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Low frequency shock waves in two-fluid quantum plasma

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An investigation is presented to study the propagation properties of low frequency ion-acoustic shocks in quantum plasma whose constituents are electrons (inertialess), positive ions and negatively charged dust grains both mobile. The Quantum hydrodynamic (QHD) model has been considered to investigate dust-ion acoustic shock structures in two-fluid quantum plasma. The reductive perturbation technique is employed to derive the Korteweg–de Vries–Burgers (KdV-B) equation whose solution has the form of shock structures and in the limiting case, solitons structures are observed. The combined effects of variation of different physical parameters on the characteristics of DIA shock and solitary structures are analyzed. The time evolution analysis of DIA shocks has also been carried out to see the occurrence of monotonic as well as oscillatory shocks in the given quantum plasma system. The results of present investigation may be useful in the understanding of fundamental plasma phenomenon in an astrophysical plasma environment.

1. Introduction

In quantum plasmas, the mean particle distance of species (say electrons/ions) is similar to the de-Broglie wavelength associated with the charged particles or smaller than it. Further, it is believed that in plasma Fermi temperature surpasses temperature of the system and such type of particles act like Fermi gas. A number of observations have confirmed that dust is ubiquitous component in most of the space/astrophysical and laboratory environments [1]. The presence of dust in e-i plasma generates new kinds of modes including dust acoustic as well as dust-ion acoustic modes. The study of linear and nonlinear dust acoustic (DA) as well as dust ion acoustic (DIA) waves has been frontline area of research in different plasma environments for the last more than four decades. [2-6].

Numerous investigations to study linear and nonlinear excitations (solitons, double layers, rogue waves etc.) in different kinds of quantum plasma environments in the framework of perturbative as well as many non-perturbative approaches have been reported. [7-13]. Nonlinear DA waves in collisionless, ultracold quantum dusty plasma consisting of inertialess electrons and ions, and inertial dust has been studied by Ali & Shukla (2006). It was reported that quantum corrections has significantly affected the nonlinear properties (amplitude and width) of DA waves. Misra (2009) studied dust ion- acoustic shocks in quantum dusty pair-ion plasmas. Masood et al. (2007) reported the linear and nonlinear properties of dust ion-acoustic waves using the two fluid quantum hydrodynamic model (QHD). It was observed that different parameters (quantum corrections and concentration of the dust particles) greatly influence the properties of DIA waves. Shock structures in the plasma system are formed due to the balance of different effects (nonlinear, dispersion, dissipation). Among different kind of nonlinear structures, researchers have also reported various kinds of investigations on the study of shock waves in the framework of different kinds of particle velocity distributions in different environments of plasma [14-19].

Over the last many years, a large number of investigations to study the characteristics of shocks in different kinds of multi component quantum plasma have been reported [20-22]. Misra (2009) addressed the requirement for the formation of DIA oscillatory and monotonic shocks in quantum dusty pair-ion plasma. Rouhani et al. (2014) reported the character- istics of IA shock waves in quantum pair plasma with dust particulates. It was observed that the quantum parameter, dust density and dissipation parameter have significant influence on the existence of monotonic and oscillatory shocks. Owing to the importance of quantum plasmas as well as the role of dust and applications of shocks in different areas motivates to study the shocks in two-fluid quantum plasma. In present investigation, we consider a two-fluid QHD model to study the nonlinear dust ion-acoustic (DIA) shock waves in an unmagnetized, collision less, three component electron-ion- dust (e-i-d) quantum plasmas. Using the reductive perturbation technique, we have derived the Korteweg–de Vries-Burgers (KdV-B) equation by incorporating the quantum-mechanical behavior in electrons and ions. Time evolution of shock waves of oscillatory shocks at later times which arises due to nonlinearity has also been studied. The manuscript is organized in the following manner. In Sec. 2, the governing model equations are presented. In Sec. 3, the derivation and solution of the KdV-Burgers equation are discussed in detail. Sec. 4 presents the numerical analysis. Sec. 5 is devoted to the conclusions of the findings.

2. Basic Equations

To study the nonlinear properties of DIA shocks in quantum plasma, we consider inertial dust as well as ions and non-inertial quantum electrons. The set of normalized fluid equations (continuity, momentum and Poisson) to study the dynamics of DIA shock waves are written as:

For dust:

Continuity equation:

$$\frac{\partial n_d}{\partial t}+\frac{\partial (n_d v_d)}{\partial x}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}=\mu_d\frac{\partial \varphi}{\partial x}+\alpha_1\eta_d\frac{\partial^2 v_d}{\partial x^2} \tag{2}$$

For ions:

Continuity equation:

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i v_i}{\partial x}=0 \tag{3}$$

Momentum equation:

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}=-\frac{\partial \varphi}{\partial x}+\frac{H_i^2}{2}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2\sqrt{n_i}}{\partial x^2}}{\sqrt{n_i}}\right)+\eta_i\frac{\partial^2 v_i}{\partial x^2} \tag{4}$$

The equation for inertialess electrons:

$$0=\frac{\partial \varphi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H_e^2}{2}\frac{\partial}{\partial x}\left(\frac{\frac{\partial^2\sqrt{n_e}}{\partial x^2}}{\sqrt{n_e}}\right) \tag{5}$$

The Poisson's equation:

$$\frac{\partial^2 \varphi}{\partial x^2}=\mu n_d+(1+\mu)n_e-n_i \tag{6}$$

The coordinates (space and time) are normalized by the Debye length $\lambda_d=\sqrt{\frac{2K_B T_{Fe}}{4\pi n_{i0}}e^2}$ and inverse of ion plasma frequency $\omega_{pi}=\sqrt{\frac{4\pi n_{i0}e^2}{m_i}}$ respectively. The electrostatic potential ϕ is normalized by $2\kappa_B T_{Fe}/e$. The fluid velocity v_j (j = i, d) is normalized by the ion sound speed $C_{si} = (2\kappa_B T_{Fe}/m_i)^{1/2}$. The ratio of unperturbed dust density to unperturbed ion density is defined as $\delta = z_{d0}n_{d0}/n_{i0}$, z_d is the charge number of the negatively charged dust, $\delta_d = z_{d0}m_i/m_d$ and $\theta = T_{Fi}/T_{Fe}$ are the ion to dust mass ratio and ratio of ion to electron Fermi temperature respectively. The non-dimensional quantum parameter H for electrons and ions are defined as $H_e=\sqrt{\omega_{pi}h^2/m_e m_i C_{si}^4}$ and $H_i=\sqrt{\omega_{pi}h^2/m_i^2 C_{si}^4}$ respectively. The dust and ion kinematic viscosities arising due to dust–dust and ion-ion collisions are normalized by $\lambda^2_{Di}n_{i0}m_i\omega_{pi}$ and $\alpha_1 = m_i n_{i0}/m_d n_{d0}$. We assume that the electrons in plasma obey the one-dimensional pressure law [23]

$$p_e=\frac{m\, v_{FE}^2}{3n_{eo}^2}n_e^3 \tag{7}$$

Where, m_e is the mass, $v_{Fe}=\frac{\sqrt{2}K_B T_{Fe}}{m_i}$ is the Fermi speed, κ_B is the Boltzmann constant, and T_{Fe} is the Fermi temperature. Furthermore, ne is the number density with its equilibrium value n_{e0}. At equilibrium, the charge neutrality condition is $n_{i0} = n_{e0} + Z_d n_{d0}$. Integrating Eq. (5) with boundary conditions ne = 1 and $\phi = 0$ at $x \to \pm\infty$ we get[12],

$$\varphi = \frac{1}{2}n_e^2 + \frac{H_e^2}{2}\left(\frac{\frac{\partial^2\sqrt{n_e}}{\partial x^2}}{\sqrt{n_e}}\right) - \frac{1}{2} \tag{8}$$

3. Derivation of the KdV-burger equation

In order to derive the KdV-Burgers (KdV-B) equation for the DIA shock waves for two fluid quantum plasma, we have employed reductive perturbation method with stretching coordinates as $\xi = \epsilon^{1/2}(x - Vt)$ and $\tau = \epsilon^{3/2}t$ .For the case of weak damping, the kinematic viscosity at arises due to dust–dust and ion-ion collisions can be considered small but finite. We assume that

$$\eta_{i,d} \approx \varepsilon^{1/2}\eta_{i,d0}$$

Where $\eta_{i,d0}$ is a parameter having finite value. Due to the minute value of η, we can use the same scaling factor as used for wave amplitude, i.e., ϵ for its scaling. We have considered the scaling of η for dust and ions in such a way that it gives significance only in dissipative term and hence, not showing any impact in nonlinear and dispersive terms. This is the magnificence of reductive perturbation method otherwise the involvement of dissipation in the wave dynamics may be absurd. The dependent physical quantities n_j , v_j (j=i,d) and ϕ are expanded about their equilibrium values in a power series as:

$$\begin{aligned} n_{j=}1 + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \cdots \\ v_{j=}1 + \varepsilon v_{j1} + \varepsilon^2 v_{j2} + \cdots \\ \varphi = \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \cdots \end{aligned} \tag{9}$$

Using stretching coordinates and Eq. (9) in Eqs. (1) - (6) and collecting terms of lowest order in ϵ, we obtain:

$$n_{d1} = -\frac{\delta_d}{V^2}\varphi_1, \quad v_{d1} = -\frac{\delta_d}{V}\varphi_1, \quad n_{e1} = \varphi_1$$

$$n_{i1} = \frac{\varphi_1}{V^2}, \quad v_{i1} = -\frac{\varphi_1}{V} \tag{10}$$

And

$$\delta n_{d1} + (1-\delta)n_{e1} - n_{i1} = 0 \tag{11}$$

From first order equations with small analytical calculations, the following dispersion relation of the DIAWs is obtained

$$V = \left(\frac{1 + \delta\delta_d}{1-\delta}\right)^{\frac{1}{2}} \tag{12}$$

From this equation, it is observed that phase velocity increases with increase in dust to ion density ratio δ. After tedious algebraic calculations, the following KdV-Burgers equation is determined as:

$$\frac{\partial\varphi_1}{\partial\tau} + A\varphi_1\frac{\partial\varphi_1}{\partial\xi} + B\frac{\partial^3\varphi_1}{\partial\xi^3} = C\frac{\partial^2\varphi_1}{\partial\xi^2} \tag{13}$$

where, the nonlinear coefficient

$$A = \frac{(1-\delta)V^4 + 3(\delta\delta_d^2 - 1)}{2(1+\delta\delta_d)V} \tag{14}$$

dispersive coefficient

$$B = \frac{(H_i^2 + H_e^2(1-\delta)V^4 - 2V^4)}{4(1+\delta\delta_d)V} \tag{15}$$

and dissipation coefficient

$$C = \frac{\eta_{d0}\delta\delta_d + \eta_{i0}}{2(1+\delta\delta_d)} \tag{16}$$

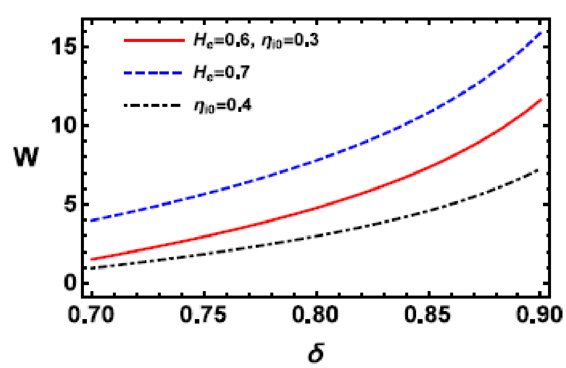

Fig. 1. (Color online) The variation of width W of shock structures with dust to ion density ratio δ (= n_{d0} n_{i0} Z_d) for different values of quantum electron Bohm potential (He) and ion kinematic viscosity (ηi0). For solid (Red) curve; He = 0.6 and η_{i0}= 0.3, dashed (Blue) curve; He = 0.7, dot-dashed (Black) curve; η_{i0}= 0.4, with fixed values of parameters η_{d0}=0.5 and H_i = 1.0

To examine the shock-like analytical solution of KdV–B Eq. (13), the tanh-method is employed to determine the shock solution of KdV–B equation as [24]

$$\phi_1(\xi,\tau) = \varphi_{max}\left[1-\frac{1}{4}\left[1+\tanh\left(\frac{\xi - V\tau}{W}\right)\right]^2\right] \tag{17}$$

Where,

$$\phi_{max} = \frac{12\,C^2}{25AB}, \quad W = \nabla^{-1} = \frac{10B}{C}, \quad u = \frac{6C^2}{25\,B}$$

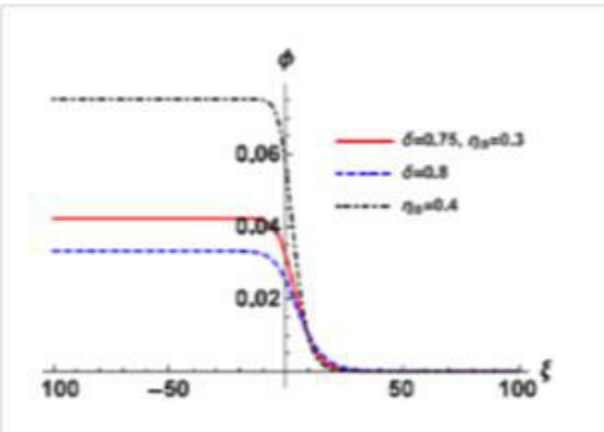

Fig.2. (Color online) The variation of shock wave profile ϕ with ξ For solid (Red) curve; $\delta = 0.75$ and $\eta_{i0} = 0.3$, dashed (Blue) curve; $\delta = 0.8$, dot-dashed (Black) curve; $\eta_{i0} = 0.4$, with fixed values of parameters as in Fig. 1.

4. Results and discussion

Owing to the dependence of nonlinear coefficient A, dispersion coefficient B and dissipation coefficient C on the various physical parameters, we have numerically analyzed the behavior of DIA shock waves in quantum dusty plasma from the KdV-Burgers Eq. (13) and its solution given by Eq. (17). The solution of KdV-B equation shows explicit dependence upon various physical parameters such as quantum electron Bohm potential (H_e), ratio of dust to ion density ratio (δ) and kinematic viscosity of ions (η_{i0}). From Eq. (12), it is seen that the phase velocity of DIA shock waves increases with the ratio of dust to ion density ratio δ. It is inferred that the shock waves tend to travel faster as the ratio of dust to ion density is increased. The dissipation term that appears due to kinematic viscosity in KdV-B equation leads to the variation of the width of shock structures. Fig. 1 illustrates the variation of width of shock structures with ratio of dust to ion density ratio (δ) with different values of quantum electron Bohm potential H_e and also along with ion kinematic viscosity increases with increase in dust to ion density ratio δ and with increase in quantum electron Bohm potential H_e. Contrast effect is seen for the variation of ion kinematic viscosity η_{i0} i.e., width decreases with increase in the value of η_{i0}. There is competition between dissipation and dispersive effects via coefficients C and B respectively. With increase in δ and He, dispersion coefficient B increases while C decreases which leads to increase in the width of shock structures. Fig. 2 presents the variation of shock profile with dust to ion density ratio (δ) and ion kinematic viscosity (η_{i0}), it is inferred that with increase in δ, the amplitude of shock structures decreases and with increase in the η_{i0}, the amplitude of the shock structures is enhanced. It is remarked that a more viscous ion fluid supports the formation η_{i0}. It is observed that the width of shock structures

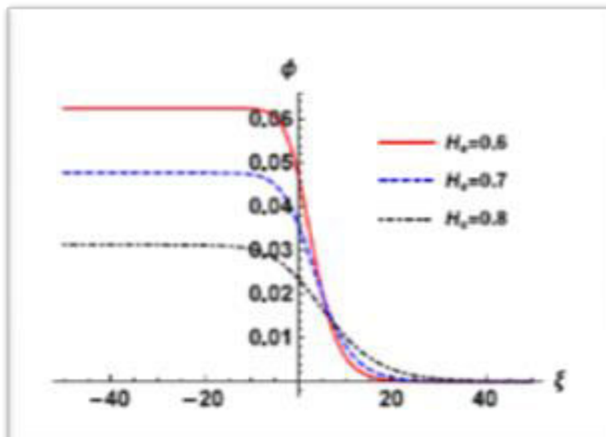

Fig.3. The variation of shock wave profile ϕ with ξ For solid (Red) curve; $H_e = 0.6$, dashed (Blue) curve; $H_e = 0.7$, dot-dashed (Black) curve; $H_e = 0.8$ for fixed values of parameters as $\eta_{d0} = 0.5$, $\eta_{i0} = 0.3$, $H_i = 1.027$ and $\delta = 0.5$.

of shocks of higher amplitude. Fig. 3 shows that for increase in the value of H_e, there is decrease in the amplitude of shock waves. In order to investigate the time evolution of the shock-like solution of the KdV-Burgers equation, we use a MATHEMATICA based finite difference scheme to numerically simulate Eq. (13). We consider the shock-like pulse as initial waveform with appropriate boundary conditions as:

$$y(0,\xi) = \Delta_{ini}(1-\tanh(k_0\xi)), \xi \in (-L, L) \tag{18}$$

Where, L depicts the spatial length and $\triangle_{ini}$ is the shock amplitude. The boundary conditions used are $y(\tau, \xi) = \triangle_{ini}(1+ \tanh(k_0 L))$ and $\phi_1(\tau, L) = 0$. Fig. 4

illustrates the evolution of oscillatory shock waves with parameters as $H_e = 0.2$, $\eta_{i0} = 0.3$ and $\delta = 0.6$. It is inferred that the initial shock-like pulse develops an oscillatory tail with increase in the amplitude, i.e., as time evolves, the dispersive effect tends to overpower the plasma dynamics and with the balance between the nonlinear, dispersive and dissipation effects, a monotonic shock transforms into an oscillatory shock profile. Fig. 5 illustrates the oscillatory shock wave profile for different value of δ, η_{i0} and H_e.

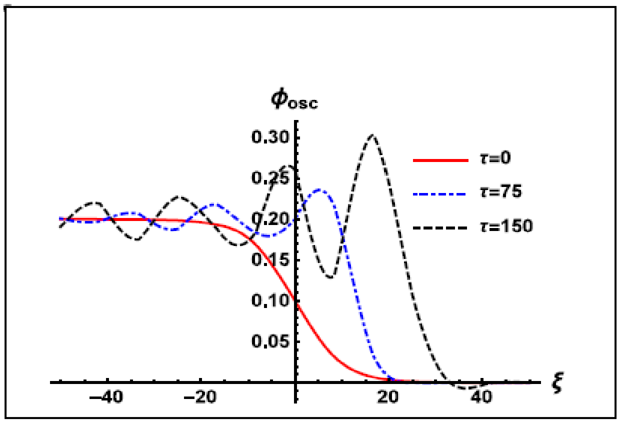

Fig. 4. The variation of oscillatory shock wave profile ϕ_{osc} with ξ at different values of time The parameters being $H_e = 0.2$, $\eta_{i0}= 0.3$, $\eta_{d0}= 0.5$, $H_i = 1.027$ and $\delta = 0.6$.

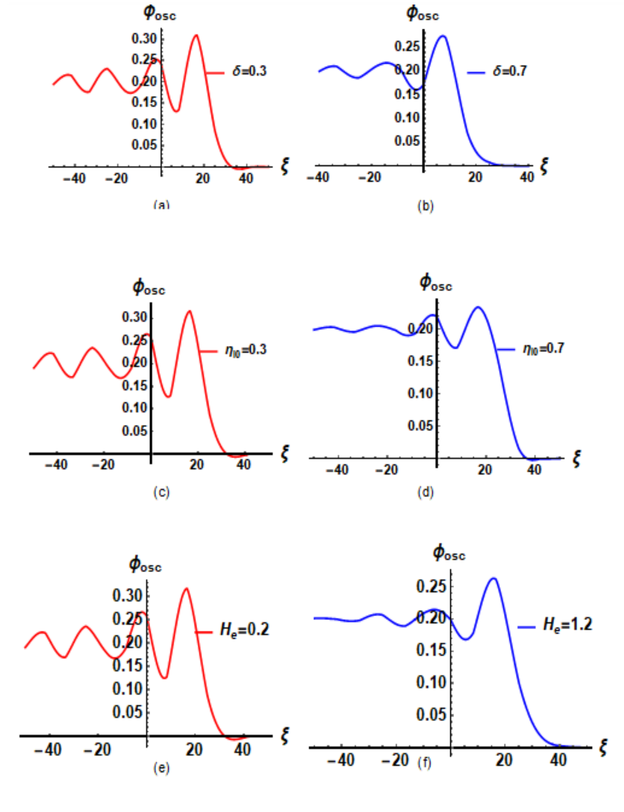

Fig. 5. The variation of oscillatory shock wave profile ϕ_{osc} with ξ for different values of δ, η_{i0} and H_e

It is observed that the amplitude of oscillatory shocks decreases with increase in δ (see Figs. 5(a) and (b)), In Figs. 5 (c) and (d), the variation of ϕ_{osc} is depicted for different values of η_{i0} and observed that ϕ_{osc} also decreases with increase in η_{i0}. Figs. 5 (e) and (f) depict the variation of ϕ_{osc} with change in H_e. The amplitude of shocks also decreases with enhancement in the value of H_e. It is emphasized that the dispersive effects become less effective and the monotonic character of the shock wave is retained for the longer time. It is revealed that monotonic shock structures are more stable as the quantum electron Bohm potential H_e is increased. Hence, all the plasma parameters have significant influence on the time evolution of nonlinear shock structures in quantum dusty plasma.

5. Conclusions

Nonlinear properties of low frequency ion acoustic shock waves propagating in an electron-dust-ion plasma with two-fluid quantum hydrodynamic model (QHD) are investigated. By employing the reductive perturbation technique, the one-dimensional KdV-B equation has been derived. Further solution of KdV-B equation is determined to highlight the effect of quantum electron Bohm potential, dust to ion density ratio and ion kinematic viscosity on the characteristics of DIA shock waves. Increase in the quantum electron Bohm potential and dust to ion density ratio tends to reduce the amplitude of the shocks and increase in ion kinematic viscosity tends to enhance the amplitude of shocks. Further, the time evolution of the shock waves is also studied as a numerical solution of KdV-Burgers equation. It is observed that an initial shock like pulse forms a oscillatory shock profile for lower values of different parameters such as dust to ion density ratio δ, ion kinematic viscosity (η_{i0}) and quantum electron Bohm potential (H_e) and the monotonic shock structures are obtained with increase in the value of δ, η_{i0} and H_e. The results of this investigation may be important for the understanding of shock wave propagation in dense astrophysical environments such as those occurring in the interior of giant planets or dwarf and neutron stars.

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