Brownian Particle's Decoherence in the Double-Well Magnetic Potential Field

M. Tchoffo*, C. G. Fouokeng, L. C.Fai, L. A. Ngoufo and J. T. Diffo
Mesoscopic and Multilayer Structure Laboratory, Department of Physics, Faculty of Science, University of Dschang, Cameroon

The influence of the magnetic confinement and the step parameters on de-coherence of a Brownian particle in a double-well magnetic potential field coupled to a bath of harmonic oscillators has been studied by applying the reduced resolvent method. The thermodynamic parameters of the system have been determined within the effective temperature and mass. Our result show that the magnetic confinement, the step parameters and the parameters of thermodynamics are the dominant factors influencing de-coherence. The entropy and internal energy are observed to increase while the specific heat capacity decreases when the magnetic confinement increases or the step parameter decreases. The motion of the particle remains constant when the coupling strength vanishes.

1. Introduction

The notion of quantum Brownian motion in the Caldera-Leggett model enables us to study the disappearance of coherence between two wave packets separated in space and driven to a statistical mixture of the Eigen state of coordinate operator x [1-4]. In fact, it has been recognized that de-coherence is of fundamental importance in understanding the nature of fuzzy boundary between quantum and classical domains. The nature of this boundary has been under scrutiny both from the theoretical and from the experimental point of view [5]. The basic physics of de-coherence is very simple: interaction with the environment tends to prevent the stable existence of a vast majority of states in the Hilbert space of macroscopic quantum systems. Thus, coherent superposition of macroscopically and microscopically distinct states tends to decay very rapidly (on a short de-coherence time-scale) into mixtures preventing the observation of delocalized (Schrödinger's cat) states.

The motion of a quantum particle in an external potential in the presence of ohmic dissipation has been studied recently [6]. The particle coordinates change classically according to the following equation of motion [1]

$$M\ddot{x} + \eta\dot{x}(t) + \partial V/\partial x = F(t) \qquad (1)$$

Where, the mass M of the particle is submitted to potential $V(x)$, η represents the phenomenological damping constant, or friction coefficient, and $F(t)$ is a fluctuating force. The study of Eqn. (1) is of great importance. Firstly, because the so-called macroscopic quantum variables such as the phase difference across a Josephson element in a SQUID are believed to obey this equation. It has been extensively discussed for instance by A. J. Leggett in the context of testing the applicability of quantum mechanics at the macroscopic level [7]. Secondly, Eqn. (1) presents some interests at a fully microscopic level since in the classical (high temperature) limit it describes the Brownian motion of the particle in potential $V(x)$. It can thus be used as a tool in the study of quantum Brownian motion [8-10]. More precisely, several different physical problems can be studied in the general framework of Eqn. (1) according to a specific type of the potential $V(x)$, such as quantum free particle [9,10], particle in a potential with a meta-stable minimum [1], particle in a potential coupled to N harmonic oscillators [11], particle in a symmetric double-well potential [12-14], particle in an asymmetric double-well potential [6], and particle in a periodic potential [15-18]. In [6], the real dynamical time of a particle is presented and the time evolution of the system is studied as well as the zero temperature relaxation [6]. The standard fundamental quantum-statistical method is investigated in [10] to describe the dynamics of a particle by investigating the critical behavior of the order parameter in an asymmetric potential [19].

The destruction of interference phenomena, due to interaction with the environment, an effect sometimes referred to as de-coherence, in general,

*mtchoffo2000@yahoo.fr

depends on the temperature. For instance, the temperature dependence of the weak localization correction to the conductivity reveals that in metals, the electron-electron interaction dominates over the phonon contribution to de-coherence at lower temperatures [11]. The question of what happens to interference phenomena at zero temperature has been hotly debated over the past few years. The debate was initiated by temperature-dependent weak localization measurements [20], reporting on a residual de-coherence in metals at zero temperature, in contradiction to theoretical predictions [21]. The subsequent theoretical debate [22,23] mainly focused on zero-temperature de-coherence induced by Coulomb interaction in disordered electron systems, but as a spin-off, it has led to a more general question: can a zero-temperature environment induce de-coherence? Recently, this issue was discussed in the framework of a well-known simple model [24, 25]: a harmonic oscillator (the "particle") coupled to a chain of harmonic oscillators (the "environment"). It was shown that the particle exchanges energy with the environment, even at zero temperature. The effect of these energy fluctuations cannot be simply captured through a renormalization of particle's parameters, but will give rise to a ground-state with non-trivial dynamics. This can have important consequences on measured thermodynamic properties of systems. An example is the suppression of the zero-temperature persistent normal or super-current in mesoscopic rings [26,27]. In Ref. [11], the authors were interested on the influence of an environment at low temperature on the behavior of a mesoscopic system; they particularly studied the effects of environment on the interference phenomena when the coupling energy between a small system and the environment is larger than the thermal energy.

In this paper, following references [9,11,28,29] we study the de-coherence of a system made of a particle in a magnetic double-well potential field coupled to an ohmic environment. This problem can be very clearly settled down, but it cannot be given an exact solution. Thus, approximate treatments have been proposed in literature [30-32]. In [11] the Reduced Resolvent Method (RRM) is used to evaluate the reduce density. This work refines and supplements previous treatments of quantum de-coherence (QD) for a harmonic oscillator in a magnetic double-well potential interacting with a thermal bath in the framework of the quantum Langevin equation for open quantum systems. We sketch the influence of the magnetic field on de-coherence of a Brownian particle coupled to a bath oscillator through thermodynamics parameters [33-35].

2. The Model

2.1. Quantum Langevin equation

Following references [13-15], we consider a particle of mass m moving in a one-dimensional symmetric electromagnetic double-well potential $V(x)=V(-x)$ with its minima located at $x=|a|$ and separated by a local maximum at $x=0$. This particle is in contact with a dissipative medium, which is generally modeled by a phonon reservoir of band width like in [1]. We consider in this paper the case of an « ohmic » dissipation in which the coordinate of the particle evolves with time according to the classical equation of motion (see Eqn. (1)).

As mentioned in [36], the Langevin equation has a very general form that can be realized with a simple and convenient model, for example, an independent oscillator [13] described by the Hamiltonian

$$H=\frac{P^2}{2m}+V(x)+\sum_{i=1}^{N}\frac{\pi_i^2}{2\mu_i}+\frac{\mu_i\omega_i^2(\varphi_i-x)^2}{2} \qquad (2)$$

The first two terms represent the "particle" (of mass m, frequency ω_0, momentum P, and coordinate x. The second two terms are the corresponding set of N independent harmonic oscillators, ω_i is the frequency of the i^{th} oscillator, and μ_i, π_i, and φ_i are its mass, momentum and position, respectively. Parameters μ_i and ω_i entirely characterize the environment.

With the help of the Hamilton-Jacobi transformation, the Langevin equation is obtain [6] as

$$m\ddot{x}=-\frac{\partial V}{\partial x}-\int_0^t\Gamma(\tau)\dot{x}(t-\tau)d\tau+F(\tau) \qquad (3)$$

Where, $\int_0^t\Gamma(\tau)\dot{x}(t-\tau)d\tau$ is the memory function and $F(\tau)$ the external force.

2.2. Temporal correlation functions in an electromagnetic double-well potential

In the confined double-well electromagnetic field potential, the Hamiltonian is as follows

$$\mathrm{H}(\mathrm{x}, \mathrm{p}, \varphi, \pi) = \mathrm{H}_{\mathrm{part}} + \mathrm{H}_{\mathrm{env}} \quad (4)$$

with

$$H_{part} = \frac{1}{2M}\left[\vec{P} + \frac{e}{c}\vec{A}(x)\right]^2 - \frac{ke^2}{x} \quad (5)$$

as the Hamiltonian of the particle. The Hamiltonian of the environment is given by

$$H_{env} = \sum_i^N \frac{1}{2\mu_i}\left[\vec{\pi}_i + \frac{e}{c}\vec{A}_i(x)\right]^2 \quad (6)$$

From Eqns. (5) and (6), the Hamiltonian (2) reads as

$$\mathrm{H} = \underbrace{\frac{\mathrm{P}^2}{2\mathrm{m}} + \frac{\mathrm{m}\,\Omega_0^2}{2}\mathrm{X}^2}_{\text{particle}} + \underbrace{\sum_{\mathrm{i}=1}^{\mathrm{N}}\left\{\frac{\pi_{\mathrm{i}}^2}{2\mu_{\mathrm{i}}} + \frac{\mu_{\mathrm{i}}\,\Omega_{\mathrm{i}}^2}{2}(\varphi_{\mathrm{i}} + \mathrm{X})^2\right\}}_{environment} \quad (7)$$

Where

$$\Omega_0^2 = \omega_0^2 + \frac{\omega_c^2}{4} \quad (8)$$

is the particle oscillation frequency and

$$\Omega_i^2 = \omega_i^2 + \frac{\omega_c^2}{4} \quad (9)$$

the environment frequency.

$$\mathrm{X} = |\mathrm{x}| \pm \mathrm{a} \quad (10)$$

Eqn. (10) above represents the new coordinate of the particle. In the above equations, ω_c is the cyclotron frequency. It follows that the role of electronic confinement may be played by the magnetic field. The aim of our investigation is to clarify the role of electromagnetic field confinement on the coherence of the system when couple to a bath oscillator.

The equation of motion is found as the memory equation

$$m\frac{d^2\hat{X}}{dt^2} = -m\Omega_0^2\hat{X} - \int_0^t \frac{d\hat{X}(t-\tau)\Gamma(\tau)d\tau}{dt} + \hat{F}(t) \quad (11)$$

In this work, we are interested in the behavior of the system when *N* is large, particularly in the continuum limit. In this case, $\Omega_i = \Omega$ is a continuous variable and the mass distribution $\mu_i(\Omega_i)$ is a smooth function of $\mu(\Omega)$, which is defined in a manner such that $\mu(\Omega)d\Omega$ is the mass of the oscillators with frequency between Ω and $\Omega + d\Omega$, whereas the distribution $\mu(\Omega)$ characterizes entirely the environment [11]. For the particular case $\mu(\Omega) = 2\eta/m\Omega^2$, the Hamiltonian leads to the following well-known classical equation of motion [11]

$$m\frac{d^2X}{dt} = -m\Omega_0^2 X \underbrace{- \eta\frac{dX}{dt} + F(t)}_{environment} \quad (12)$$

The electromagnetic environment induces a dissipative force $-\eta\dot{X}$ and a fluctuating force $F(t)$. The parameter $\eta/m\Omega_0$ represents the strength of the coupling. For this particular environment, $F(t)$ is considered white noise.

Historically, the classical equation (Eqn. (12)) was derived by Lamb in 1900. Since then, it has also been studied in the quantum case. We have [37,38] as a first point in the huge literature available on this topic.

From Eqn. (12) and with the help of the Kubo's formula (Eqn. (13))

$$\phi(t) = \left\langle \frac{[\hat{\upsilon}(0), \hat{y}(t)]}{i\hbar} \right\rangle Y(t) \quad (13)$$

Where, $Y(t)$ is the Heaviside function, the equation of motion is derived and given as

$$i\hbar m\ddot{\phi} = -m\Omega_0^2 i\hbar\phi - \eta\dot{\phi} + i\hbar\delta \quad (14)$$

Here ϕ and δ represent, respectively, the impulsion response of the observable $\hat{y}$ with respect to υ and the Dirac delta function. Eqn. (14) is a memory-less equation and the solution is found in the form

$$\phi = \phi_0 e^{-i\Omega_0 t} \quad (15)$$

The imaginary part of the above solution may serve to evaluate the temporal correlation function

$$\mathrm{Im}\,\phi = \chi_{AB}(\Omega) = \frac{i\eta\Omega}{m^2\left(\Omega_0{}^2 - \Omega^2\right) + \eta^2\Omega^2} \quad (16)$$

Where, indices *AB* represent the adjoint operators of a pair of observables. According to the fluctuation-dissipation theorem, Eqn. (16) can be equally written as

$$\chi_{AB}(\Omega)=\frac{i}{\hbar}Tr([B,\rho_0]A_0)=Tr(\rho_0[A_0,B]) \quad (17)$$

Where, A_0 is a defined operator at equilibrium and ρ_0 the equilibrium state density matrix. From Eqn. (17) we have the temporal correlation function

$$G_{AB}=\langle\hat{y}(t),\hat{y}(0)\rangle=\frac{\hbar}{2\pi}\int_{-\infty}^{\infty}\mathrm{Im}\,\chi(\Omega)\coth\left(\frac{\beta\hbar\Omega}{2}\right)e^{i\Omega t}d\Omega \quad (18)$$

2.3. Reduced density operator of the particle

We now consider the quantum version of the system defined by Eqn. (11), i.e., a quantum oscillator coupled to a double quantum well environment. Considering that the entire system is in equilibrium (i.e., $t\rightarrow 0$) at temperature *T*, then from Eqn. (18), following the fluctuation of the coordinate and the momentum of the particle, as presented in Ref. [11], we can write the reduced density operator $\hat{\sigma}$ in a canonical form with unknown coefficients $\tilde{T}$ and $\tilde{m}$ as

$$\hat{\sigma}=\frac{1}{\tilde{Z}}\exp\left(-\frac{1}{K\tilde{T}}H_{eff}\right) \quad (19)$$

Where

$$\hat{H}_{eff}=\frac{\hat{p}^2}{2\tilde{m}}+\frac{\tilde{m}\Omega_0^2}{2}X^2 \quad (20)$$

is the effective Hamiltonian, $\tilde{m}$ the effective mass, $\tilde{Z}$ the effective partition function, and $\tilde{T}$ the effective temperature. In particular it has already been shown in [11] that even at zero temperature, i.e., $T=0$, we may have $\tilde{T}\neq 0$. Thus, $\hat{\sigma}$ is not a pure state and the particle cannot be in its ground state. Here, $\tilde{Z}$ is a normalization constant that ensures $tr\hat{\sigma}=1$. The above condition shows that the system is incoherent. We can explicitly calculate $\tilde{T}$ and $\tilde{m}$ as functions of the real temperature T and the mass distribution $\mu(\Omega)$ of the environment (see Appendix A in [11]).

The relevant intrinsic quantity describing the particle is the reduced density operator $\hat{\sigma}$. It has been shown [11,38] with the RRM that if

$$\underbrace{tr}_{part}\,\hat{\rho}(t=0)=\frac{e^{H_{env}/KT}}{Z_{env}} \quad (21)$$

and $\mu(\Omega)$ has no gap, then

$$\lim_{t\to\infty}\underbrace{tr}_{env}\,\hat{\rho}(t)=\hat{\sigma} \quad (22)$$

In the next section, following [11], we suppose that $\mu(\Omega)$ is such that the reduced resolvent of the system has no poles on the real axis. Furthermore, we choose an ohmic environment given as

$$\mu(\Omega)=\begin{cases}2\eta/m\Omega^2 & \Omega<\Omega_C\\ 0 & \text{otherwise}\end{cases} \quad (23)$$

Where, Ω_c is the cut-off frequency (that is, when $T=0$). It is interesting to note that for $\Omega_C >> \Omega$, the classical behavior of the particle is described by Eqn. (1) [1,30,39]. It is obvious that when $\eta=0$, $\mu(\Omega)=0$ does not correspond to the classical case but should be understood as the case where a free particle moving in a magnetic field has a constant motion. Thus both the position and the momentum are averagely constant independently of the magnetic field frequency. This important result is shown in Figs. 1 and 2.

In order to evaluate the influence of magnetic confinement on the evolution of the system and its coupling with that of an ohmic environment, the temperature and the effective mass of the total system are obtained, respectively, in Eqns. (24) and (25).

$$K\tilde{T}=\frac{\hbar\Omega_0}{2}\frac{1}{Argth\sqrt{\frac{\hbar^2}{4\left(\langle X^2\rangle+a^2\rangle\langle p^2\rangle\right)}}} \quad (24)$$

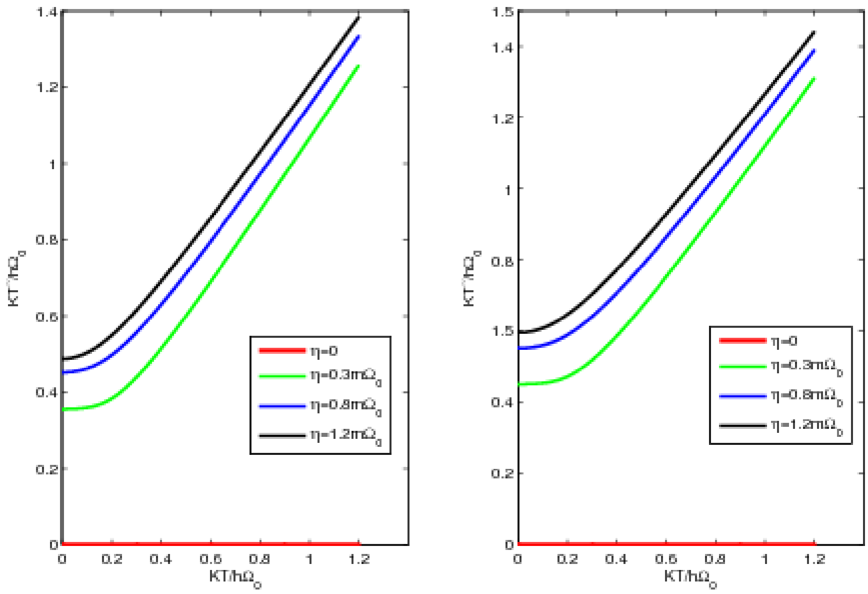

Fig.1: Effective temperature $\tilde{T}$ versus temperature T for different values of coupling constant η , respectively, with a given values of the step parameter a = 0.01 (left) and a = 0.1 (right).

$$\tilde{m} = \sqrt{\frac{\langle p^2 \rangle}{\Omega_0^2 \left(\langle X^2 \rangle + a^2 \right)}} \tag{25}$$

The above curves show the influence of the magnetic field and the step parameter a on respective behaviors of the effective temperature and the effective mass. In the same manner as the effective mass the temperature of the system also decreases when the magnetic field increases. This result is foreseeable as the confinement due to the magnetic field potential strongly localized the system and therefore reduces both its temperature and mass. On the other hand, when the step parameter increases, the degree of freedom of the system also increases and so do the temperature and the mass. De-coherence of the system may be controlled by increasing the magnetic field and reducing the step parameter. Furthermore, we study this de-coherence by evaluating in the following section the thermodynamic parameters.

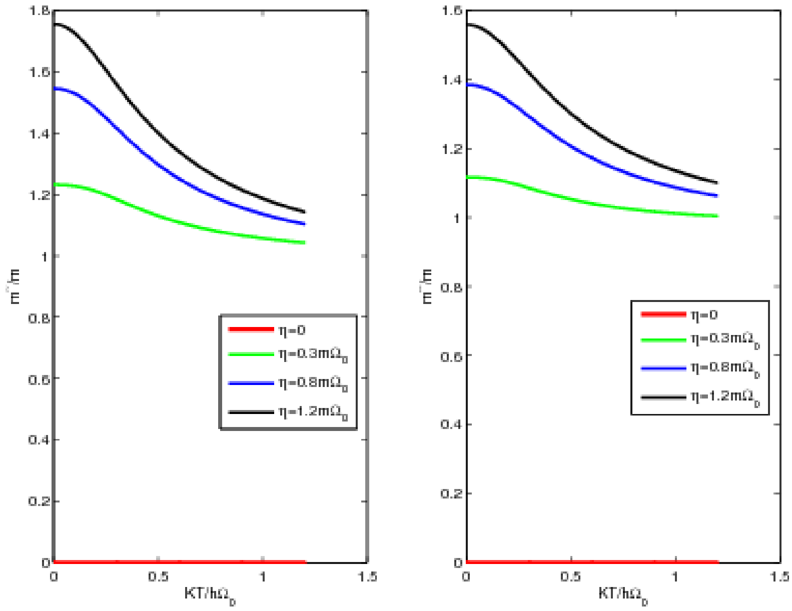

Fig.2: Effective mass $\tilde{m}$ versus temperature T for different values of the coupling constant η, respectively, with a given values of the step parameter a = 0.01 (left) and a = 0.1 (right).

3. Thermodynamic Parameters in an Ohmic Environment

In contrast to $\tilde{T}$ and $\tilde{m}$, the entropy S and the specific heat capacity C_v of the system have intrinsic definitions. We evaluate thermodynamic parameters using the Shannon relation of the form

$$S = -K\, tr(\hat{\rho}\, \ln\, \hat{\rho}) \qquad (26)$$

The partial entropy of the system is given in Eqn. (27)

$$\frac{S}{K} = \frac{\tilde{\beta}\hbar\Omega_0}{2}\coth\left(\frac{\tilde{\beta}\hbar\Omega_0}{2}\right) - \ln\left(2sh\frac{\tilde{\beta}\hbar\Omega_0}{2}\right) \qquad (27)$$

From the Gibbs thermodynamic relations, we derive the expressions for internal energy and specific heat of the system

$$E = \frac{\hbar\Omega_0}{2}\coth\left(\frac{\tilde{\beta}\hbar\Omega_0}{2}\right) \qquad (28)$$

The specific heat capacity of the system

$$C_V = \frac{K\tilde{\beta}^2(\hbar\Omega_0)^2}{4\left(\sinh\left(\frac{\hbar\tilde{\beta}\Omega_0}{2}\right)\right)^2} \qquad (29)$$

According to the plots of Figs. 3 to 5 we find that the entropy and the internal energy increase while the specific heat capacity decreases even at $T = 0$. In this way, we can say at this stage that there is a persistence of de-coherence. These statements are not really in contradiction with the ordinary statistical mechanics or with the third principle of thermodynamics; indeed in statistical mechanics one neglects the coupling energy between the particle and the bath. This approximation is good at high temperatures when $T >> \tilde{T}(0)$. When the system is strongly localized (high magnetic frequency), its entropy and energy decrease while the heat capacity increases. Obviously, the de-coherence aspect is observed when the electromagnetic field frequency decreases and with the increase of the step parameter. This sentence is confirmed by the curves obtained in Figs. 3 to 5.

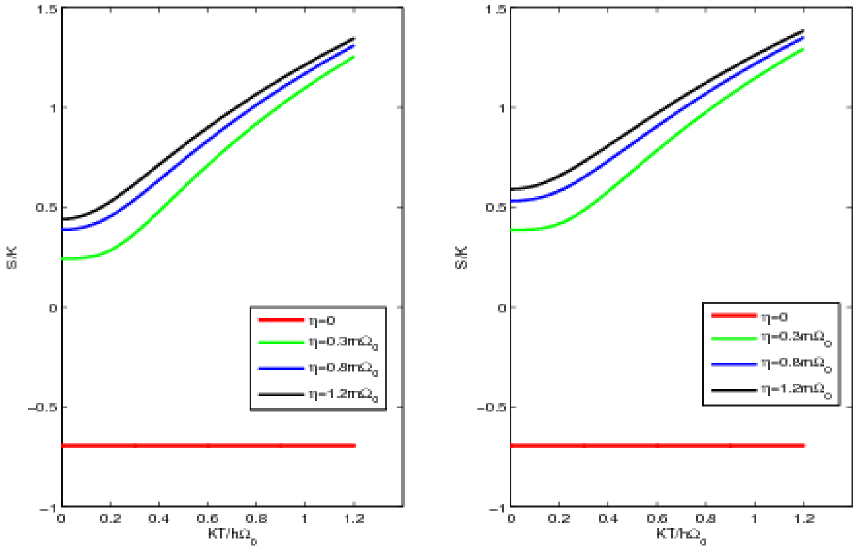

Fig.3: Entropy S versus temperature T for different values of the coupling constant η , respectively, with given values of the step parameter as: a = 0.01 (left) and a = 0.1 (right).

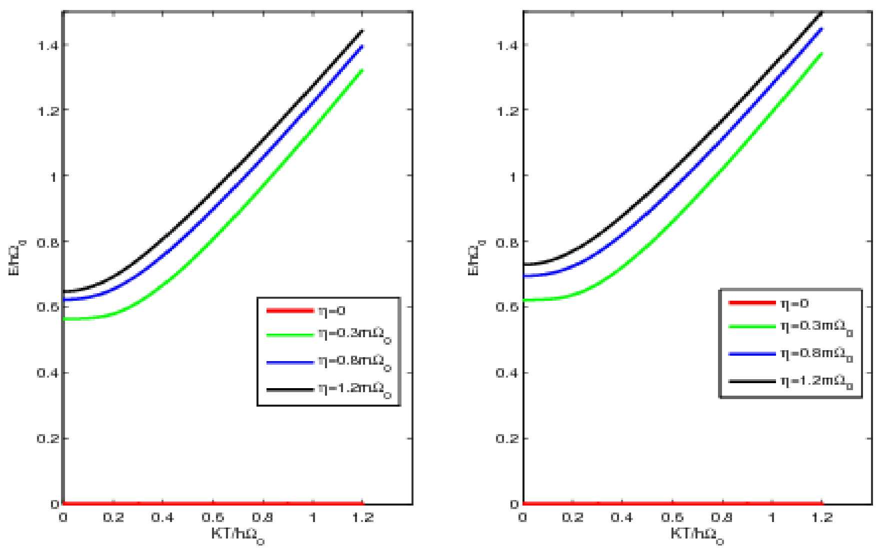

Fig. 4: Internal energy E versus temperature T for different values of the coupling constant η , respectively, with given values of the step parameter: a = 0.01 (left) and a = 0.1 (right).

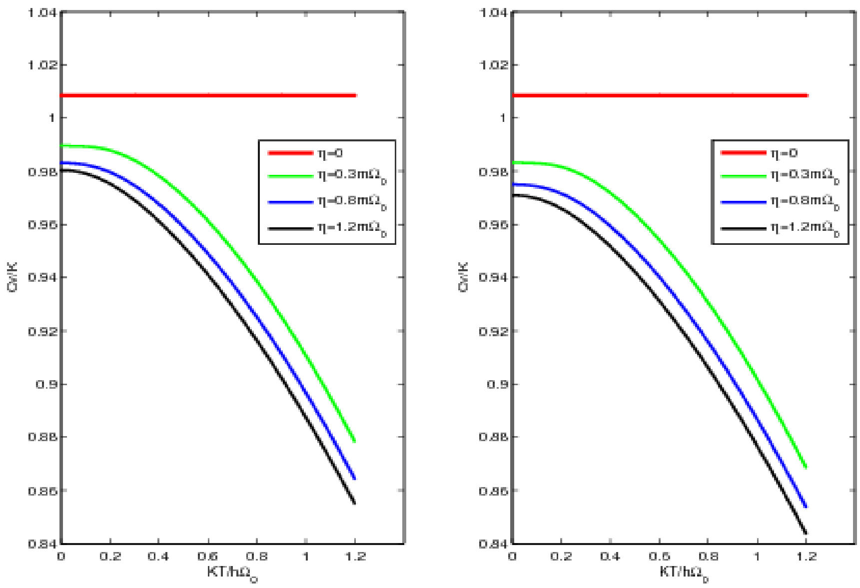

Fig.5: Specific heat capacity C_v versus temperature $\tilde{T}$ for different values of the coupling constant η, respectively, with given values of the step parameter: a = 0.01 (left) and a = 0.1 (right).

4. Concluding Remarks

In the present paper, we have discussed the influence of confinement and step parameter on de-coherence of a Brownian particle in a double-well magnetic potential field coupled to an oscillator bath.

The results obtained are in accordance with those presented in literature. That is, for adiabatic quantum de-coherence processes such as studied in this work, the thermodynamic parameters of the system are strongly dependent on the effective temperature, the step parameter and on the confinement. In the zero temperature regime the effective temperature is different from zero (see Fig. 1) and it leads to the behaviors of the entropy, the internal energy and the specific heat capacity, as discussed in Sec. 3, which shows that de-coherence exists even at zero temperature. Nevertheless, this de-coherence can be reduced in a double-well potential field with a small value of the step parameter by simply increasing the magnetic confinement of the system. Thus, the quantum Brownian particle coupled to its environment and subjected to a constant electromagnetic field looks classical if we reduce the magnetic confinement or increase the step parameter and the temperature. It is therefore obvious that the magnetic confinement plays an important role on the de-coherence aspect of a Brownian particle in a double-well potential coupled to its environment by reducing the effects of the environmental degrees of freedom.

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Received: 18 March, 2014
Accepted: 18 July, 2014