposition of the $|\mathit{undecayed\ atom} > \otimes|\mathit{untriggered\ counter} > \otimes|\mathit{clock\ running} >$ and of the $|\mathit{decayed\ atom} > \otimes|\mathit{decay\ products} > \otimes|\mathit{triggered\ counter} > \otimes|\mathit{clock\ stopped} >$, i.e. of the first state of Fig. 2 and of the state we have represented in Fig. 3 (see p.294).

7. Yun's Proposal and the Different Interpretations of the Theory

In this section we will analyze the implications of Yun's variant of Schrödinger's cat experiment from the point of view of the various positions concerning the real status and meaning of quantum theory mentioned in Secs. 2, 3 and 4.

7.1. The Copenhagen Interpretation 1

Under this perspective, when an entangled superposition of the system and the apparatus occurs, and the apparatus states are macroscopically (or classically) distinguishable, reduction takes place, full stop. Accordingly, with reference to Yun's example it is the system made by the counter and the scissor that, when triggered by the decay products, either snaps or fails to do so (with the appropriate probabilities) the power line.

Let us be more specific concerning the situation under consideration. We have our decaying atom whose lifetime has been assumed to be an hour, which emits decay fragments with an appropriate probability. The fragments propagate in the region between the system and the Geiger counter in a small fraction of the lifetime of the atom, just to fix our ideas and to parallel the discussion by Yun, let us say in one minute[8].

It is obvious that, due to the not infinitely sharp extension of the wave function of the decay products and to the specific features of the counter, the exact time at which the reduction - leaving unaffected or closing the scissors - actually occurs, will

[8] As we have discussed in our papers, the fact that many reduction processes occur during the life time is a typical and essential characteristics of all actual set ups to determine the decay law of an unstable system.

be characterized by a non-zero distribution along the time axis, i.e., the instants in which the reduction processes actually take place will be to some extent random with (in our case) an appropriate mean frequency of 1 minute[9]. This is precisely the attitude we have taken in our papers on decay processes, but here, for the case under consideration and in order to simplify our treatment, we can assume - without any loss of validity of the conclusions - that we have a quantum unstable system with a life time of one hour, which is subjected, at equal intervals of 1 minute, to a test aiming to ascertain whether it is decayed or not, with the subsequent reduction to the "unstable state, untriggered counter, open scissors and running clock" or to the "decayed atom, triggered counter, closed scissors and stopped clock". Let us recall that, as we have proved, the decay law turns out to be purely exponential with a mean life of 1 hour. We then have:

1. At time $t = 11.01$, the first measurement occurs, the probability of the system being found undecayed is $e^{-1/60}$ and the one that reduction has led to the detection of the decay products and the stopping of the clock is $1 - e^{-1/60}$. It is extremely relevant to stress that within the Copenhagen Interpretation 1, the process we have just described **leads from a pure state to a statistical mixture**.

2. We can now consider the situation at $11:02$. In the minute between the first reduction - in the case it has led to the unstable state - and $11:02$, the probability that the undecayed system, under a measurement, is still found undecayed, is $e^{-1/60}$ which, combined with the probability that it has been found undecayed before, tells us that there is a probability $e^{-2/60}$ that the system is still undecayed. On the other hand, let us call x the probability that the second reduction leads to the triggering of the Geiger counter and the stopping of the clock. It is obvious that the sum of the probabilities (i.e., the probability of all possible occurrences) that the system has been found decayed at $11:01$, the probability x and the probability that the system is undecayed at $11:02$, must equal 1, so that $1-e^{-1/60}+x+e^{-2/60} = 1$, implying that the probability that the system is found decayed, and, accordingly, the clock stops precisely at $11:02$, is $x = e^{-1/60} - e^{-2/60}$.

3. One can go on and prove in a trivial way that the probability that the system is found undecayed in the measurement at $11:n$, with $n < 60$, is $P_{UN}(n) = e^{-n/60}$, while the probability that the clock stops at $11:n$ is $P_n = e^{-(n-1)/60} - e^{-n/60}$. Note that the sum of all probabilities that the clock has stopped, coincides, as it must, with $1 - P_{UN}(n)$, i.e., the probability that the system has been found decayed before $t = n$.

4. At $12:00$ the probability of finding the system undecayed is e^{-1}, as it must be, and the sum of the probabilities that the clock indicates one of the times $11:n$ $(n = 1, 2, ..., 60)$ amounts precisely to $1 - e^{-1}$.

We want to stress once more that here, in the spirit of the Copenhagen Interpretation 1, no conscious observers are involved and the physical situation corresponds to a very rich statistical mixture of states in each of which the clock marks a different time. The **emergence of one of the members of the statistical mixture at $12:00$ is conceptually radically different from what happens if one, at this time, has a superposition and performs a measurement**. Thus, according to the description we have given, one can very well discover, at $12:00$, that the clock stopped at $11:30$, and no contradiction arises. We have also given the probabilities that the clock ends up indicating any one of the possible times (at steps of one minute between $11:00$ and $12:00$, or, according to the more general description we have given in the last footnote, at steps compatible with the clock's scale).

[9] Actually, according to our simplified description, if at a certain time reduction on the unstable states occurs, it will take 1 minute before the wave function of the decay products which are emitted subsequently reaches the counter and a new reduction process takes place. Obviously, the actual situation is richer than the one we have sketched here because the reduction does not leads rigorously to the unstable state, but to a superposition of the undecayed state and (with weight almost equal to 0) of the decayed state plus the decay products confined in the region between the atom and the counter. It is possible to deal rigorously [41,42] also with this more complex situation and, as the reader will easily understand, if one proceeds in this way the clock can end up in any one of the times between $11:00$ and $12:00$, in dependence of its possibility of distinguishing extremely small intervals. But the picture does not change, from the conceptual point of view, from the one ensuing from the drastic oversimplification of the process we will make assuming equally (time) spaced reductions.

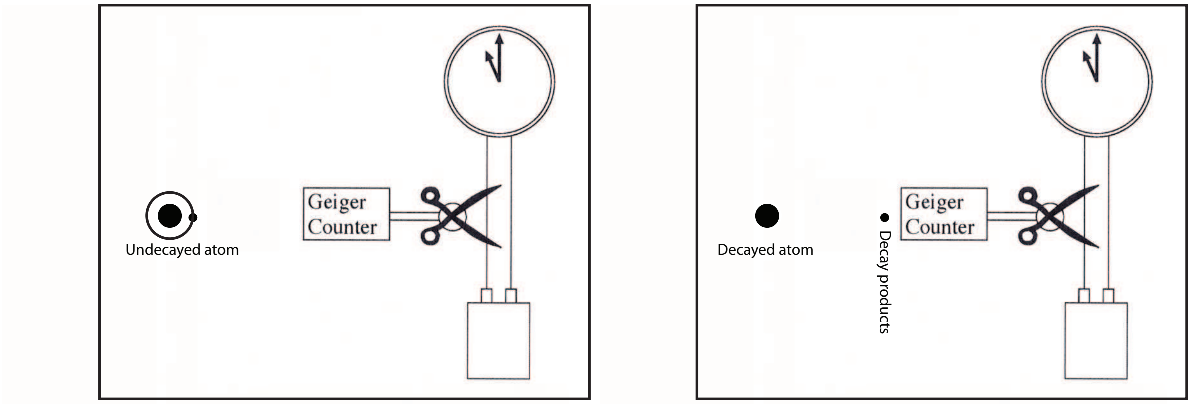

FIG. 2: The two states of the superposition in the extremely short time interval in which the decay products have not yet reached the counter.

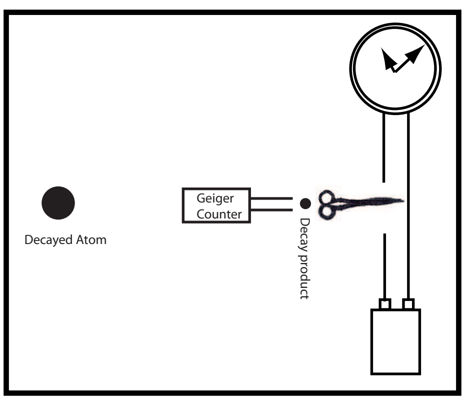

FIG. 3: The state occurring when, due to the fact that the counter has registered the decayed fragments of the superposition, the reduction of the wave packet leads to the stopping of the clock.

The crucial question can now be raised: does this mean that one should follow this line of explanation for the dilemma presented by Yun? Absolutely not, for the simple reason that the above position about quantum theory is self-contradictory and conceptually quite vague, since it assumes two incompatible evolution principles (one linear and deterministic and the other nonlinear and stochastic) to describe physical objects, according to their being microscopic or macroscopic, but, at the same time, it does not contain any criterion to identify in which cases one should resort to one or the other of the principles[10].

This point of view has been made crystal clear in the words of J. Bell [48]:

> *There is a fundamental ambiguity in quantum mechanics, in that nobody knows exactly what it says about any particular situation, for nobody knows exactly where the boundary between the wavy quantum world and the world of particular events is located.... every time we put that boundary - we must put it somewhere - we are arbitrarily dividing the world into two pieces, using two quite different descriptions ...*

Thus, for what concerns the Copenhagen Interpretation 1 of quantum mechanics, Yun's example does not raise new difficulties, simply it leads us back to the universally well known problems related to the measurement process and nothing more, since it is fundamentally based on the assumption that at the macro level reduction takes place, full stop.

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[10] Typically, Bohr has not been fully clear on this problem since he has insisted on the necessity of basing the theory on a clear distinction between quantum microsystems and classical objects, but, at the same time he has claimed in various occasions that even some macroscopic parts of the apparatus (such as a moving macroscopic screen with two slits) must be treated as quantum systems and that only at the end of the process, when the diffracted particle hits the detecting screen, reduction takes place.

7.2. The Copenhagen Interpretation 2

In this case, with reference to the ideal experiment proposed by Yun, there is no doubt that things go in a way quite similar to the one discussed in the previous section, but with the important distinction that **no reduction takes place before** 12 : 00. Accordingly, the natural dynamics of the unstable system and the ensuing behavior of the Geiger counter, the scissors and the clock, do not lead to a statistical mixture but to a superposition of the above mentioned states. In brief, we will have a state, at 12.00, of the kind envisaged by the author in Eqn. (1), which we repeat here to be more precise concerning the relative coefficients of the various states:

$$|\Psi, 12:00\rangle = a|undecayed\rangle \otimes |clock\ running\rangle + \sum_{j=1}^{60} a_j|decayed\rangle \otimes |clock\ pointing\ at\ 11:j\rangle \qquad (10)$$

where the moduli squared of the coefficients coincide with the probabilities we have associated in the previous section to all possible outcomes which can occur at 12 : 00. In this case, therefore, the situation is precisely the one anticipated by Yun after Eqn. (1), i.e., when we open the lid at 12 : 00, the state collapses to one of the terms of the superposition.

As anticipated in Sec. 2.2, I have great difficulties in swallowing this position (which Yun considers as the standard one) mainly due to its fundamental ambiguity concerning what one means when making reference to 'consciousness'. Once more the interpretation should be supplemented with specifications which make it absolutely precise and meaningful if one wants to consider it as the general and universal formulation of our basic theory. Without this step the theory represents one of those verbal and shifty solutions of the problem which, e.g., J. Bell has so many times classified as unprofessional. He also clearly stated that theoretical physicists should go significantly further in making the scheme consistent and acceptable.

Concluding, also the Copenhagen Interpretation 2 does not solve Yuns' dilemma, just for the same reasons for which it does not solve the basic dilemma afflicting quantum theory, i.e., the one of reconciling, in a clear way, the linear and deterministic evolution governed by Schrödinger's equation and the non linear and stochastic reduction process leading to our definite perceptions.

7.3. Many Worlds

We believe that our sketchy presentation of the Many Worlds Interpretation has made clear that the position taken by Yun with respect to it is fully inappropriate, since it has not made clear whether he adheres to the DwWitt or to the Everett interpretation and, even more important, because it is not true that the proponents of this position have strictly related the splitting of the universe or the possibility of taking different viewpoints about the universe to conscious perceptions. At any rate, as we have already anticipated, even if one would assume, with Yun and in disagreement with the proponents of the considered interpretation, that only the act of conscious perception leads to the multiplication of the universes, all remarks made in subsection 7.2 hold unchanged. In this case we will have the generation of alternative universes precisely when the observer (who has also replied many times) uncovers the lid.

This might seem reasonable, but for sure Yun's approach is not the correct one and, at any rate, it meets all the problems we have raised concerning both formulations of the Many World interpretation.

7.4. Collapse theories

Concerning this position, we stress that collapse theories imply (to an extremely high degree of accuracy) the quantum behavior of microsystems while they lead, in a perfectly consistent way and on the basis of a universal dynamics governing all physical processes, to wave packet reduction (for all practical purposes with the quantum probabilities) when superpositions of macroscopic situations corresponding to different locations of a macro-object occur. Since in Yun's proposed experiment the difference of the two superposed states (clock running and clock stopped) corresponds precisely to different locations of the scissors and of the macroscopic pointers of the clock, we can repeat here the same argument presented in subsection 7.1. Accordingly, no problem arises from the peculiar outcome pointed out by Yun, with the remarkable advantage that all the process is accounted for by a consistent, precise, non contradictory and universal description of the dynamics of physical systems. To sum up, it seems that, as in many other situations, the detailed consideration of Yun's experiment leads to the conclusion that, presently, the only viable interpretations of quantum mechanics, within an Hilbert space formulation of the theory, is represented by Collapse Theories.

7.5. Bohmian mechanics

This theory, as we have discussed in Subsec. 3.2, coincides fully with quantum theory for what concerns the positions of all particles of the universe - and consequently the positions and setting of macroscopic objects - and it leads, with reference to these variables, to deterministic results without requiring any intervention of the observer. Since the crucial point of the proposal by Yun has precisely to do with the location of macroscopic objects (the scissors and the pointers of the clock), Bohmian Mechanics perfectly predicts that, when looking at the clock at 12:00, one can find it pointing at 11:30, precisely with the quantum probability.

The fact that the clock stops at a certain time and the precise time at which this occurs, depends, within this theory, from the initial wavefunction and the initial distribution of the particles of the constituents of the composite system, atom, counter, scissors and clock. One can be more or less satisfied with this perspective, but one cannot deny that, just as it happens for any experiment whose final outcome is related to the distribution of the positions of particles, the theory does not meet any trouble and, actually, it perfectly agrees with the predictions of quantum mechanics with wave packet reduction, i.e., with the Copenhagen Interpretation 1. However, contrary to this interpretation, Bohmian mechanics does not present any problem of internal consistency.

8. On Feynman's Approach

The author of the paper we are considering stresses repeatedly that, if one resorts to Feynman's path integral formulation of the process, one does not encounter any difficulty concerning wave packet reduction. The implicit suggestion of these statements is that Feynman's path integral formulation accounts, consistently, also for wave packet reduction (an essential step, as we have seen, to overcome Yun's problems concerning his Schrödinger's cat paradox). This is, for all interested people, an absolutely absurd position to take. Nobody, during the lively debate on quantum mechanics and, more specifically, on the macro-objectification problem, has ever claimed that Feyman's path integral formulation is of any use in solving such a problem.

In a precise sense, Feynman's path integral is equivalent to the (linear and deterministic) unitary evolution of any quantum system - no matter how complex - and in no way whatsoever is able to account for the nonlinear and stochastic evolution which takes place during a measurement process. This makes evident that all claims of the author and his resorting to alternative implications of Feyman's theory according to the connection between the initial and the final states which one is taking into account, are simply misleading and fundamentally wrong.

I consider it important to close this subsection by mentioning that in [49] it has been proved rigorously, by resorting precisely to a path-integral formulation of classical mechanics, that the superposition principle cannot hold for "classical systems". This relevant conclusion has to be taken into account to evaluate, in the appropriate way, Yun's arguments concerning Feynman's formulation of quantum theory.

9. Conclusion

Our conclusion is quite simple and natural. The consideration of the modified Schrödinger's cat experiment proposed by Yun meets precisely the same problems of standard quantum theory in connection with wave packet reduction when a micro system interacts with a macro apparatus or stimulates a perception of a conscious observer. The analysis of this paper should have made clear that the (presently) unique consistent and natural account of what happens, if one wants to stay within a strict Hilbert space formulation of the theory, is the one ensuing from Collapse Theories.

Within such theories one can safely state that an event occurred precisely at 11 : 30 simply because, actually, a random localization of one of the constituents of the macroscopic objects which enter into play (the scissors and the clock, respectively) has led to its stopping at 11 : 30. This holds completely in general and does not require a vague separations between two levels of reality. In particular, the theory accounts in a perfectly consistent way of the fact that looking at the clock at 12 : 00 one can read 11 : 30, and this occurs precisely with the standard quantum probability.

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