

Mix-master Universe Model with Decaying Vacuum Density

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Mix-master space-time is considered in the presence of cosmic string with decaying Λ term in Einstein's general theory of relativity. To obtain a deterministic solution, we have assumed the conditions: $\Lambda = aR^{-m}$ (where R is scale factor, a and m are constants) and $\rho + \lambda = 0$. The physical properties of the model are discussed. The model provides an alternate way to study Mix-master space-time and shows No-Hair conjecture.

1. Introduction

Mixmaster Universe (Bianchi type-IX) models have drawn considerable attention among researchers in the sense that these models allow not only expansion but also rotation and shear, and in general are anisotropic. Many relativists have taken keen interest to study Mixmaster universes because popular solutions such as Robertson Walker universe with positive curvature, the de-sitter universe and the Taub-Nut are of Mixmaster space-time. In these models, neutrino viscosity does not guarantee isotropy at the present epoch [1-5].

On the other hand, the collapse conjecture for a Mixmaster space-time has been proved by Lin and Wald [6,7]. The collapse conjecture states that an initially expanding Mixmaster space-time cannot expand for an infinite time. It should, in a finite time, reach a maximum of expansion and then begin to contract. The proof given in reference [6] that there do not exist solutions which expand forever is divided in two steps. In the first step, it is proved that for any solution in which the universe expands forever, the dynamical trajectory in the anisotropy plane must escape to infinity along one of the channels of the mix-master potential. The second step uses detailed properties of the equation of motion to show that such escape along a channel is impossible.

The Bianchi type-IX model (dubbed mix-master) has been intensively studied over the years. This model was first investigated by Belinski, Khalatnikov and Lifschitz [8, 9] and Misner [10]. One of the most appealing properties is its chaotic motion dynamics. The emergence of chaos helped in the understanding of the singularities in general relativity [11-14], the readers are referred to [12].

For historical reasons, the approach to study Bianchi model was to rewrite Einstein equations in the Hamiltonian form (with constrain). This was intensively pursued by Misner [10,11] and several authors for which a nice Hamiltonian system was constructed, although this specific form of Hamiltonian was not used in the proof of the collapse conjecture by Lin and Wald [6,7].

Kibble [15] has investigated that strings are one of the sources of density perturbation, which is required for the formation of large scale structure of the universe. These strings possess stress energy and are coupled to the gravitational field. Latelier [16] and Satchel [17] have initiated the general relativistic treatment of the strings. Consequently, many relativists have obtained exact solutions which describe homogeneous string cosmological models with different Bianchi symmetries. Bali and Anjali [18] have investigated Bianchi type-I magnetized string cosmological model in general relativity. Rao et al. [19] studied exact Bianchi type II, VIII, and IX string cosmological models in Saez-Ballester theory of gravitation. Singh [20] also studied string cosmology with electromagnetic fields in Bianchi type II, VIII, and IX space-times. Bali and Dave [21], Bali, Pradhan and Amirhashchi [22], Bali and Pradhan [23], Wang [24], Adhav, Dawande and Raut [25] have investigated Bianchi type-IX string cosmological models with bulk viscosity. The study of cosmic strings has received considerable attention in cosmology since they played an important role in structure formation and evolution of the universe [26,27]. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [28] and Goetz [29] in general relativity.

In the present paper, we obtain Mix-master universe with the help of relations, $\Lambda = aR^{-m}$ (where R is scale factor, a and m are constants) and

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$\rho + \lambda = 0$. The paper is organized as follows: Sec. 2 contains metric and field equations; Sec. 3 displays the solution of field equations with cosmic string as a source; and finally Sec. 4 and Sec. 5 show the discussion and conclusion parts of the model, respectively.

2. Metric and Field Equations

The metric in Mix-master universe is of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + (B^2 \sin^2 y + A^2 \cos^2 y) dz^2 - 2A^2 \cos y dx dz \quad (1)$$

Where, A and B are functions of t alone.

The energy-momentum tensor for a cloud of strings with perfect fluid distribution is taken as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j \quad (2)$$

Where, v^i and x^i satisfy the condition

$$v_i v^i = -1 = -x_i x^i \quad (3)$$

and

$$v_i x^i = 0 \quad (4)$$

Where, p is the pressure, ρ is the proper density for cloud of strings with particles attached to them, λ is the tensor string density, v^i is the four velocity vector of the particles and x^i is a unit space like vector representing the direction of the string.

In a co-moving coordinate system, we have

$$v^i = (0, 0, 0, 1); \quad x^i = \left(\frac{1}{A}, 0, 0, 0\right) \quad (5)$$

If the particle density of configuration is denoted by ρ_p , then

$$\rho = \rho_p + \lambda \quad (6)$$

The Einstein field equations (in gravitational units, $8\pi G = c = 1$) are

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda g_i^j \quad (7)$$

For the line element (Eqn. (1)), Eqn. (7) leads to

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} = \rho + \Lambda \quad (8)$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} = -p + \lambda + \Lambda \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{A^2}{4B^4} = -p + \Lambda \quad (10)$$

Where, an overhead dot here and everywhere stands for derivative with respect to t .

The average scale factor (R) for Mixmaster space-time is defined as

$$R = (AB^2)^{\frac{1}{3}} \quad (11)$$

The expansion scalar (θ) and shear scalar (σ) are defined as

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \quad (12)$$

$$\sigma^2 = \frac{2}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 \quad (13)$$

3. Solution of Field Equations with Cosmic String as a Source

The Field equations (8)-(10) are a system of three equations with six unknowns A, B, ρ , p , λ and Λ . For complete determinacy of the system, we need three extra conditions. Firstly, we assume that the expansion scalar θ in the model is proportional to the shear scalar σ .

This condition leads to

$$A = B^n \quad (14)$$

Where, n is a constant.

Secondly, we assume that

$$\Lambda = aR^{-m} \quad (15)$$

Where, m is a positive constant.

From Eqns. (11) and (14), we obtain

$$R = B^{\frac{(n+2)}{3}} \quad (16)$$

Eqn. (16) together with Eqn. (15) yields,

$$\Lambda = aB^{\frac{-m(n+2)}{3}} \quad (17)$$

In order to overcome the indeterminacy of six unknowns involved in three independent field equations, we consider the Reddy String equation

$$\rho + \lambda = 0 \quad (18)$$

i.e., the sum of the rest energy density and tension density for a cloud of strings vanishes.

From Eqns. (8)-(10), (14) and (15), we obtain

$$(n-1)\frac{\dot{B}}{B} + (n^2 - 2n - 2)\frac{\dot{B}^2}{B^2} - \frac{2}{B^2} + \frac{5}{4}B^{2n-4} \\ = aB^{\frac{-m(n+2)}{3}} \quad (19)$$

Integrating Eqn. (19) gives us

$$\dot{B} = \left[\frac{2}{(n^2-2n-2)} - \frac{5}{4} \frac{B^{2n-2}}{(2n^2-4n-1)} + \frac{6aB^{-m(\frac{n+2}{3})+2}}{(n-1)(6n-mn-2m)} + \alpha B^{\frac{-2(n^2-2n-2)}{n-1}} \right]^{\frac{1}{2}} \quad (20)$$

Where, α is constant of integration.

Using Eqn. (20), after some suitable transformation of coordinates, the metric (Eqn. (1)) reduces to

$$ds^2 = - \left[\frac{2}{(n^2-2n-2)} - \frac{5}{4} \frac{T^{2n-2}}{(2n^2-4n-1)} + \frac{6aT^{-m(\frac{n+2}{3})+2}}{(n-1)(6n-mn-2m)} + \alpha T^{\frac{-2(n^2-2n-2)}{n-1}} \right]^{-1} dT^2 + T^{2n} dX^2 + T^2 dY^2 + (T^2 \sin^2 Y + T^2 \cos^2 Y) dZ^2 - 2T^{2n} \cos Y dX dZ \quad (21)$$

The pressure p and energy density ρ for the model (Eqn. (21)) are given by

$$p = \frac{2a}{1-n} T^{-m(\frac{n+2}{3})} \left[1 + \frac{3(4n+1)}{(n-1)(6n-mn-2m)-18} \right] + \frac{2}{1-n} T^{-2} \left[(1+n) + \frac{4n+1}{n^2-2n-2} \right] + \frac{T^{2n-4}}{4(n-1)} [(n+4) + 5 \frac{(4n+1)}{(2n^2-4n-1)}] + \frac{m(4n+1)}{1-n} T^{\frac{-m(n+2)}{3}} \quad (22)$$

$$\rho = -\lambda = T^{-2} \left[1 + \frac{2(2n+1)}{n^2-2n-2} \right] - \frac{T^{2n-4}}{4} \left[1 + \frac{5(2n+1)}{2n^2-4n-1} \right] + aT^{\frac{-m(n+2)}{3}} \left[-1 + \frac{6(2n+1)}{(n-1)(6n-mn-2m)-18} \right] + \alpha(2n+1)T^{\frac{-2(n^2-n-3)}{n-1}} \quad (23)$$

The expansion scalar θ and particle density ρ_p for the model are obtained as

$$\theta = (n+2) \left(\frac{2T^{-2}}{(n^2-2n-2)} - \frac{5}{4} \frac{T^{2n-4}}{(2n^2-4n-1)} + \frac{6aT^{\frac{-m(n+2)}{3}}}{(n-1)(6n-mn-2m)-18} + \alpha T^{\frac{-2(n^2-n-3)}{n-1}} \right)^{\frac{1}{2}} \quad (24)$$

$$\rho_p = T^{-2} \left[1 - \frac{2(2n+1)}{n^2-2n-2} \right] + \frac{T^{2n-4}}{4} \left[1 + \frac{5(2n+1)}{2n^2-4n-1} \right] + aT^{\frac{-m(n+2)}{3}} \left[\frac{\sigma(2n+1)}{(n-1)(6n-mn-2m)-18} \right] + \alpha(2n+1)T^{\frac{-2(n^2-n-3)}{n-1}} \quad (25)$$

The cosmological constant Λ and Shear scalar σ are

$$\Lambda = aT^{\frac{-m(n+2)}{3}} \quad (26)$$

$$\sigma^2 = \frac{2}{3} (n-1)^2 \left[\frac{2T^{-2}}{(n^2-2n-2)} - \frac{5}{4} \frac{T^{2n-4}}{(2n^2-4n-1)} + \frac{6aT^{\frac{-m(n+2)}{3}}}{(n-1)(6n-mn-2m)-18} + \alpha T^{\frac{-2(n^2-n-3)}{n-1}} \right] \quad (27)$$

Eqn. (24) together with (27) gives

$$\frac{\sigma^2}{\theta^2} = \frac{2}{3} \frac{(n-1)^2}{(n+2)^2} \quad (28)$$

4. Discussion

In this model, we observe that the model (Eqn. (21)) starts with a big-bang at $T = 0$, where $m > 0$ and $n < 2$, the expansion in the model decreases as time increases. The expansion in the model stops at $T = \infty$, when, $-2 < n < 2$. It can also be seen that at $T = 0$, pressure p , energy density ρ , expansion scalar θ , particle density ρ_p , vacuum density Λ and Shear σ are all infinite when $-2 < n < 2$, which means that at the initial time the whole universe is concentrated to one point mass, which is a sign of big-bang at $T = 0$. Also at $T = 0$, the anisotropy $(\sigma/\theta) \neq 0$. Thus the model is not isotropic at initial times. Again, at $T = \infty$; pressure p , energy density ρ , expansion scalar θ , and particle density ρ_p , are all zero when, $-2 < n < 2$. Also, anisotropy $\sigma/\theta \neq 0$ at $t = \infty$ implies that the model does not approach isotropy at late times when $-2 < n < 2$. Furthermore, for $n = 1$, the shear scalar σ is zero and $\sigma^2/\theta^2 \rightarrow 0$ as $T \rightarrow \infty$. The model becomes isotropic for $n = 1$. Thus the model (Eqn. (21)) becomes isotropic at late times when $-2 < n < 2$, which agrees with the Cosmic No-Hair conjecture [30].

5. Conclusion

In this paper, we have investigated Mixmaster String Cosmological model in the presence of decaying vacuum density in general relativity. To get the deterministic model we have assumed three conditions: $A = B^n$, $\Lambda = aR^{-m}$ (where R is the Scale factor, a and m are constants) and $\rho + \lambda = 0$.

It has been shown that the universe starts with a big-bang at initial times having anisotropic behavior and still remain anisotropic at late times. However, for $n = 1$, the model isotropizes showing No-Hair conjecture studied by Wald [30].

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