

Dark Energy Model of the Universe Inspired by Conharmonically Flat Geometry

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Con-harmonically flat space time is studied in the frame work of FRW cosmological model. Einstein's field equations with variable cosmological constant are solved by using the law of variation of cosmological constant that scale as $\mathbf{a}^{-\mathbf{m}}$, where $\mathbf{a}$ is a scale factor and $\mathbf{m}$ is a positive constant. A new class of exact solutions of the field equations has been obtained in which cosmological constant decreases with cosmic time. Physical and geometrical properties are also discussed in detailed.

1. Introduction

The importance of con-harmonic curvature tensor is very well known [1,2] which is given as

$$L_{ijk}^h = R_{ijk}^h - \frac{1}{(n-2)} (g_{ij} R_k^h - g_{ik} R_j^h + \delta_k^h R_{ij} - \delta_j^h R_{ik}) \quad (1)$$

Where, R_{ijk}^h is the Riemannian curvature tensor and R_{ij} , is the Ricci tensor. A space-time whose con-harmonic curvature tensor vanishes at each point is called con-harmonically flat space time. Con-harmonic curvature tensor represents the deviation of the space-time from con-harmonically flatness. Ahsan and Siddiqui, [3], Siddiqui and Ahsan [4], have studied the significance of the con-harmonic curvature tensor in general theory of relativity. The significance of the space of constant curvature is very well known in cosmology. Due to the cosmological principle we have in the rest system of matter no preferred point and no preferred direction, the three-dimensional space being constituted in the same way everywhere. Narlikar [5] defined those cosmological solutions of Einstein field equations which contain a three-dimensional space-time surface of a constant curvature as the Robertson-Walker metrics. While a four-dimensional space of constant curvature is the deSitter model of the universe, the deSitter model has a three-dimensional space of constant curvature and this belongs to Robertson- Walker metrics.

Recently Kumar and Shrivastava [6] studied FRW cosmological model for con-harmonically flat space time. Einstein's field equations with variable cosmological term were solved by using the law of variation of Hubble Parameter. In this paper, we consider FRW space-time filled with a perfect fluid in presence of the time-dependent cosmological constant. To solve the Einstein field equations we assume that cosmological constant Λ scale with $\mathbf{a}^{-\mathbf{m}}$, where $\mathbf{a}$ is a scale factor and $\mathbf{m}$ is a positive constant, as in [7]. This work is organized as follows: in Sec. 2, the model and field equations are presented. The field equations are solved in Sec. 3 by using the decaying law of Λ . Exact R-W models are obtained in subsections 3.1, 3.2 and 3.3 and the physical behavior of these models is discussed in detail. In Sec. 4, we give concluding remark.

2. Field Equations

As we know, the Friedman-Robertson-Walker (FRW) universe is in four-dimensional space-time which is described by the following metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right] \quad (2)$$

Where, $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and $a(t)$ represents the scale factor. The θ and ϕ parameters are the usual azimuthal and polar angles of spherical coordinates, with $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The coordinates (t, r, θ, ϕ) are called co-moving coordinates. The constant k demotes the curvature of the space. When $k=1$, universe is closed and is open for $k = -1$, and $k = 0$ represent flat universe.

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The Einstein field equations are given by

$$R_{\mu\nu} - \frac{1}{2} R_k^k g_{\mu\nu} = -8\pi T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (3)$$

We assume that the energy momentum tensor of a perfect field is given by

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} \quad (4)$$

with the equation of state

$$p = \omega \rho, \quad 0 \leq \omega \leq 1 \quad (5)$$

Where, p and ρ are, respectively, pressure and energy density of the cosmic fluid and U_u is unit flour vector such that $U_u = (1, 0, 0, 0)$.

The con-harmonic curvature tensor for four-dimensional space-time is

$$L_{ijk}^h = R_{ijk}^h - \frac{1}{2} (g_{ij} R_k^h - g_{ik} R_j^h + \delta_k^h R_{ij} - \delta_j^h R_{ik}) \quad (6)$$

For con-harmonically flat space time we have

$$g_{ij} R_k^h - g_{ik} R_j^h + \delta_k^h R_{ij} - \delta_j^h R_{ik} = 2R_{ijk}^h \quad (7)$$

Contracting this with $h = j$ and summing over j , we have

$$R_{ik} = -\frac{1}{4} R_l^l g_{ik} \quad (8)$$

Thus for con-harmonically flat space-time, the Einstein field equation (3) reduces to

$$R_{\mu\nu} = -\frac{8\pi}{3} T_{\mu\nu} + \frac{\Lambda}{3} g_{\mu\nu} \quad (9)$$

For the metric in Eqn. (2) and the energy momentum tensor in Eqn. (4), the Einstein field equations (9) reduces to

$$8\pi\rho + \Lambda = -9(\dot{H} + H^2) \quad (10)$$

$$8\pi p - \Lambda = 3(\dot{H} + 3H^2 + \frac{2k}{a^2}) \quad (11)$$

Where, $H = \frac{\dot{a}}{a}$, is the Hubble parameter. Here and elsewhere an over head dot denotes ordinary differentiation with respect to time t .

3. Solution of the Field Equation

The system of Eqns. (5), (10) and (11) supply only three equations in four unknowns: $(a, \rho, p \text{ and } \Lambda)$. One extra equation is needed to solve the system completely. The phenomenological Λ decay scenarios have been considered by a number of researchers such as Chen and Wu [8] considered $\Lambda \propto a^{-2}$ (a is scale factor of the Robertson-Walker model), Hoyle et. al. [9] considered $\Lambda \propto a^{-3}$ while $\Lambda \propto a^{-m}$ was considered by Olsan and Jordan [10]; Pavon [11], Maia and Silva [12], Silveira and Waga [13,14]. Thus we take the decaying vacuum energy density

$$\Lambda = \frac{\sigma}{a^m} \quad (12)$$

Where, σ and m are positive constants. For $k = 0$, from Eqns. (5), (10), (11) and (12), one finds that

$$\frac{\ddot{a}}{a} + \frac{2}{1+3\omega} \left(\frac{\dot{a}}{a}\right)^2 + \frac{(1+\omega)}{3(1+3\omega)} \sigma a^{-m} = 0 \quad (13)$$

Now, integrating (13), we have the scale factor

$$a = \left[\frac{m}{2} \sqrt{\frac{2\sigma(1+\omega)}{3(3m\omega - 6\omega + m - 6)}} t + t_0 \right]^{\frac{2}{m}} \quad (14)$$

Where, t_0 is a constant of integration. The integration constant is related to the choice of origin of time.

Now, we analyze scenarios for different values of ω .

3.1. Matter dominated solution (cosmology for $\omega = 0$)

For $\omega = 0$ from Eqn. (14) we obtain the scale factor

$$a(t) = \left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^{2/m} \quad (15)$$

In this case the spatial volume V , matter density ρ , pressure p and cosmological constant Λ are given by

$$V = \left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^{6/m} \quad (16)$$

$$\rho = \frac{m\sigma}{4\pi(m-6)} \left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^{-2} \quad (17)$$

$$p = 0 \quad (18)$$

$$\Lambda = \frac{\sigma}{\left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^2} \quad (19)$$

The expansion scalar θ and density parameter Ω are given by

$$\theta = 3 \sqrt{\frac{2\sigma}{3(m-6)}} \frac{1}{\left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]} \quad (20)$$

$$\Omega = \frac{8\pi\rho}{3H^2} = m \quad (21)$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1 \quad (22)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{\Lambda}{8\pi} = \frac{\sigma}{8\pi \left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^2} \quad (23)$$

$$\rho_c = \frac{3H^2}{8\pi} = \frac{\sigma}{4\pi(m-6) \left[\frac{m}{2} \sqrt{\frac{2\sigma}{3(m-6)}} t + t_0 \right]^2} \quad (24)$$

Here we observe that the spatial volume V is zero at $t=t'$, where $t' = -\frac{2t_0}{\sqrt{\frac{2\sigma}{3(m-6)}}}$ and expansion scalar

θ is infinite, which shows that the universe starts evolving with zero volume at $t=t'$ with an infinite rate of expansion. At $t = t'$, the scale factor vanishes, and therefore the space time exhibits point type singularity at initial epoch. The energy density and cosmological constant diverge at the

initial singularity. As t increases, the scale factor and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increasing time. Also, the cosmological parameters ρ , ρ_v , ρ_c and Λ decreases as t increase. As, $t \rightarrow \infty$, the scale factor and volume become infinite, ρ , ρ_v , ρ_c and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t .

Furthermore, we observe that $\Lambda \propto \frac{1}{t^2}$ which follows from the model of Kalligas et al. [15]; Berman and Som [16]; Berman [17]; Berman et al. [18] and Bertolami [19,20]. This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

3.2. Zeldovich fluid distribution (cosmology for $\omega = 1$)

In this case, from Eqn. (14) the scale factor $a(t)$ becomes

$$a(t) = \left[\frac{m}{2} \sqrt{\frac{\sigma}{3(m-3)}} t + t_0 \right]^{2/m} \quad (25)$$

Here, spatial volume V , matter density ρ , pressure p and cosmological constant Λ are

$$V = \left[\frac{m}{2} \sqrt{\frac{\sigma}{3(m-3)}} t + t_0 \right]^{6/m} \quad (26)$$

$$\rho = p = \frac{m\sigma}{6\pi(m-3)} \left[\frac{m}{2} \sqrt{\frac{\sigma}{3(m-3)}} t + t_0 \right]^{-2} \quad (27)$$

$$\Lambda = \frac{\sigma}{\left[\frac{m}{2} \sqrt{\frac{\sigma}{3(m-3)}} t + t_0 \right]^2} \quad (28)$$

The expansion scalar θ and deceleration parameter q are given by

$$\theta = 3 \sqrt{\frac{\sigma}{3(m-3)}} \frac{1}{\left[\frac{m}{2} \sqrt{\frac{\sigma}{3(m-3)}} t + t_0 \right]^2} \quad (29)$$

$$q = \frac{m}{2} - 1 \quad (30)$$

In this case, we observe that the spatial volume V is

$$\text{zero at } t = \frac{-2t_0}{\sqrt[m]{\frac{2\sigma}{3(m-3)}}} = t'', \text{ and expansion scalar } \theta$$

is infinite at $t = t''$, which shows that the universe starts evolving with zero volume and infinite rate of expansion at initially. At $t = t''$, the energy density ρ , pressure p , cosmological constant Λ are infinite. As t increases, the spatial volume increases but the expansion scalar decreases. Thus the expansion rate decreases as time increases. As t tends to ∞ the spatial volume V becomes infinitely large. As t increases, all the parameters ρ, p, Λ, θ decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty for large values of t .

3.3. Radiation dominated solution (cosmology for $\omega = 1/3$)

In this case behavior of the cosmological model is same as in Sec.3.2, which we have already discussed.

4. Conclusion

We have investigated co-harmonically flat space time in the framework of FRW cosmological model. Einstein field equations with variable cosmological constant are solved by using the law of variation of cosmological constant that scale as a^{-m} (where a is scale factor and m is a positive constant) suggested by Olsan and Jordan [10], Pavon [11], Maia and Silva [12], Silveira and Waga [13,14]. In all these cases, we have found that the cosmological term Λ being very large at initial time and decreases as time increases, which is in accordance with the observations.

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