

Bianchi Type-V Universe with Anisotropic Dark Energy in Lyra's Geometry

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The present study deals with Bianchi type-V cosmological models with anisotropic dark energy in Lyra's geometry. The field equations are solved explicitly by a particular form of Hubble parameter that represents an early deceleration and late times acceleration. It has been found that displacement vector (β) is a decreasing function of time and it approaches to small positive value at late time, which is consistent with recent observations. Physical and kinematical behaviors of the model are discussed.

1. Introduction

Recent observations like type Ia supernova (SN Ia) [1-4], CMB anisotropy [5] and large scale structure [6] strongly indicate that our universe is spatially flat and there exists an exotic cosmic fluid called dark energy (DE) with negative pressure, which constitutes about 70 percent of the total energy of universe. Dark energy dominates the present universe, causing cosmic acceleration. This acceleration is realized with negative pressure and positive energy density that violate the strong energy condition. This violation gives a reverse gravitational effect. Due to this effect, the universe got a jerk and the transition from the earlier deceleration phase to the recent acceleration phase took place [7]. The cause of this sudden transition and the source of accelerated expansion are still unknown. In physical cosmology and astronomy, the simplest candidate for the dark energy is the cosmological constant Λ , but it needs to be extremely fine-tuned to satisfy the current value of the dark energy density, which is a serious problem. Alternatively, to explain the decay of density, the different form of dynamically changing dark energy with an effective equation of state $\omega = p^{(de)} / \rho^{(de)} < -1/3$ were proposed instead of the constant vacuum energy density. Other possible forms of dark energy include quintessence ($\omega > -1$) [8] and phantom ($\omega < -1$) [9]. While the possibility $\omega \ll -1$ is ruled out by the current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) [4,10], CMBR (WMAP, BOOMER-ANG)

[11,12] and large scale structure (Sloan Digital Sky Survey) [13] data, but the dynamically evolving dark energy crossing the phantom divide line (PDL) ($\omega = -1$) is mildly favored. Some other limits obtained from the observational results coming from SN Ia data [14] collaborated with CMBR anisotropy and galaxy clustering statistics [15] are $-1.67 < \omega^{(de)} < -0.62$ and $-1.33 < \omega^{(de)} < -0.79$, respectively.

Bianchi type-V universe is a generalization of open universe in FRW cosmology and hence its study is important in the context of dark energy models in the universe with non-zero curvature [16]. A number of authors such as Collins [17], Maartens and Nel [18], Wainwright et al. [19], Singh et al. [20], and Yadav [21] have studied Bianchi type-V model in different physical contexts. Recently Kumar and Yadav [22] have studied anisotropic dark energy model with a variable equation of state parameter in Bianchi type-V space-time and found that the universe is dominated by dark energy at present epoch, and after the dominance of dark energy the universe achieves flatness. This motivates the researcher to consider the model of universe with anisotropic dark energy.

The isotropic models are considered to be most suitable to study large scale structure of the universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the universe with models having anisotropic background. Thus, it would be worthwhile to explore anisotropic dark energy models in the context of modified theories of gravity. Among the various modification of general relativity, Lyra geometry is a well known example of scalar tensor

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theory. Lyra [23] suggested a modification of Riemannian geometry by introducing a gauge function into the structure less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a close resemblance to Weyl's [24] geometry. In consecutive investigations, Sen [25] and Sen and Dunn [26] have proposed a new scalar tensor theory of gravitation and constructed an analogue of the Einstein field equation based on Lyra's geometry. Halford [27] has pointed out that the constant displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal relativistic treatment. According to Halford [28] the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits as in Einstein's theory. Soleng [29] has pointed out that the constant displacement field in Lyra's geometry will either include a creation field and be equal to Hoyle's creation field cosmology or contain a special vacuum field, which together with the gauge vector form may be considered as a cosmological term. Subsequent investigations were done by several authors in cosmology and topological defects within the frame work of Lyra's geometry [30-36]. In this paper, we have considered minimally interacting perfect fluid and dark energy components with time dependent deceleration parameter within the framework of a Bianchi type-V space-time in Lyra's geometry. This paper is organized as follows. In Sec. 2, the metric and field equations are presented. Sec. 3 deals with the exact solutions of the field equations and physical behavior of the model. Finally, concluding remarks are given in Sec. 4.

2. Metric and Field Equations

Spatially homogeneous and anisotropic Bianchi type-V space-time is given by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x} (B^2 dy^2 + C^2 dz^2) \quad (1)$$

Where, A, B, C are metric functions of cosmic time t and α is a constant.

We define $S = (ABC)^{\frac{1}{3}}$ as the average scale factor of the space-time (Eqn. (1)) so that the average Hubble's parameter is written as

$$H = \frac{\dot{S}}{S} \quad (2)$$

Where, an over dot denotes derivative with respect to cosmic time t .

The field equations on Lyra's manifold as obtained by Sen [25] are (units: $8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -T_{ij} \quad (3)$$

Where, R_{ij} is the Ricci tensor, R is the Ricci scalar, ϕ_i is the displacement vector defined as $\phi_i = (0, 0, 0, \beta(t))$ and $T_{ij} = T_{ij}^{(m)} + T_{ij}^{(de)}$ is the overall energy momentum tensor, and $T_{ij}^{(m)}$ and $T_{ij}^{(de)}$ are energy momentum tensors of ordinary matter and dark energy, respectively. These are given by

$$\begin{aligned} T_j^{(m)i} &= \text{diag}[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}] \\ &= \text{diag}[-1, \omega^{(m)}, \omega^{(m)}, \omega^{(m)}] \rho^{(m)} \end{aligned} \quad (4)$$

and

$$\begin{aligned} T_j^{(de)i} &= \text{diag}[-\rho^{(de)}, p^{(de)}, p^{(de)}, p^{(de)}] \\ &= \text{diag}[-1, \omega^{(de)}, \omega^{(de)}, \omega^{(de)}] \rho^{(de)} \end{aligned} \quad (5)$$

Where, $\rho^{(m)}$ and $p^{(m)}$ are the energy density and pressure of the perfect fluid component or ordinary baryonic matter, respectively, while $\omega^{(m)} = \frac{p^{(m)}}{\rho^{(m)}}$

is its equation of state. Similarly, $\rho^{(de)}$ and $p^{(de)}$ are the energy density and pressure of the dark energy component while $\omega^{(de)} = \frac{p^{(de)}}{\rho^{(de)}}$ is the

corresponding equation of state parameter.

The field equation (Eqn. (3)), for the Bianchi type-V space-time (Eqn. (1)), in case of (Eqn. (4)) and (Eqn. (5)) leads to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} + \frac{3}{4} \beta^2 = -\omega^{(m)} \rho^{(m)} - \omega^{(de)} \rho^{(de)} \quad (6)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} + \frac{3}{4} \beta^2 = -\omega^{(m)} \rho^{(m)} - \omega^{(de)} \rho^{(de)} \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = -\omega^{(m)}\rho^{(m)} - \omega^{(de)}\rho^{(de)} \quad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3\alpha^2}{A^2} - \frac{3}{4}\beta^2 = \rho^{(m)} + \rho^{(de)} \quad (9)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (10)$$

The conservation of the left hand side of (Eqn. (3)) leads to

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \quad (11)$$

The energy conservation equation $T_{;j}^{ij} = 0$ yields

$$\begin{aligned} \dot{\rho}^{(m)} + 3(1 + \omega^{(m)})\rho^{(m)}H + \dot{\rho}^{(de)} \\ + 3(1 + \omega^{(de)})\rho^{(de)}H = 0 \end{aligned} \quad (12)$$

3. Solution of Field Equations and its Physical Significance

From Eqns. (6), (7), (8) and (10), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} \quad (13)$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} - \frac{k}{S^3} \quad (14)$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} + \frac{k}{S^3} \quad (15)$$

Where, k is a constant of integration.

Eqns. (13) to (15), on integration give

$$A = m_1 S \quad (16)$$

$$B = m_2 S \exp\left(-k \int \frac{dt}{S^3}\right) \quad (17)$$

$$C = m_3 \exp\left(k \int \frac{dt}{S^3}\right) \quad (18)$$

Where, m_1 , m_2 and m_3 are constants of integration satisfying $m_1 m_2 m_3 = 1$.

In order to solve the field equations completely, we first assume that the perfect fluid and dark energy component interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation, $T_{;j}^{(m)ij} = 0$, of perfect fluid leads to

$$\dot{\rho}^{(m)} + 3(1 + \omega^{(m)})\rho^{(m)}H = 0 \quad (19)$$

Whereas, the energy conservation equation, $T_{;j}^{(de)ij} = 0$, of the dark energy leads to

$$\dot{\rho}^{(de)} + 3(1 + \omega^{(de)})\rho^{(de)}H = 0 \quad (20)$$

Following Akrasu and Kilinc [37], we assume that the equation of state parameter of a perfect fluid to be constant. So,

$$\omega^{(m)} = \frac{p^{(m)}}{\rho^{(m)}} = \text{cons.} \quad (21)$$

While, $\omega^{(de)}$ has been allowed to be a function of time.

Now integration of Eqn. (19) leads to

$$\rho^{(m)} = c_0 S^{-3(1+\omega^{(m)})} \quad (22)$$

Where, c_0 is a constant of integration.

We now introduce volume expansion θ and shear scalar σ as usual

$$\theta = v_{;i}^i \quad \text{and} \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad (23)$$

Where, σ_{ij} is the shear tensor and semicolon (;) stands for covariant differentiation. For the Bianchi type-V model, expansion scalar θ and shear scalar σ are given by

$$\theta = \frac{3\dot{S}}{S} \quad (24)$$

$$\sigma = \frac{k}{S^3} \quad (25)$$

In analogy with FRW universe, we define a generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{S}}{S} \quad (26)$$

$$q = -\frac{\ddot{S}}{SH^2} \quad (27)$$

In order to get an expanding model of the universe consistent with recent observations, one needs a Hubble parameter H such that the model starts with decelerating expansion followed by an accelerated expansion at late times. Following the lines floated by Ellis and Madsen [38], we choose a functional form for H , which despite being quite simple describes both the deceleration and the accelerating phase of the universe. These conditions are satisfied by the one-parameter function,

$$H(S) = a(S^{-n} + 1) \quad (28)$$

Where, $a(>0)$ and $n(>1)$ are constants. For this choice, the deceleration parameter q turns out to be

$$q = \frac{n}{(S^n + 1)} - 1 \quad (29)$$

A similar form for q has also been proposed by Banerjee and Das [39] in the case of Robertson-Walker space-time. From Eqn. (29), we observed that when $S = 0$, $q = n - 1 > 0$, for $S^n = n - 1$, $q = 0$ and for $S^n > n - 1$, $q < 0$. We assume that the value $S = 0$ for $t = 0$. Therefore, the universe begins with a decelerating expansion and the expansion changes from the decelerating phase to accelerating phase. Integrating Eqn. (28), we obtain

$$S^n = e^{na(t+t_1)} - 1 \quad (30)$$

Where, t_1 is a constant of integration. Assuming $S = 0$ for $t = 0$, we get $t_1 = 0$. Therefore

$$S^n = e^{nat} - 1 \quad (31)$$

The expressions for physical parameters such as the Hubble parameter, (H), expansion scalar (θ), shear scalar (σ), spatial volume V , anisotropy parameter A_m and deceleration parameter q are given by

$$H = \frac{a}{1 - e^{-nat}} \quad (32)$$

$$\theta = \frac{3a}{1 - e^{-nat}} \quad (33)$$

$$\sigma = \frac{k}{(e^{nat} - 1)^{\frac{3}{n}}} \quad (34)$$

$$V = S^3 = (e^{nat} - 1)^{\frac{3}{n}} \exp(2\alpha x) \quad (35)$$

$$A_m = \frac{2k^2}{3a^2 e^{2nat} (e^{nat} - 1)^{\frac{6}{n}-2}} \quad (36)$$

$$q = \frac{n}{e^{nat}} - 1 \quad (37)$$

Also,

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \frac{k}{a(e^{nat} - 1)^{\frac{2}{n}}} = 0 \quad (38)$$

It is observed that at $t = 0$ the spatial volume vanishes and other parameters θ, σ, H diverge. Hence, the model starts with a big bang singularity at $t = 0$. We observe that A_m decreases with time and tends to zero as $t \rightarrow \infty$ for $n < 3$. Hence, the present model is isotropic at late time, which is consistent to the current observations.

From Eqns. (31) and (11), we have

$$\beta = \frac{k_1}{(e^{nat} - 1)^{\frac{3}{n}}} \quad (39)$$

Where, k_1 is a constant of integration. The energy density ($\rho^{(m)}$) of perfect fluid, the pressure ($p^{(de)}$) of dark energy component, dark energy density ($\rho^{(de)}$), and equation of state parameter ($\omega^{(de)}$) of dark energy, for this model are given by

$$\rho^{(m)} = c_0 (e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})} \quad (40)$$

Where, c_0 is a constant of integration.

$$p^{(de)} = -2a^2 n(e^{nat} - 1)^{-1} - a^2(3 - 2n)e^{2nat}(e^{nat} - 1)^{-2} \\ + \frac{\alpha^2}{m_1^2}(e^{nat} - 1)^{\frac{-2}{n}} - \frac{3}{4}k_1^2(e^{nat} - 1)^{\frac{-6}{n}} \\ - \omega^{(m)}c_0(e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})} \quad (41)$$

$$\rho^{(de)} = 3a^2 e^{2nat}(e^{nat} - 1)^{-2} - (k^2 + \frac{3}{4}k_1^2)(e^{nat} - 1)^{\frac{-6}{n}} \\ - \frac{3\alpha^2}{m_1^2}(e^{nat} - 1)^{\frac{-2}{n}} - \omega^{(m)}c_0(e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})} \quad (42)$$

$$\omega^{(de)} = \frac{-2a^2 n(e^{nat} - 1)^{-1} - a^2(3 - 2n)e^{2nat}(e^{nat} - 1)^{-2} \\ + \frac{\alpha^2}{m_1^2}(e^{nat} - 1)^{\frac{-2}{n}} \\ - \frac{3}{4}k_1^2(e^{nat} - 1)^{\frac{-6}{n}} - \omega^{(m)}c_0(e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})}}{3a^2 e^{2nat}(e^{nat} - 1)^{-2} - (k^2 + \frac{3}{4}k_1^2)(e^{nat} - 1)^{\frac{-6}{n}} \\ - \frac{3\alpha^2}{m_1^2}(e^{nat} - 1)^{\frac{-2}{n}} - \omega^{(m)}c_0(e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})}} \quad (43)$$

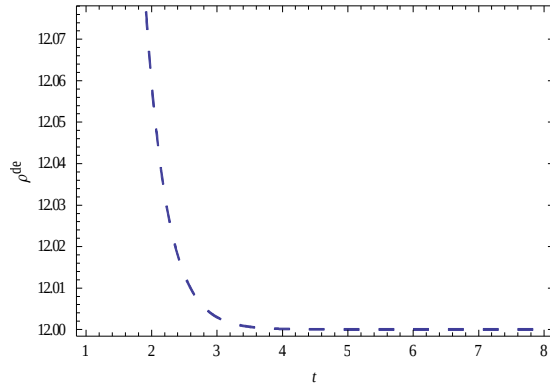


Fig.1: Variation of ρ^{de} vs. t choosing the parameters $a=0.5$, $n=2$, $\alpha=0.02$, $k=0.17$, $k_1=0.07$, $c_0=1$, $m_1=0.87$ and $\omega^{(m)}=0.012$.

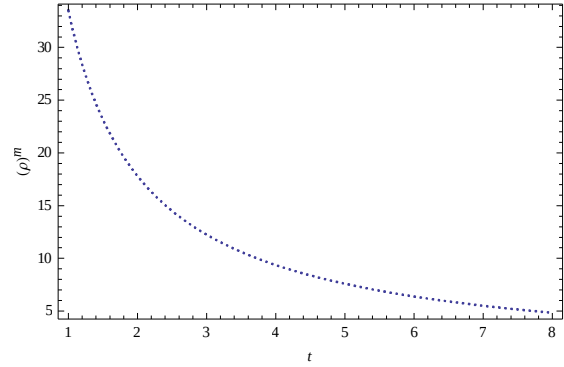


Fig.2: Matter energy density versus t with, $a=0.01$, $n=2$, $c_0=1$ and $\omega^{(m)}=0.4$.

From Fig. 1 and Fig. 2, we observe that $\rho^{(de)}$ and $\rho^{(m)}$ remain positive during cosmic evolution. Therefore, the weak energy conditions (WEC) as well as the null energy condition (NEC) are obeyed in the derived model. Further, $\rho^{(m)}$ and $\rho^{(de)}$ decrease with time and approaches to a small positive values at present epoch.

$$(\rho + p)^{(de)} = -2a^2 n(e^{nat} - 1)^{-2} + 2a^2 ne^{2nat}(e^{nat} - 1)^{-2} \\ - \frac{2\alpha^2}{m_1^2}(e^{nat} - 1)^{\frac{-2}{n}} - \left(k^2 + \frac{3}{4}k_1^2\right)(e^{nat} - 1)^{\frac{-6}{n}} \\ - 2\omega^{(m)}c_0(e^{nat} - 1)^{\frac{-3}{n}(1+\omega^{(m)})} \quad (44)$$

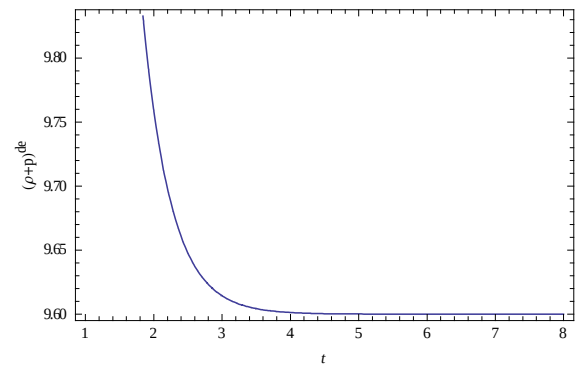


Fig.3: Variation of $(\rho + p)^{de}$ vs. t by choosing the parameters $a=0.5$, $n=2$, $\alpha=0.73$, $m_1=0.04$, $k=0.32$, $k_1=0.13$ and $\omega^{(m)}=0.98$.

$$\begin{aligned}
 (\rho + 3p)^{(de)} = & -6a^2 n (e^{nat} - 1)^{-2} - 6a^2 (1-n) e^{2nat} (e^{nat} - 1)^{-2} \\
 & - (k^2 + 3k_1^2) (e^{nat} - 1)^{-\frac{6}{n}} - (k^2 + 3k_1^2) (e^{nat} - 1)^{-\frac{6}{n}} \\
 & - 4\omega^{(m)} c_0 (e^{nat} - 1)^{-\frac{3}{n}(1+\omega^{(m)})} \quad (45)
 \end{aligned}$$

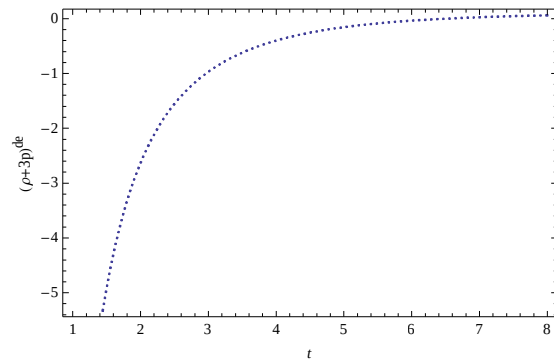


Fig.4: Variation of $(\rho + 3p)^{(de)}$ vs. t by choosing the parameters $a = 0.5$, $n = 3$, $c_0 = 1$, $k = 0.001$, $k_1 = 0.6$ and $\omega^{(m)} = 0.12$.

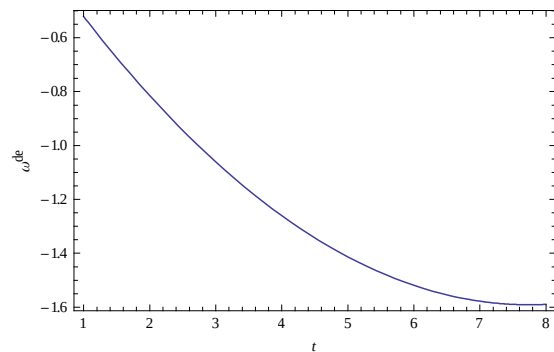


Fig.5: Variation of ω^{de} vs. t by taking parameters $a = 0.5$, $k = 0.07$, $k_1 = 0.13$, $m_1 = 0.56$, $c_0 = 1$ and $n = 2$.

From the above Figs. 1, 3 and 4, we observe that

$$\rho^{(de)} \geq 0, (\rho + p)^{(de)} \geq 0 \quad \text{and} \quad (\rho + 3p) < 0$$

Thus, the model violates strong energy condition (SEC) as the same predicted by current astronomical observations.

The perfect fluid density parameter $(\Omega^{(m)})$ and dark energy parameter $(\Omega^{(de)})$ are given by

$$\Omega^{(m)} = \frac{c_0}{3a^2} e^{-3at(1+\omega^{(m)})} (1 - e^{-nat})^{\frac{-3}{n}(1+\omega^{(m)})+2} \quad (46)$$

$$\begin{aligned}
 \Omega^{(de)} = & 1 - \left(\frac{k^2 + \frac{3}{4}k_1^2}{3a^2} \right) e^{-6at} (1 - e^{-nat})^{\left(2 - \frac{6}{n}\right)} \\
 & - \frac{\alpha^2}{m_1^2 a^2} e^{-2at} (1 - e^{-nat})^{\left(2 - \frac{2}{n}\right)} \\
 & - \frac{\omega^{(m)} c_0}{3a^2} e^{-3at(1+\omega^{(m)})} (1 - e^{-nat})^{\frac{-3}{n}(1+\omega^{(m)})+2} \quad (47)
 \end{aligned}$$

Thus, the overall density parameter (Ω) is obtained as

$$\begin{aligned}
 \Omega = & \Omega^{(m)} + \Omega^{(de)} \\
 = & 1 - \left(\frac{k^2 + \frac{3}{4}k_1^2}{3a^2} \right) e^{-6at} (1 - e^{-nat})^{\left(2 - \frac{6}{n}\right)} \\
 & - \frac{\alpha^2}{m_1^2 a^2} e^{-2at} (1 - e^{-nat})^{\left(2 - \frac{2}{n}\right)} \\
 & + (1 - \omega^{(m)}) \frac{c_0}{3a^2} e^{-3at(1+\omega^{(m)})} (1 - e^{-nat})^{\frac{-3}{n}(1+\omega^{(m)})+2} \quad (48)
 \end{aligned}$$

From Eqn. (48), it is observed that the overall density parameter (Ω) approaches to 1 at late times. Thus, the derived model predicts a flat universe at present epoch.

4. Concluding Remarks

In this paper, we have studied Bianchi type-V cosmological models with anisotropic dark energy in the Lyra's geometry. The field equations are solved explicitly by particular form of Hubble's parameter, which represents an early deceleration and late time acceleration. We observe that spatial volume of the model is zero at $t = 0$. Therefore, the model starts evolving at $t = 0$ and expands with cosmic time. At $t = 0$, the spatial volume vanishes and other parameters θ, σ, H diverge. Hence, the model starts with a big bang singularity at $t = 0$. At $t = 0$, the deceleration parameter is $q = n - 1 (> 0)$. Therefore, the expansion in the model decelerates. In the limit $t \rightarrow \infty$, we obtain $q = -1$, which characterizes the de Sitter solution [40]. Therefore, the model starts with a decelerating expansion and expansion in the model changes from the decelerating phase to an accelerating one. For large

values of t , $\frac{\sigma}{\theta}$ tends to zero. Therefore, the model approaches isotropy at late times. From Eqn. (39), it has been found that the displacement vector (β) is a decreasing function of time and it approaches to small positive value at late times, which is consistent with recent observations.

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Received: 18 December, 2013

Accepted: 2 May, 2014