

LRS Bianchi Type-I Model with Time Dependent Cosmological Term Λ

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In this paper, we investigate LRS Bianchi type-I cosmological model for stiff matter and vacuum solution with a cosmological term proportional to $\frac{\ddot{R}}{R}$, where R is the scale factor. The cosmological term decreases with increasing time. We show that the present universe is accelerating with a large fraction of cosmological density in the form of cosmological term.

1. Introduction

The decrease of cosmological constant with time is considered an interesting problem at present. Recent observations [29-34] indicate that Λ is very small. In earlier literature, cosmological models where the cosmological term is proportional to scale factor have been studied by many researchers [3-9]. Chen and Wu [10] considered Λ varying as R^{-2} , where R is the scale factor, Carvalho et al. [11] generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$, where R is the scale factor of Robertson-Walker metric and H is the Hubble parameter, and α, β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background. Another solution to field equations may be generated using the law of variation of scale factor that was proposed by D. Pavon [2]. The behavior of cosmological scale factor $R(t)$ in the solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies.

A number of researchers investigated FRW models and Bianchi type models using a different approach [12-24]. Borges and Carneiro [25] have considered the case where the cosmological term is proportional to the Hubble parameter in FRW model with Λ as a variable. Recently, we considered the case where cosmological term is proportional to Hubble parameter in Bianchi type-I model with varying G and Λ [26] and $\Lambda \propto R^m$ (m is a constant) [35,36].

At present, the universe seems to be homogeneous and isotropic at a large scale. But many authors have considered anisotropic cosmological solutions as possible models for the beginning of the universe, which isotropize very rapidly due to particle production in the course of their expansion [16-23]. Therefore, it would be worthwhile to study the decay of Λ in an anisotropic background. Bianchi type-I cosmological models, which are anisotropic generalization of FRW models with flat space slices and have been studied by a number of workers.

In this paper, we investigate an LRS Bianchi type-I model containing stiff matter and the vacuum solution with a cosmological term proportional to $\frac{\ddot{R}}{R}$.

2. Field Equations

We consider the LRS Bianchi type-I metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2) \quad (1)$$

Where, A and B are function of cosmic time t only.

Matter components of cosmic fluid consist of perfect fluid represented by energy momentum tensor

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \quad (2)$$

with the equation of state,

$$p = \omega \rho, \quad 0 \leq \omega \leq 1 \quad (3)$$

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Here p and ρ are pressure and energy density, respectively, and v_i is the unit flow vector satisfying $v_i v^i = -1$. For stiff fluid ($p = \rho$), we have $\omega = 1$. The Einstein's field equation (in gravitational units $8\pi G = c = 1$) with time dependent cosmological term $\Lambda(t)$ is

$$R_{ij} - \frac{1}{2} R g_{ij} = -(T_{ij} - \Lambda g_{ij}) \quad (4)$$

Since, the vacuum has symmetry of the background, its energy-momentum tensor has the form, $T_{ij}^{vac} = \Lambda g_{ij}$, where Λ is an invariant function of time in homogeneous space.

In co-moving coordinates, this corresponds to a perfect fluid with energy density $\epsilon_v = \Lambda$ and pressure $p_v = -\Lambda$ satisfying the equation of state

$$p_v = -\epsilon_v = -\Lambda \quad (5)$$

For the metric (Eqn. (1)) and energy-momentum tensor (Eqn. (2)), the field equations (Eqn. (4)) yield

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p + \Lambda \quad (6)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \rho + \Lambda \quad (7)$$

Where, the dot means derivation with respect to the cosmological time.

Eliminating p and Λ from (6), we get

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^{\bullet} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \quad (8)$$

Integrating Eqn. (8) gives

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k}{AB^2} \quad (9)$$

Where, k is a constant of integration.

Computing the divergence of Eqn. (4) leads to

$$\dot{\rho} + 3(\rho + p)H + \dot{\Lambda} = 0 \quad (10)$$

Where, $R^3 = AB^2$ and, $H = \frac{\dot{R}}{R} = \frac{\theta}{3}$, θ being the volume expansion scalar.

When Λ is constant, we get the continuity equation for matter. In view of energy conservation, equation (10) shows that a decaying vacuum term leads to matter production. Equations (6), (7) and (10) can also be written in terms of Hubble parameter H , deceleration parameter q and shear σ as

$$H^2(2q - 1) - \sigma^2 = p - \Lambda \quad (11)$$

$$3H^2 - \sigma^2 = \rho + \Lambda \quad (12)$$

$$\frac{\dot{\sigma}}{\sigma} = -3H \quad (13)$$

Where, $q = -\frac{\ddot{R}}{RH^2}$ and $\sigma^2 = \frac{k^2}{3R^6}$.

Thus, energy density ϵ_β associated with anisotropy given by $\epsilon_\beta = \sigma^2$ decays very rapidly due to the expansion by converting in to photons. From Eqn. (12), we get

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\Lambda}{\theta^2}$$

This implies that for $\Lambda \geq 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, \quad 0 < \frac{\rho}{\theta^2} < \frac{1}{3}$$

Therefore, the presence of positive Λ lowers the upper limit of anisotropy. From Eqns. (11) and (12), we get

$$\dot{\theta} = \frac{-3\omega\rho}{2} - 3\sigma^2 = -\frac{9}{2}\omega H^2 + \frac{3}{2}(\omega - 2)\sigma^2 + \frac{3}{2}\omega\Lambda \quad (14)$$

This means that the rate of volume expansion decreases during time evolution and the presence of positive Λ arrests this decrease, whereas negative Λ promotes it. It is also to mention that the rate of decrease of volume expansion is faster in anisotropic space in comparison to isotropic one. Also Eqns. (3), (11) and (12) gives

$$\frac{\Lambda}{H^2} = 2 - q + (\omega - 1)\frac{\rho}{2H^2} \quad (15)$$

We can determine the present value of the cosmological constant.

If, $q < 2 + (\omega - 1)\frac{\rho}{2H^2}$, $\Lambda > 0$, whereas $\Lambda < 0$

for $q > 2 + (\omega - 1)\frac{\rho}{2H^2}$.

Following Cooperstock [2], Al-Rawaf et al. [37,38] and Pradhan et al. [39], we consider the decaying vacuum energy density

$$\Lambda = a \frac{\ddot{R}}{R} \quad (16)$$

Where, $a (\neq 1)$ and m are positive constants.

Let

$$\Omega = \frac{\rho}{\rho_c} \quad (17)$$

be the density parameter.

For stiff fluid ($\rho = p$) Eqns. (11), (12) and (16) lead to the differential equation

$$(1-a) \frac{\ddot{R}}{R} + 2 \left(\frac{\dot{R}}{R} \right)^2 = 0 \quad (18)$$

This determines the scale factor. Apart from an integration constant related to the choice of the origin of time, its general solution is

$$R = \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{\frac{1-a}{3-a}} \quad (19)$$

Where, k_1 and k_2 are the constant of integration.

For this solution metric (Eqn. (1)) assumes the form

$$ds^2 = -dt^2 + \left(\frac{3-a}{1-a} k_1 t + k_2 \right)^{\frac{(1-a)}{k_1(3-a)}} \exp \left[\frac{k(3-a)}{3a} \left\{ \frac{3-a}{1-a} k_1 t + k_2 \right\}^{\frac{2a}{3-a}} \right] dx^2 \\ + \left(\frac{3-a}{1-a} k_1 t + k_2 \right)^{\frac{(1-a)}{k_1(3-a)}} \exp \left[\frac{-k(3-a)}{6a} \left\{ \frac{3-a}{1-a} k_1 t + k_2 \right\}^{\frac{2a}{3-a}} \right] (dy^2 + dz^2) \quad (20)$$

3. Discussion

For model given by Eqn. (20), spatial volume V , cosmological energy density ρ , pressure p , and cosmological constant Λ are given by

$$V = \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{\frac{3(1-a)}{3-a}} \quad (21)$$

$$\rho = \frac{k_1}{\left(\frac{3-a}{1-a} k_1 t + k_2 \right)^2 \left(\frac{3-a}{1-a} \right)} \\ + \frac{k^2}{3 \left(\frac{3-a}{1-a} k_1 t + k_2 \right)^{\frac{6(1-a)}{3-a}}} - \frac{2kk_1}{\left(\frac{3-a}{1-a} k_1 t + k_2 \right)^{\frac{6-4a}{3-a}}} \quad (22)$$

$$\Lambda = \frac{2a}{a-1} k_1^2 \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{-2} \quad (23)$$

Expansion scalar θ and shear σ are given by

$$\theta = k_1 \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{-1} \quad (24)$$

$$\sigma = \frac{k}{\sqrt{3}} \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{-3 \left(\frac{1-a}{3-a} \right)} \quad (25)$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \left[\frac{3(3-a)}{(1-a)} - \frac{k^2}{k_1^2} \left(\frac{3-a}{1-a} k_1 t + k_2 \right)^{\frac{4a}{3-a}} \right] \quad (26)$$

The deceleration parameter q for the model is

$$q = \frac{a+17}{a-1} - 1 \quad (27)$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{2a}{a-1} k_1^2 \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{-2} \quad (28)$$

$$\rho_c = \frac{k_1^2}{3} \left[\frac{3-a}{1-a} k_1 t + k_2 \right]^{-2} \quad (29)$$

In the model, we observe that the spatial volume V is zero at $t = \frac{-k_2}{k_1} \left(\frac{1-a}{3-a} \right) = t_0$ and expansion scalar

θ is infinite at $t = t_0$, which shows that the universe starts evolving with zero volume and infinite rate of expansion. The scale factors also vanish at $t = t_0$ and hence the model has a point type singularity at the initial epoch. Initially at $t = t_0$ the cosmological energy density ρ , pressure p , cosmological constant Λ , and shear scalar σ are infinite. As t increases, the spatial volume increases but the expansion scalar decreases. Thus, the expansion rate decreases as time increases. As t tends to infinity the spatial volume V becomes infinitely large. As t increases, parameters p , ρ , Λ , θ , ρ_c , and ρ_v all decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t . For $a > 3$, the ratio $\frac{\sigma}{\theta} \rightarrow 0$ as

$t \rightarrow \infty$, which shows the model approaches isotropy for large values of t . Also we see that, $\Lambda \propto \frac{1}{t^2}$,

which follows from the model of Kalligas *et al.* [17], Berman [26], Berman and Som [27], Berman *et al.* [28] and Bertolami [29]. This form of Λ is physically reasonable as observation suggest that Λ is very small in the present universe.

Furthermore, for $a = 0$, $q < 0$ and $q > 0$ for $a > 1$. Thus the universe begins with a decelerating expansion change and then the expansion changes from decelerating phase to accelerating one. This cosmological scenario is in agreement with SNe Ia astronomical observational [29-34] and it presents a unified description of the evolution of the universe.

4. Physical and Geometrical Features

In this paper, we have studied a spatially homogeneous and anisotropic LRS Bianchi type-I space time with variable cosmological constant $\Lambda(t)$. The Einstein's field equations have been solved by using a law of variation of scale factor with a variable cosmological term i.e., a cosmological term that scales as

$$\Lambda \propto \frac{\ddot{R}}{R}$$

Where, R is scale factor. Expressions for some important cosmological parameters have been obtained and the physical behavior of this model is discussed in detail. Clearly, the model represents a shearing, non-rotating and expanding universe with a big-bang start. Model has point type singularity at initial epoch and approaches to isotropy at late times. It is interesting that the proposed variation law provides an alternative approach to obtain exact solution of Einstein's field equations (see, e.g., [1]). It presents a unified description of the evolution of the universe, which starts with a decelerating expansion and expands with acceleration at late times. Recent observational data [30-34] strongly suggest this acceleration. The cosmological constant varies as $\Lambda(t) \propto \frac{1}{t^2}$, which

corresponds to the model of Kalligas *et al.* and others [17,15]. This form of Λ is physically reasonable as observation suggests that Λ is very small in the present universe. Finally, the solution obtained in this paper is new and useful for better understanding of the evolution in LRS Bianchi type-I universe with Λ as a variable.

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