

Oscillating Universe: From Big Bang to Big Crunch with Linear Equation of State

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We have studied the oscillating universe from the big bang to big crunch with linear equation of state in an anisotropic and homogeneous locally rotationally symmetric Bianchi type-IX cosmological model. We have used the linear equation of state (EoS) $p = \alpha\rho - \beta$, where α and β are constants, in general relativity. In order to obtain the solution of these field equations, we have used a relation between shear scalar σ and expansion scalar θ . The geometrical and physical aspects of this model are also studied.

1. Introduction

The development of theoretical and observational cosmologies and the discovery of cosmic acceleration [1-6] in particular have elevated the study of cosmological models where new types of singularities are described.

Various types of solutions of the Einstein field equation lead to a future space-like singularity known as the big crunch. The energy density and curvature invariants become singular on an entire space-like hyper-surface. Several authors have speculated about the meaning of this singularity. Khouri, Seiberg, Liu and Horowitz and Polchinski [7-12] have revived the most popular scenario in the context of string theory and pronounced that quantum effects would lead to a “bounce” followed by re-expansion of the universe. The big bang and the big crunch types of singularities occur not at zero but at finite or even infinite values of the cosmological radius. As per Caldwell and Starobinsky [13,14] the most famous amongst these singularities is known as the ‘Big Rip’ singularity. The big rip singularity arises when the absolute value of negative pressure p of the dark energy will be higher than the energy density ρ .

It is known that Bianchi universe anisotropies give rise to CMB anisotropies depending upon the type of model [15]. The anisotropy plays a significant role in the early stages of evolution of the universe and hence the study of anisotropic and homogeneous cosmological models becomes important. In the present paper, we study the oscillating universe from the big bang to the big crunch with linear equation of state in an anisotropic and homogeneous locally rotationally

symmetric Bianchi type-IX cosmological model. We have used the linear equation of state (EoS) $p = \alpha\rho - \beta$, where α and β are constants, in general relativity. In order to obtain the solutions of these field equations, we have used the relation between shear scalar σ and expansion scalar θ . We studied both the geometrical and physical aspects of this model.

2. Metric and Field Equations

We consider a spatially homogeneous Locally Rotationally Symmetric (LRS) Bianchi type-IX metric in the form

$$ds^2 = dt^2 - B^2 dx^2 - A^2 dy^2 - [A^2 \sin^2 y + B^2 \cos^2 y] dz^2 + 2B^2 \cos y dx dz \quad (1)$$

Where, A and B are the scale factors (metric potentials) and functions of cosmic time t only.

The energy momentum tensor is given by

$$T_v^u = \text{diag}[T_4^4, T_1^1, T_2^2, T_3^3] = \text{diag}[\rho, -p, -p, -p] \quad (2)$$

Where, ρ is the energy density of the fluid and p is its pressure.

Here we use linear equation of state [16] as

$$p = \alpha\rho - \beta \quad (3)$$

Where, α and β are constants.

The Einstein field equations, in natural units ($8\pi G = 1$ and $c = 1$), are

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$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -T_{\mu\nu} \quad (4)$$

Where, $g_{\mu\nu} u^\mu u^\nu = 1$; $u^\mu = (1, 0, 0, 0)$ is the four-velocity vector, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, and $T_{\mu\nu}$ is the energy-momentum tensor

Using Eqn. (4) the corresponding field equations for metric (Eqn. (1)) with the help of Eqns. (3) and (4) can be written as

$$2 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} + \frac{1}{A^2} - \frac{B^2}{4A^4} = \rho \quad (5)$$

$$2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{1}{A^2} - \frac{3B^2}{4A^4} = -(\alpha\rho - \beta) \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = -(\alpha\rho - \beta) \quad (7)$$

Where, the overhead dot (·) denotes derivative with respect to the cosmic time t .

3. Solutions of the Field Equations

The field equations (5)-(7) are non-linear differential equations. In order to get a deterministic solution, we consider the physical condition that shear scalar σ is proportional to scalar expansion θ , which leads to a linear relationship between the metric potentials (scale factors) A and B .

i.e., $B = A^n$, where n is an arbitrary constant (8)

From Eqns. (6), (7) and (8), we obtain

$$\frac{\ddot{A}}{A} + (1+n) \frac{\dot{A}^2}{A^2} + \frac{1}{(1-n)A^2} - \frac{A^{2n-4}}{(1-n)} = 0, \quad n \neq 1 \quad (9)$$

We can solve the above equation with suitable choice of substitution. We then get

$$\dot{A}^2 = w^2 - \gamma^2 A^2 \quad (10)$$

Where, $w^2 = \frac{1}{3}$ and $\gamma^2 = \frac{1}{4}$.

From Eqn. (10), we get

$$A = \left(\frac{w}{\gamma} \right) \sin(\gamma t) \quad (11)$$

From Eqns. (8) and (11), we get

$$B = \left(\frac{w}{\gamma} \right)^2 \sin^2(\gamma t) \quad (12)$$

The metric (Eqn. (1)) with the help of Eqns. (11) and (12) can be written as

$$ds^2 = dt^2 - \left(\frac{w}{\gamma} \right)^4 \sin^4(\gamma t) dx^2 - \left(\frac{w}{\gamma} \right)^2 \sin^2(\gamma t) dy^2 - 2 \left(\frac{w}{\gamma} \right)^4 \sin^4(\gamma t) \cos y dx dz - \left[\left(\frac{w}{\gamma} \right)^2 \sin^2(\gamma t) \sin^2 y + \left(\frac{w}{\gamma} \right)^4 \sin^4(\gamma t) \cos^2 y \right] dz^2 \quad (13)$$

The spatial volume is given by

$$V = \left(\frac{w}{\gamma} \right)^2 \sin^2(\gamma t) \quad (14)$$

The component H_x , H_y and H_z of the Hubble parameter are given by.

$$H_x = H_y = \frac{\dot{A}}{A} = \gamma \cot(\gamma t) \\ H_z = \frac{\dot{B}}{B} = 2\gamma \cot(\gamma t) \quad (15)$$

The mean Hubble parameter H is found to be

$$H = \frac{1}{3} \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = \frac{4\gamma}{3} \cot(\gamma t) \quad (16)$$

The expansion scalar θ is found to be

$$\theta = 3H = 4\gamma \cot(\gamma t) \quad (17)$$

The deceleration parameter q is found to be

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = \frac{3}{4} \sec^2(\gamma t) - 1 \quad (18)$$

The anisotropic parameter of the expansion (Δ) is found to be

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{1}{8} \quad (19)$$

Where, $H_i (i=1,2,3)$ represent the directional Hubble parameters in the direction of x , y and z -axis, respectively.

The shear scalar σ^2 is found to be

$$\sigma^2 = \frac{3}{2} \Delta H^2 = \frac{1}{3} \gamma^2 \cot^2(\gamma t) \quad (20)$$

Using Eqns. (11) and (12) in Eqn. (5), we get the energy density as

$$\rho = 5\gamma^2 \cot^2(\gamma t) + \left(\frac{\gamma}{\omega}\right)^2 \operatorname{cosec}^2(\gamma t) - \frac{1}{4} \quad (21)$$

Using Eqns. (21) and (3), we get the pressure as

$$p = \alpha \left[5\gamma^2 \cot^2(\gamma t) + \left(\frac{\gamma}{\omega}\right)^2 \operatorname{cosec}^2(\gamma t) - \frac{1}{4} \right] - \beta \quad (22)$$

Using Eqns. (17), (20) and (21), we get the rotation tensor as

$$\frac{\sigma^2}{\theta} = \frac{\gamma}{12} \cot(\gamma t) \quad (23)$$

and

$$\frac{\sigma^2}{\rho} = \frac{4\omega^2 \gamma^2 \cot^2(\gamma t)}{60\omega^2 \gamma^2 \cot^2(\gamma t) + 12\gamma^2 \operatorname{cosec}^2(\gamma t) - 3\omega^2} \quad (24)$$

The Ricci scalar is given by

$$R = \frac{1}{2} \left[7\gamma^2 \cot^2(\gamma t) - \left(\frac{\gamma}{\omega}\right)^2 \operatorname{cosec}^2(\gamma t) - 4\gamma^2 - 1 \right] \quad (25)$$

4. Discussion

For this cosmological model, we observe the following:

i) The volume is zero at $t=0$ and as time increases $0 < t < t_{\max}$ the volume also increases, and it reaches to finite maximum value at $t_{\max}$. In the interval, $t_{\max} < t < t_c$, it starts decreasing and becomes zero at $t = t_c$, as shown in Fig. 1.

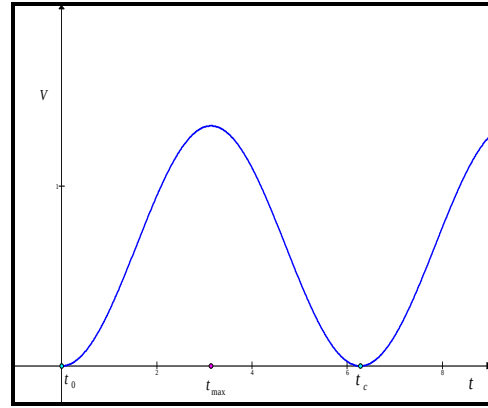


Fig.1: Evolution of volume V .

ii) The energy density is infinite at $t=0$, and as time t increases $0 < t < t_{\max}$ it decreases and reaches to finite constant value at $t_{\max}$. In the interval $t_{\max} < t < t_c$, it starts increasing and becomes infinite at $t = t_c$, as shown in Fig. 2.

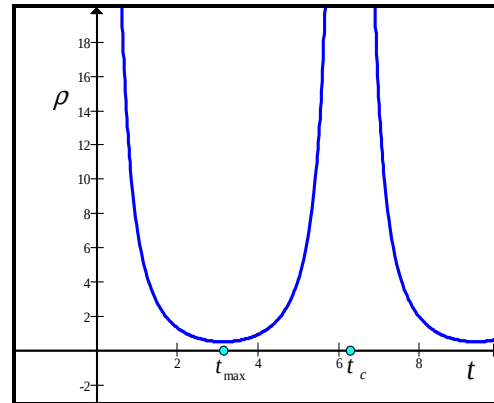


Fig.2: Evolution of energy density ρ .

iii) The universe fluctuates from accelerated expansion to decelerated expansion between the interval, $0 < t < t_c$, as shown in Fig. 3. The acceleration is due to the dominance of dark energy (negative pressure as shown in Fig. 4) and deceleration due to a decrease in the dark energy.

iv) Also, we observe that our model reduces to Strange Quark Matter (SQM) model for $\alpha = 1/3$ and $\beta = (4/3)B_c$, where B_c is Bag constant or vacuum energy density of the Bag model of quark matter.

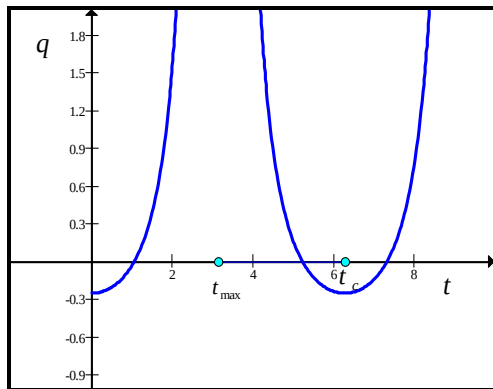


Fig.3: Dynamics of the deceleration parameter.

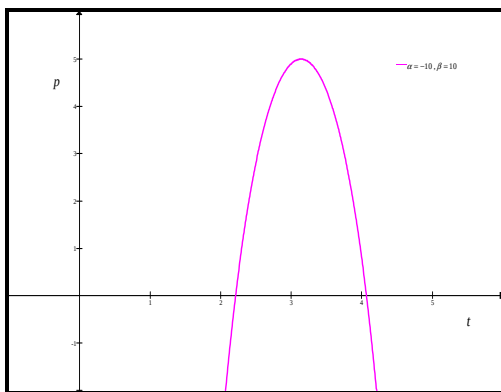


Fig.4: Evolution of pressure p.

v) Also the anisotropy is maintained throughout the evolution of the universe as $\Delta = \text{constant}$.

5. Conclusion

In this paper, we have studied anisotropic and homogeneous LRS Bianchi type-IX cosmological models with linear equation of state. In the region, $0 < t < t_{\text{max}}$, we have a universe expanding from a big-bang singularity, whereas in the region, $t_{\text{max}} < t < t_c$, we have a collapsing universe approaching to a big-crunch singularity. Near these singularities, and in between them, we should expect quantum mechanics to be relevant. Thus, the universe begins with a big bang and ends with a big crunch. If we consider the singularity at $t = t_c$ (bounce) as a transition from big crunch to big bang due to the quantum effect at the singularity, then the universe again starts expanding and eventually one can obtain the oscillating universe, and then the universe has no beginning or end. We agree with the statement made by [17] that “If the universe existed for a long time before the big

bang, many questions in cosmology can have new solutions and have to be revisited”.

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