

Cosmological Models with Strange Quark Matter Attached to String Cloud in Self-creation Theory

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The anisotropic Bianchi type-VI₀ space-time with strange quark matter attached to string cloud has been studied in Barber's (1982) second self-creation theory and general relativity. The field equations have been solved and some important features of the models thus obtained have been discussed. We noticed that the presence of scalar field does not affect the geometry of the space-time but changes the matter distribution.

1. Introduction

Barber [1] proposed two self-creation cosmologies by modifying the Brans-Dicke [2] theory and general relativity. These modified theories create the universe out of self contained gravitational and matter fields. Brans [3] has pointed out that Barber's first theory is not only in disagreement with experimental results but it is also inconsistent. Barber's second theory is a modification of general relativity. In this theory, the scalar field does not directly gravitate, but simply divides the matter tensor and acting as a reciprocal gravitational constant.

The field equations of Barber's second self-creation theory are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} \quad (1)$$

and the scalar field ϕ satisfies the equation

$$\square\phi = \phi^k; \quad k = \frac{8}{3}\pi\eta T \quad (2)$$

Where, $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, T_{ij} is the stress energy tensor of the matter, ϕ is the Barber's scalar field, and η is a coupling constant. The measurements of the deflection of light restricts the value of the coupling constant to $|\eta| < 10^{-1}$.

A comparison with Einstein's equations shows that the Barber's theory goes over to general relativity in all aspects in case of coupling constant $\eta \rightarrow 0$. For small values of η there would have been a violent period of matter creation in the early stages

of the Big-Bang. For detailed discussion of the self-creation theories of gravitation one may refer to [1]. Barber [1] and Soleng [4] have discussed the FRW models, while Reddy and Venkateswarlu [5] have studied the Bianchi type-VI₀ cosmological model in Barber's self-creation theory of gravitation. Venkateswarlu et al. [6] have investigated Bianchi type-I, II, VIII & IX string cosmological solutions in this theory. Mohanty and Mahanta [7] have discussed the plane symmetric cosmological model in the case when $\rho = \lambda$. Rao et al. [8] have discussed Bianchi type – II, VIII and IX string cosmological models and Rao and Vinutha [9] have discussed plane symmetric string cosmological models in this theory. Recently, Rao and Sireesha [10] have discussed axially symmetric string cosmological model with bulk viscosity in self-creation theory of gravitation.

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Because, one of such transitions during various phase transitions of the universe could be Quark Gluon Plasma (QGP) harden gas (called quark-hadron phase transition) when cosmic temperature was $T \approx 200$ Mev. Strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). In the framework of this model, the quark matter is composed of massless u and d quarks, massive s quarks and electrons. In a simplified version of the bag model, it is assumed that quarks are massless and non-interacting.

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Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3} \quad (3)$$

Where, ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c \quad (4)$$

But the total pressure is

$$p = p_q - B_c \quad (5)$$

For more information and review of strange quark matter attached to string cloud one can refer to [11]. Khadekar et al. [12] have confined their work to the quark matter that is attached to the topological defects in general relativity. Khadekar and Rupali Wanjari [13] have discussed the geometry of quark and strange quark matters in higher dimensional general relativity. Rao and Neelima [14] have discussed Bianchi type-VI₀ with strange quark matter attached to the string cloud in the Saez-Ballester theory of gravitation. Rao and Neelima [15] have discussed axially symmetric space-time with strange quark matter attached to the string cloud in self creation theory and general relativity and established that the additional condition, special law of variation of Hubble parameter proposed by [16], taken by [17] in general relativity, is superfluous. Recently, Rao and Sireesha [18,19] have discussed axially symmetric and Bianchi type-II, VIII & IX space-times with strange quark matter attached to the string cloud, respectively, in Brans-Dicke theory of gravitation.

In continuation of the above mentioned works, we will now study Bianchi type-VI₀ space time with strange quark matter attached to the string cloud in Barber's second self-creation theory and general relativity.

2. Metric and Field Equations

We consider the Bianchi type-VI₀ line element given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2 \quad (6)$$

Where, A, B and C are the functions of time t only.

The energy momentum tensor for string cloud ([20]) is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j \quad (7)$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s \quad (8)$$

Where, ρ_p is the particle energy density. We know that string is free to vibrate. The different vibration models of the string represent different types of particles because these different models are seen as different masses or spins. Therefore, we will take here quarks instead of particles in the string cloud. Hence we consider quark matter energy density instead of particle energy density in the string cloud. In this case from (8), we get

$$\rho = \rho_q + \rho_s + B_c \quad (9)$$

Using Eqns. (8) and (9) ([21]), the energy momentum tensor for strange quark matter attached to the string cloud can be taken as

$$T_{ij} = (\rho_q + \rho_s + B_c) u_i u_j - \rho_s x_i x_j \quad (10)$$

Where, u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string.

We have u_i and x_i with

$$u_i u^i = -x_i x^i = 1 \quad \text{and} \quad u^i x_i = 0 \quad (11)$$

We have taken the direction of string along x-axes. Then the components of energy momentum tensor are

$$T_1^1 = \rho_s, T_2^2 = T_3^3 = 0, T_4^4 = \rho \quad (12)$$

Where, ρ and ρ_s are functions of t only.

Now the field equations for the metric (Eqn. (6)) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = 8\pi\phi^{-1}\rho_s \quad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = 0 \quad (14)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 0 \quad (15)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = 8\pi\phi^{-1}\rho \quad (16)$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \quad (17)$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{8\pi\eta}{3} (\rho_s + \rho) \quad (18)$$

Where, the overhead dot (.) denotes the differentiation with respect to t .

From Eqn. (17), we get

$$C = \alpha B$$

Without loss of generality we can take $\alpha = 1$ so that we have

$$C = B \quad (19)$$

Using Eqn. (19), the field equations (Eqns. (13)-(18)) reduce to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = 8\pi\phi^{-1}\rho_s \quad (20)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 0 \quad (21)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = 8\pi\phi^{-1}\rho \quad (22)$$

and

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) = \frac{8\pi\eta}{3} (\rho_s + \rho) \quad (23)$$

The field equations (Eqns. (20)-(23)) are only four independent equations with five unknowns, A, B, ρ, ρ_s , and ϕ . Since these equations are non-linear in nature, in order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to

scalar expansion θ , which leads to the linear relationship between the metric potentials A and B , i.e.,

$$A = B^m \quad (m \neq 0) \quad (24)$$

From Eqns. (21) and (24), we get

$$\frac{\ddot{B}}{B} + \left(\frac{m^2}{m+1} \right) \frac{\dot{B}^2}{B^2} = \frac{1}{(m+1)} B^{-2m}$$

From the above equation, we get

$$B = (mt + n)^{\frac{1}{m}} \quad (25)$$

From Eqns. (24) and (25), we get

$$A = (mt + n) \quad (26)$$

From Eqn. (24), we get

$$C = (mt + n)^{\frac{1}{m}} \quad (27)$$

Then the metric (Eqn. (6)) can be written as

$$ds^2 = dt^2 - (mt + n)^2 dx^2 - (mt + n)^{\frac{2}{m}} e^{2x} dy^2 - (mt + n)^{\frac{2}{m}} e^{-2x} dz^2 \quad (28)$$

From Eqns. (20), (22) and (23), we get

$$\left[(mt + n)^2 \ddot{\phi} + (m+2)(mt + n)\dot{\phi} \right] - \frac{4\eta}{3} \phi = 0 \quad (29)$$

From Eqn. (29), we get

$$\phi = (mt + n)^r \quad \text{Where, } r = \frac{4\eta}{3m}, \frac{4\eta}{3m} - \frac{2}{m} \quad (30)$$

Since Barber's theory has to go back to Einstein's theory in all aspects when $\eta \rightarrow 0$ then r

should be of the form: $r = \frac{4\eta}{3m}$.

Hence from (Eqn. 22), we get the string energy density

$$\rho = \frac{m}{4\pi} (mt+n)^{r-2} \quad (31)$$

From Eqn. (20), we get the string tension density

$$\rho_s = \frac{1}{4\pi} \{ (2-m)(mt+n)^{r-2} \} \quad (32)$$

The string particle density is given by

$$\rho_q + B_c = \rho_p = \rho - \rho_s = \frac{(m-1)}{2\pi} (mt+n)^{r-2} \quad (33)$$

And the quark energy density is given by

$$\rho_q = \rho - B_c = \frac{m}{4\pi} (mt+n)^{r-2} - B_c \quad (34)$$

The quark pressure is given by

$$p_q = \frac{\rho_q}{3} = \frac{m}{12\pi} (mt+n)^{r-2} - \frac{B_c}{3} \quad (35)$$

Hence the metric (Eqn. (28)) together with Eqns. (30)-(35) represents Bianchi type-VI₀ cosmological model with strange quark matter attached to string cloud in self creation theory of gravitation.

Bianchi type-VI₀ cosmological model with strange quark matter attached to string cloud in general relativity

It is interesting to note that when $\eta \rightarrow 0$; $r = 0$ and $\phi = \text{constant}$. In this case, the present model (Eqn. (28)) together with Eqns. (31)-(35) represents the Bianchi type-VI₀ cosmological model with strange quark matter attached to string cloud in general relativity. Also, the solution satisfies the divergence equation identically, i.e.,

$$T^{ij}; j = \dot{\rho} + \rho \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) - \rho_s \left(\frac{\dot{A}}{A} \right) = 0$$

3. Some Important Features of the Models

The spatial volume of the metric (Eqn. (28)) is given by

$$V = \sqrt{-g} = (mt+n)^{\frac{m+2}{m}} \quad (36)$$

The expansion scalar θ is given by

$$\theta = u^i; i = \frac{(m+2)}{(mt+n)} \quad (37)$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{7}{18} \frac{(m+2)^2}{(mt+n)^2} \quad (38)$$

The mean Hubble parameter H is given by

$$H = \frac{(m+2)}{3(mt+n)} \quad (39)$$

The mean anisotropy parameter Δ is given by

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{2(m-1)^2}{(m+2)^2} \quad (40)$$

The density parameter Ω is given by

$$\Omega = \frac{\rho}{3H^2} = \left[\frac{3m(mt+n)^r}{8\pi(m-1)^2} \right] \Delta \quad (41)$$

The deceleration parameter

$$q = -3\theta^{-2} [\theta_{;\alpha} u^\alpha + \frac{1}{3} \theta^2] = \frac{2(m-1)}{(2+m)}, \quad m \neq -2 \quad (42)$$

The deceleration parameter $q > 0$ for $-\infty < m < -2$ and $m > 1$ and $q < 0$ for $-2 < m < 1$.

If $q < 0$ the model accelerates and when $q > 0$, the model decelerates. Here the presented models sometimes decelerate in the standard way and later accelerate, which is in accordance with the present day scenario. It may be noted that Bianchi models represent cosmos in its early stage of evolution. However, in spite of the fact that the universe, in this case, decelerates in the standard way and it will accelerate in finite time due to cosmic re-collapse, where the universe in turns inflates “decelerates and then accelerates” ([22]).

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

Also, since the mean anisotropy parameter $\Delta \neq 0$ for $m \neq 1$, the models do not approach isotropy for $m \neq 1$. Various experiments show that there is a certain amount of anisotropy in the universe and hence anisotropic space-times are important. For $m = 1$ the mean anisotropy parameter is zero and hence the model (Eqn. (28)) together

with Eqns. (31)-(35) represents isotropic cosmological model with strange quark matter attached to string cloud in self-creation theory of gravitation.

4. Conclusions

In this paper, we have presented homogeneous Bianchi type-VI₀ cosmological models with strange quark matter attached to string cloud in self-creation theory as well as in general relativity.

The following are the observations and conclusions.

1) The models have point type singularity at $t = \frac{-n}{m}$ for $m > 0$ and cigar type singularity for $m < 0$.

2) At, $t = \frac{-n}{m}$, the proper volume will be zero whereas when $t \rightarrow \infty$, the spatial volume becomes infinitely large for $m > 0$. Thus the derived model starts expanding from the singular point.

3) At singular point the expansion scalar θ , shear scalar σ tends to infinity whereas when $t \rightarrow \infty$, they tend to zero. This seems to be consistent with the results of [23,24,25] at the Brookhaven National Laboratory.

4) For $m = 2$ we get dust quark matter solution i.e., $\rho_s = 0$ and $\rho_q + B_c = \rho_p = \rho = \frac{1}{2\pi} (2t + n)^{r-2}$.

5) The deceleration parameter $q > 0$ for $-\infty < m < -2$ and $m > 1$ and $q < 0$ for $-2 < m < 1$ and hence these models sometimes decelerate and sometimes accelerate.

6) Eqn. (41) shows that the density parameter Ω depends on the anisotropy parameter Δ .

7) When the mean anisotropy parameter $\Delta \neq 0$ the models do not approach isotropy for $m \neq 1$.

8) For $m = 1$, the mean anisotropy parameter $\Delta = 0$, and hence the model (Eqn. (28)) will become isotropic. Also, since $q = 0$, the present isotropic universe expands at a constant rate.

9) From Eqn. (33), we observe that the matter disappears and we will get back the geometric string model for $m = 1$, i.e., $\rho_p = 0$ and

$$\rho = \rho_s = \frac{1}{4\pi} (t + n)^{r-2}.$$

10) The Hubble parameter H decreases with the increase of time.

11) For our model, $\frac{\sigma}{\theta} \approx 0.6236$, which is greater

than the present upper limit $(10)^{-5}$ as obtained by Collins [26] from indirect arguments concerning the isotropy of the primordial black body radiation.

12) Finally, we can conclude that our models represent not only the early stages of evolution but also the present stage of the universe, and the presence of scalar field does not affect the geometry of the space-time but changes the matter distribution.

Acknowledgments

The second author (D. Neelima) is grateful to the Department of Science and Technology (DST), New Delhi, India for providing INSPIRE fellowship.

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Received: 29 October, 2013
Accepted: 21 February, 2014