

Introduction to the Effective Mass Tensor in GR

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There have been many proposals to use alternative theories of gravity in lieu of the general theory of relativity (GR). In this paper, we propose something different: a new interpretation of the GR. We propose here to use the concept of effective mass tensor to describe gravitational phenomena instead of usual metric tensor. This paper discusses the implications arising from the use of this concept.

1. Introduction

In this paper, we propose a new physical interpretation of general theory of relativity [1]. The starting point for our consideration is the assumption that in the absence of the gravitational field all bodies have bare mass m (and more generally, the bare mass tensor $m_{\mu\nu}^{bare}$), but in the presence of the gravitational field all bodies have an effective mass m^* (and more generally the effective mass tensor (EMT) $m_{\mu\nu}^*$).

We expect that the concept of the EMT will satisfy the classical tests of general theory of relativity, such as the perihelion shift, the deflection of light by the Sun and the gravitational redshift, but their physical interpretation should be different. These physical phenomena will not be generated by the curvature of the space-time but by the EMT of the body.

This paper briefly describes the implications of the application of the concept of the effective mass tensor $m_{\mu\nu}^*$ in GR. The full description of this concept was published in my previous papers [1-4].

2. The Concept of the EMT in GR

The consequence of our above assumption is the idea that there exists a relation between the metric tensor $g_{\mu\nu}$ and the EMT of the body $m_{\mu\nu}^*$, expressed by the simple formula

$$g_{\mu\nu} = \frac{m_{\mu\nu}^*}{m} \quad (1)$$

Where the space-time components $\mu, \nu = 0, 1, 2, 3$. Therefore, the metric

$$ds^2(g_{\mu\nu}) = ds^2(m_{\mu\nu}^*) \quad (2)$$

Where, $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$ and

$$ds^2(m_{\mu\nu}^*) = \frac{m_{\mu\nu}^*}{m} dx^\mu dx^\nu.$$

As we know from GR that in a weak gravitational field we can decompose the metric tensor $g_{\mu\nu}$ into two parts: flat Minkowski metric $\eta_{\mu\nu}$ and a small “perturbation” $h_{\mu\nu}$ according to the equation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (3)$$

Where, $|h_{\mu\nu}| \ll 1$, $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ is the Minkowski tensor. Similarly, in a weak gravitational field we can decompose the EMT of the body to the simple form:

$$m_{\mu\nu}^* = m_{\mu\nu}^{bare} + m_{\mu\nu}^* \quad (4)$$

Where the bare mass tensor $m_{\mu\nu}^{bare} = m \cdot \eta_{\mu\nu} = \text{diag}(-m, +m, +m, +m)$, and $m_{\mu\nu}^* = m \cdot |h_{\mu\nu}| \ll 1$ is a small EMT “perturbation”. In the absence of the gravitational field the EMT becomes the bare mass tensor and $m_{\mu\nu}^* \rightarrow m_{\mu\nu}^{bare}$.

A short discussion of the concept of effective mass tensor follows.

3. Clocks and Rods with the Bare or Effective Mass

Clocks measure the time and the rods measure the length. In the GR clocks and rods are massless. The key question is whether the clocks and rods with the effective mass m^* will measure the same times

and the same lengths as clocks and rods with the bare mass m . Our considerations will be realized in the Schwarzschild metric.

The line element in the Schwarzschild metric has the form

$$c^2 d\tau^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (5)$$

Where, τ is a proper time, c is the speed of light, t is the time coordinate, M is the mass of star, r – distance from the star, G is the gravitational constant, θ is the co-latitude (angle from North, in units of radians), and ϕ is the longitude (also in radians).

The same line element with the effective mass m^* has the form

$$c^2 d\tau^2 = -\left(1 - \frac{m^*}{m}\right) c^2 dt^2 + \left(1 - \frac{m^*}{m}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (6)$$

If we take a slice given by $t = \text{constant}$, then we obtain a 3-dimensional manifold with line element

$$ds^2 = \left(1 - \frac{m^*}{m}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (7)$$

Eqn. (7) we obtained by putting $dt = 0$ in Eqn. (6). Thus, the linear element is

$$ds^2 = \frac{m_{ij}}{m} dx^i dx^j \quad (8)$$

Where the components are $i, j = 1, 2, 3$, ($x^1 = r$, $x^2 = \theta$, $x^3 = \phi$).

Let's go back to Eqn. (7). If we assume that θ and ϕ are constants, then we have the infinitesimal radial distance dR of the form

$$dR = dr \sqrt{\frac{m_{rr}}{m}} = \frac{dr}{\sqrt{1 - \frac{m^*}{m}}} \quad (9)$$

If we assume that $m^* \uparrow$ then rods with the effective mass will measure different length than the rods with the bare mass and $dR > dr$. But if we assume that $m^* \rightarrow 0$, then rods with the effective mass and the rods with the bare mass will measure the same length: $dR = dr$.

The measurement results are consistent with GR but their physical meaning was changed. Under the influence of the gravitational field only physical properties of the rods was changed, but not the space properties.

Let us now turn our attention to the time. According to Eqn. (6) we have

$$d\tau = dt \sqrt{\frac{\tilde{m}_{tt}}{m}} = dt \sqrt{1 - \frac{m^*}{m}} \quad (10)$$

Where we assumed that $dr = d\theta = d\phi = 0$ and

$$\frac{\tilde{m}_{tt}}{m} = -\frac{m_{tt}}{m} = \left(1 - \frac{m^*}{m}\right) \quad (11)$$

If we assume that $m^* \uparrow$ then clocks with the effective mass will measure different time than the clocks with the bare mass and $d\tau < dt$. But if we assume that $m^* \rightarrow 0$, then clocks with the effective mass and the clocks with the bare mass will measure the same time $d\tau = dt$.

The measurement results are consistent with GR but their physical meaning was changed. Under the influence of the gravitational field only physical properties of the clocks was changed, but not the time properties.

4. The EMT and the Equation of Motion

Let us consider the Lagrangian function for the body with the EMT

$$L = \frac{1}{2} m_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \quad (12)$$

The equation of motion has form

$$\frac{d^2 x^\beta}{dt^2} + \Gamma_{\mu\nu}^{*\beta} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = 0 \quad (13)$$

Where the term $\Gamma_{\mu\nu}^{*\beta}$ we will call the modified Christoffel symbols of the second kind and

$$\Gamma_{\mu\nu}^{*\beta} = \frac{1}{2} m^{\beta\alpha} \left(\frac{\partial m_{\alpha\mu}}{\partial x^\nu} + \frac{\partial m_{\alpha\nu}}{\partial x^\mu} - \frac{\partial m_{\mu\nu}}{\partial x^\alpha} \right) \quad (14)$$

Here, $\Gamma_{\mu\nu}^{*\beta}$ describes the anisotropy of the EMT. For the bare mass tensor, $\Gamma_{\mu\nu}^{*\beta} = 0$. We can rewrite Eqn. (13) in a slightly different form as

$$\frac{dp_\beta}{d\tau} + \Gamma_{\beta\mu\nu}^* \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \quad (15)$$

Where, $p_\beta = m_{\beta\nu} \frac{dx^\nu}{d\tau}$ is the four-momentum and the term $\Gamma_{\beta\mu\nu}^*$ we will call the modified Christoffel symbols of the first kind and expressed by the following formula

$$m_{\mu\nu}^{\text{Schwarzschild}} = m \begin{bmatrix} -\left(1 - \frac{2GM}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{bmatrix} \quad (17)$$

The well known calculation for $m_{\mu\nu}^{\text{Schwarzschild}}$ satisfies the classical tests of GR, such as the perihelion shift, the deflection of light by the Sun and the gravitational redshift, but their physical interpretation is different.

The concept of the EMT correctly describes gravitational phenomena known from GR. However, these physical phenomena are not generated by the curvature of space-time, but by the effective mass tensor of the body.

6. The Mass Anisotropy in the Solar System

To test whether the concept is observable or not we suggest that we should search for mass anisotropy in the Solar System.

For the $m_{\mu\nu}^{\text{Schwarzschild}}$ (see Eqn. (17)) components of m_{tt} and m_{rr} depends on the distance r between the planet and the star. It is known that in the Solar System the distance r between the Earth and the Sun is changing (the orbit is an ellipse). For relative changes in the mass anisotropy of components m_{tt} and m_{rr} , one should make observations during the whole one year period measuring from perihelion (aphelion) to perihelion (aphelion).

$$\Gamma_{\beta\mu\nu}^* = \frac{1}{2} \left(\frac{\partial m_{\beta\nu}}{\partial x^\mu} + \frac{\partial m_{\beta\mu}}{\partial x^\nu} - \frac{\partial m_{\mu\nu}}{\partial x^\beta} \right) \quad (16)$$

Here, $\Gamma_{\beta\mu\nu}^*$ describes “the four-gradient” of the EMT. The dimension of this term is [kg/m].

According to Eqn. (16), we can say that the effective mass of the body is not an intrinsic property but is a result of interactions between this body and the effective mass of other bodies in the Universe.

This is the Mach’s Principle in a new perspective.

5. The EMT in the Schwarzschild World

Let us consider the Lagrangian function (Eqn. (12)) with the Schwarzschild EMT

The component m_{tt} is the smallest in the perihelion while the component m_{rr} is the biggest. The biggest relative changes annually (for example, from perihelion to perihelion) for m_{tt} and for m_{rr} should be measured in the Solar System, and the estimated value is equal to

$$\begin{aligned} \frac{|\delta m_{tt}|}{m} &= \frac{|\delta m_{rr}|}{m} = \frac{1}{m} |(m_{tt})_{\text{perih}} - (m_{tt})_{\text{aphel}}| \\ &= \frac{1}{m} |(m_{rr})_{\text{perih}} - (m_{rr})_{\text{aphel}}| \\ &= \frac{2GM}{c^2} \left| \frac{1}{r_{\text{perih}}} - \frac{1}{r_{\text{aphel}}} \right| \approx 6,6 \cdot 10^{-10} \end{aligned} \quad (18)$$

7. The Concept of the EMT and the Gravitational Waves

In the Einstein’s GR gravitational waves are fluctuations of the gravitational fields (or the ripples in the curvature of the space-time), which are propagated as a wave and are generated mainly by moving massive bodies.

Let's assume that

$$g_{\mu\nu} = \frac{\rho_{\mu\nu}}{\rho} \quad (19)$$

Where, $\rho_{\mu\nu}$ is the effective mass density tensor (EMDT), ρ is the bare mass density. The metric can be expressed as

$$ds^2(g_{\mu\nu}) = ds^2(\rho_{\mu\nu}) \quad (20)$$

Where, $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$ and $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho} dx^\mu dx^\nu$.

In the weak gravitational field we can decompose the EMDT of a body into following simple form

$\rho_{\mu\nu} = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^*$, where, $\rho_{\mu\nu}^{bare} = \rho \cdot \eta_{\mu\nu} = \text{diag}(-\rho, +\rho, +\rho, +\rho)$ will be called the bare mass density tensor, $\eta_{\mu\nu}$ is the Minkowski tensor, $\rho_{\mu\nu}^* = \rho \cdot |h_{\mu\nu}| \ll 1$ is a small EMDT perturbation. Note that in the absence of the gravitational field the EMDT becomes the bare mass density tensor and $\rho_{\mu\nu} \rightarrow \rho_{\mu\nu}^{bare}$.

The wave equation in the vacuum has the form

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) \frac{\bar{\rho}_{\mu\nu}^*}{\rho} = 0 \quad (21)$$

Where, $\bar{\rho}_{\mu\nu}^* = \rho_{\mu\nu}^* - \frac{1}{2} \rho \cdot \eta_{\mu\nu}$ and the gauge condition, $\partial_\mu \bar{\rho}_{\mu\sigma}^* = 0$.

In general relativity theory the gravitational waves are the ripples in the curvature of the space-time that propagate as a wave. The concept of EMDT predicts that the gravitational waves are the perturbation (fluctuation) in the effective mass density that propagates as a wave. Although, in both cases the gravitational waves are propagated with the speed of light, the way they are detected should be different from the currently accepted method.

8. Conclusion

The concept of the EMT is a new physical interpretation of the general theory of relativity, where the curvature of space-time has been replaced by the

EMT. This concept opens up new possibilities for the study of the gravitational phenomena. However, the concept of the EMT requires experimental verification.

References

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Received: 11 November, 2013

Accepted: 2 April, 2014