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Stationary Formations of Dust-Ion Acoustic Waves in Degenerate Dusty Plasma at Critical Regime

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The theoretical investigation of double-layer structures in a dense, quantum plasma containing electrons, ions, and mobile dust particles, has been carried out. A linear dispersion relation is derived for the corresponding Dust-Ion Acoustic Waves. We have considered the collisions to be absent. Furthermore, with the help of a standard reductive perturbation technique and using the one–dimensional quantum hydrodynamic (QHD) model, an Evolutionary profile formation of Ion Acoustic Waves with mobile dust particles in a dissipative, dispersive medium has been examined in a 3 – component ultra – relativistically degenerate super dense quantum plasma and analyzed numerically. The relativistic effects significantly alter the linear and non–linear properties of Dust-Ion plasma waves. The importance of the results of the studies of both linear and non-linear characteristics and their parametric dependence studied graphically, have also been pointed out.

1. Introduction

The investigation of linear and non-linear phenomena in various media is the thrust area of research in science and technology. Ion acoustic waves (IAW) and Electron acoustic waves (EAW) with two and multi-component plasmas are being studied continuously for the last four decades. The emergence of relativistic and quantum effects in plasmas unfolded a new dimension in the studies of solitary waves. Space plasmas have created vast interest among the plasma workers due to the occurrence of dust particles. This interest is large because the presence of particles significantly alters the charged particle equilibrium leading to different phenomena.

A plasma that contains millimetre (10^{-3}) to nanometre (10^{-9}) sized suspended particles is called dusty plasma. Dust particles are charged. The study of plasmas with this charged dust particles creates an additional complexity, therefore dusty plasmas are also known as complex plasmas. Astrophysical environments such as in interstellar medium, in cometary tails, in asteroid zones, in planetary rings, in the earth's magnetosphere, in the neighbourhood of stars [1], [2], and radiofrequency discharge contain abundant dusty plasma. After a lot of bewilderment, scientists realized that the spokes of Saturn were possibly minute specks of dust moving around the rings because of the electric and magnetic forces which result from electrically charged gases known as plasma. The damping source to dust acoustic waves is the fluctuation in the dust charge.

Again, the physics of quantum plasmas is now a very rapidly growing subject of plasma physics. Various linear and nonlinear properties of plasma waves are significantly altered by quantum effects observed in white dwarfs and neutron stars. The quantum hydrodynamic (QHD) model, is derived from the Wigner equations by taking velocity space moments, generalizes the classical fluid model for plasma with the inclusion of a quantum correction term also known as the Bohm potential [3].

The QHD model has been widely used by several authors [4-8] because of simplicity, straight forward approach, and numerical efficiency. The role of quantum diffraction in the propagation of ion-acoustic waves has been investigated by Haas et al [9]. The linear dispersion character of compressional magneto-acoustic waves in a quantum magneto-plasma has been studied by Shukla [10] by taking into account the quantum Bohm potential and the magnetization of electrons owing to electron spin effects.

In some plasmas, the particle velocities may become very high; and in some cases, it may even approach the speed of light and therefore it becomes important to consider the relativistic effects for such plasmas. Relativistic plasmas can be formed in many practical situations such as in white dwarfs, plasma phenomena [11], the plasma sheet boundary of Earth's magnetosphere [12], Van Allen radiation belts [13], and laser plasma interaction experiments [14]. The degenerate electron number density is extremely high in such compact objects which results in the electron Fermi energy being comparable to the electron Mass energy and the electron speed being comparable to the speed of light in a vacuum. The equation of state for these degenerate electrons in such interstellar compact objects is mathematically explained by Chandrasekhar for two limits, namely, non-relativistic and ultra-relativistic limits. The degenerate electron equation of state of Chandrasekhar is P_e for the ultra-relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. So we can study these compact objects at extremely high densities (degenerate state) for which quantum, as well as relativistic effects, become important.
Elementary electrodynamics shows us that charged particles can never be accelerated by magnetic fields. Electric fields are necessary, either electrostatic or induced by time varying magnetic fields. Since energetic electrons and ions are often seen in plasmas, it is important to realize the mechanisms that can generate and maintain magnetic fields in plasmas. Many such mechanisms have been discussed by Block and Fälthammar (1976). One of these mechanisms is the Double Layer mechanism, which is electrostatic.
The present paper summarizes the various properties of importance in quantum dusty plasmas. The paper is organized in the following way : Initially the basic set of quantum hydrodynamic equations are presented, including relativistic effects. Then we derived the Evolutionary equation in a dispersive , dissipative medium using the standard multiple scale perturbation technique by taking into account ultra-relativistic effects. We analytically solved this equation to get an exponentially decaying Double Layer Structure. In the last section we discuss the results and give some concluding remarks of the work.

2. Basic formulations

Equations governing the Dynamics of Motion

We consider the propagation of Dust-Ion acoustic waves in an unmagnetized , ultra-relativistically degenerate , 3-component super dense quantum plasma . In order to investigate the formation and propagation of Double-layered structures in a homogenous , un-magnetized plasma ; we start with a set of inter-penetrating fluid characterized by the equations of continuity and motion of the negatively charged dust particles, positively charged ions and electrons with the Poisson's Equation. The dynamics of such a plasma is governed by the equations following the one-dimensional quantum hydrodynamic model .

Following Chandrasekhar (1939) , the electron degeneracy pressure in fully degenerate and relativistic configuration can be expressed as follows :

$$\mathrm{P_j} = \frac{\pi m_e{}^4 c^5}{3h^3}\left[R_j\left(2R_j{}^2 - 3\right)\sqrt{1 + R_j{}^2} + 3\sinh^{-1} R_j\right],$$

In which,

$$R_j = \frac{P_{F_j}}{m_e c} = \left[3h^3 n_j / 8\pi m_e{}^3 c^3\right]^{1/3} = R_{j_0} n_j{}^{1/3}$$

Where, R_j is the Relativity parameter .

Using , $R_{j_0} = \left(\frac{n_{j_0}}{n_{e_0}}\right)^{1/3}$ with $n_{e_0} = \frac{8\pi m_e{}^3 c^3}{3h^3} \approx 5.9 \times 10^{29}$ and for $R_j \to \infty$, we get $P_j = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3} hcn_j{}^{4/3}$.

This is the Ultra-relativistic degeneracy pressure.

We use : $P_j = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3} hcn_j{}^{4/3}$

Where, j = e for electrons and j = i for ions .

Assuming that the Bohm potential is independent of any thermal fluctuations at finite temperature situation , the set of QHD equations governing the dynamics of the dust-ion plasma waves in the Model Plasma under consideration are given by -

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \quad (1)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \quad (2)$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0 \quad (3)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{Q_d}{m_d}\frac{\partial \varphi}{\partial x} + \frac{1}{m_d}\eta_d \frac{\partial^2 u_d}{\partial x^2} \quad (4)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{Q_i}{m_i}\frac{\partial \varphi}{\partial x} - \frac{1}{m_i n_i}\frac{\partial p_i}{\partial x} + \frac{1}{m_i}\eta_i \frac{\partial^2 u_i}{\partial x^2} \quad (5)$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{Q_e}{m_e}\frac{\partial \varphi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x}$$

$$+\frac{\hbar^2}{2{m_e}^2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \qquad (6)$$

$$\frac{\partial^2\phi}{\partial x^2} = Q_e n_{e+} Q_d n_d - Q_i n_i \qquad (7)$$

Where,

$j \rightarrow$ e for electrons, d for dust and i for ions
$n_j \rightarrow$ number density of the j^{th} species
$Q_j \rightarrow$ charge ($Z_j |e|$) of the j^{th} species
$\varphi \rightarrow$ electrostatic wave potential
$P_j \rightarrow$ Degeneracy Pressure of the j^{th} species

To achieve the dimensionless form of the above equations, we used standard normalization conditions , which are –

$$x \rightarrow \frac{x\omega_{pe}}{v_{Fe}}\, t \rightarrow t\omega_{pe}\, n_d \rightarrow \frac{n_d}{n_{d_0}}\, u_d \rightarrow \frac{u_d}{v_{F_e}}$$

$$\varphi \rightarrow \frac{|e|\varphi}{2k_B T_{F_e}}\, \eta \rightarrow \frac{\eta}{m}\frac{\omega_{pe}}{{v_{Fe}}^2} \text{where} ,\ \omega_{pe} = \sqrt{\frac{4\pi n_{e_0}e^2}{m_e}}$$

3. Analytical studies

3.1 Derivation of Linear Dispersion Relation:

In order to investigate the linear behaviour of the Dust Ion Acoustic waves, we obtain the Linear Dispersion Relation and study its parametric dependence. We assumed that all field variables vary as $e^{i(kx-wt)}$ and accordingly for normalized wave frequency (ω) and complex wave number (k) [which contains both real and imaginary part] , the Linear Dispersion equation is obtained .

Here the viscous term plays a pivotal role. The Dispersion Relation has an exponentially decaying complex part in addition to the real part. In this case , we substitute the wave number with a real plus an imaginary part (k= k_x +i k_y) . The imaginary part of the dispersion relation amounts to collision-less damping. On the other hand the real dispersion relation follows that of a standard dust ion acoustic wave. The complex part of the viscosity is due to its dynamic nature. In this section we derived the complex dispersion relation for dust ion acoustic waves including dynamic viscosity.

If we take the coefficient of the imaginary part of the complex wave no 'k' to be 0 i.e. $k_y = 0$, then we get a simplified Real Dispersion Relation expression:

$$\frac{-\mu Z_d^2\delta_1}{k_x^4\eta_d^2+(\omega-u_0k_x)^2}+\frac{4Z_e^2}{-4(\omega-u_0k_x)^2+4k_x^2\beta+H^2k_x^4} +\frac{\rho Z_i^2\delta_2\left[(\omega-u_0k_x)^2+k_x^2\alpha\right]}{\left[\eta_i k_x^2(\omega-u_0k_x)\right]^2+\left[(\omega-u_0k_x)^2+k_x^2\alpha\right]}=-1 \qquad (8)$$

Similarly, If we take the coefficient of the real part of the complex wave no 'k' to be 0 i.e. $k_x = 0$, then we get a simplified Imaginary part of the linear dispersion relation expression:

$$\frac{\mu Z_d^2\omega k_y(2u_0+\eta_d)\delta_1}{(\omega^2+k_y^2)\left[(k_y^2\eta_d+k_yu_0)^2+\omega^2\right]}+ \frac{4Z_e^2(8\omega u_0k_y)}{\left(-4\omega^2-4k_y^2(\beta-u_0^2)+H^2k_y^4\right)^2+(8\omega u_0k_y)^2}+ \frac{\rho Z_i^2\left[-\omega\eta_i k_y^2-2\omega u_0k_y\right]\delta_2}{\left[-\omega\eta_i k_y^2-2\omega u_0k_y\right]^2+\left[\left(-\omega^2+u_0\eta_i k_y^3+(\alpha+u_0^2)k_y^2\right]^2}=0 \qquad (9)$$

3.2 Derivation of evolutionary equation in a dispersive dissipative medium and its solution

In order to study the nonlinear nature of Dust-Ion acoustic wave, we derive the KdV equation from the basic equations. For the description of the propagation of dust ion acoustic waves, we expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter ε as follows:

$$\begin{bmatrix} n_j \\ u_j \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 \\ u_0 \\ \varphi_0 \end{bmatrix} + \epsilon^1 \begin{bmatrix} {n_j}^{(1)} \\ {u_j}^{(1)} \\ \varphi^{(1)} \end{bmatrix} + \epsilon^3 \begin{bmatrix} {n_j}^{(2)} \\ {u_j}^{(2)} \\ \varphi^{(2)} \end{bmatrix} + \epsilon^5 \begin{bmatrix} {n_j}^{(3)} \\ {u_j}^{(3)} \\ \varphi^{(3)} \end{bmatrix} .. \qquad (10)$$

From the Standard Reductive Perturbation Technique, we use the following stretching of space and time variables:

$$\xi = \epsilon(x - v_0 t)\tau = \epsilon^3 t\eta = \epsilon\eta_0$$

Where,
$v_0 \rightarrow$ Normalized Linear wave Phase velocity
$\varepsilon \rightarrow$ Smallness parameter
ξ and τ are stretched space and time coordinates respectively.

The stretching in η is due to small variation in the perpendicular direction.
Now solving for the lowest order equation with boundary conditions for all variables:. $n_d^{(1)}$, $n_e^{(1)}$, $n_i^{(1)}$, $u_d^{(1)}$, $u_e^{(1)}$, $u_i^{(1)}$ and $\varphi^{(1)} \to 0 \;\; as|\xi| \to \infty$, the following solutions are obtained :

$$n_d{}^{(1)} = \frac{\mu Z_d}{-(v_0-u_0)^2}\varphi^{(1)} = P_d\,\varphi^{(1)}(say)\ , \quad (11)$$

$$n_e{}^{(1)} = \frac{Z_e}{\beta-(v_0-u_0)^2}\varphi^{(1)} = P_e\,\varphi^{(1)}(say)\ , \quad (12)$$

$$n_i{}^{(1)} = \frac{\rho Z_i}{-[(v_0-u_0)^2+\alpha]}\varphi^{(1)} = P_i\,\varphi^{(1)}(say), \quad (13)$$

Going for the next higher order terms in ϵ and following the usual method, we obtain the desired Evolutionary Equation:

$$\frac{\partial\varphi}{\partial\tau} + A\frac{\partial^3\varphi}{\partial\xi^3} - B\frac{\partial^2\varphi}{\partial\xi^2} = 0\ , \qquad (14)$$

(Here we see that the non-linear term is absent and the dispersive effect is predominant. The dissipative term relating to the viscous coefficient η of ions and dust, is also present.)

To find the solution of the KdV Burgers equation, we transform the independent co-ordinates ξ and τ into coordinate η = ξ − Mτ. Here M is the normalized constant speed of the wave frame, also known as Mach number.

Applying the boundary conditions that:

$$as|\eta| \pm \infty\ ;\ \varphi, \frac{\partial\varphi}{\partial\eta}, \frac{\partial^2\varphi}{\partial\eta^2} \to 0$$

the following solution is obtained:

$$\varphi = \varphi_0\, e^{\left[(B-\sqrt{B^2+4AM})/2A\right]\eta}\ , \qquad (15)$$

Where, φ_0 is the Amplitude of the Electrostatic Potential wave.

4. Numerical studies: results and discussions

4.1 Real part of the complex linear dispersion relation

The wave frequency ω is found to increase with the increase in wave number k for a definite value of Quantum diffraction parameter H and the increase of ω with k is even more if we increasingly vary H from 1 to 3. Higher the value of H steeper is the curve.

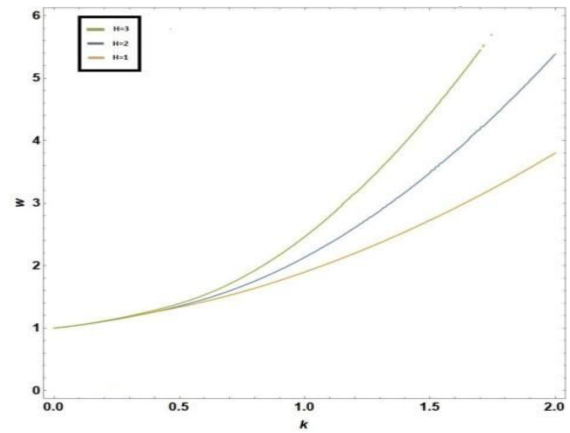

Fig. 1(a): Dispersion Curves for different values of Quantum Diffraction Parameter (H).

The H dependent term, called Bohm potential term, plays a role similar to that of pressure and is responsible for typical quantum phenomena such as tunnelling and wave packet spreading. H is a normalizing factor of the energy of a Plasmon (here we take only electrons).The electron plasma oscillation frequency evidently contributes to the frequency of the dust ion acoustic wave making the quantum effects generate high frequency oscillations in the high wave number range. Quantum effects in plasma become significant when the fermi energy of the plasma species exceeds the thermal energy which requires high density and low temperature.

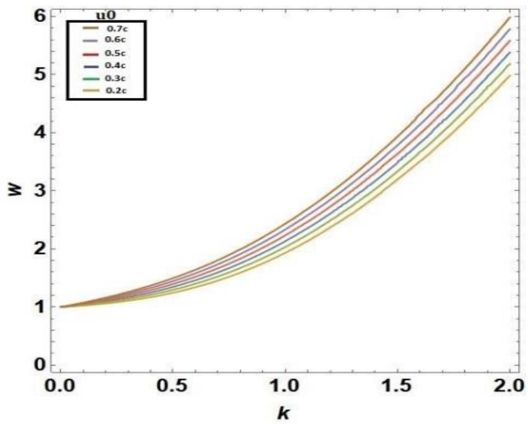

Figure 1(b): Dispersion Curves for different values of Streaming Velocity (u_0)

For a definite value of the Streaming Velocity u_0, the wave frequency ω increases sharply with the wave number k. The steepness of the curve increases with u_0.

If an energetic particle stream is injected in a plasma, a current will be set up along the plasma. So the different plasma species (ions, electrons and dust) will have different relative drift velocities. The energy from the particles can lead to plasma wave excitation thus leading to an increase in group velocity of the wave which results in the increased steepness of the dispersion curve.

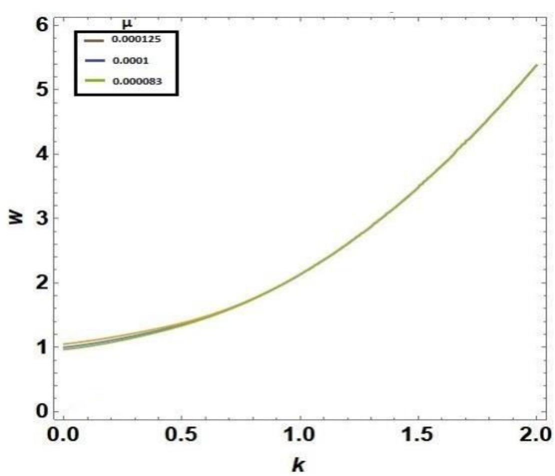

Fig. 1(c): Dispersion Curves for different values of Electron: Dust Mass ratio (μ)

When we consider a particular value of Electron – to- Dust mass ratio μ (standard 0.0001 as the mass of a Dust particle is approximately 10,000 times that of an Electron) , it is observed that ω increases with k. For long wavelength range (when k<0.5), it is seen that from μ=1/8000 (0.000125) to μ=1/12000 (0.000083), the value of ω decreases very minutely. For higher values of k, we see an asymptotic convergence of the three curves.

μ is the ratio of mass of electron to that of dust. Since both are negatively charged we can consider their reduced mass. With decrease in μ, the reduced mass of the dust and electron also increases. Now we know from the expression of plasma frequency that it is inversely proportional to the mass of the species. Therefore with decrease in μ, we observe a decrease in frequency of the curves.

In the low energy region of the curve (low wave number), the amount of energy required to set the dust particles of different mass(due to different inertia) in oscillations of same frequency is slightly different, which explains the slight deviation of frequency with μ_d in the low k region. In the high energy region of the curve (high wave number), the differences in the ratio of the masses do not alter the wave frequency of the compact dense plasma.

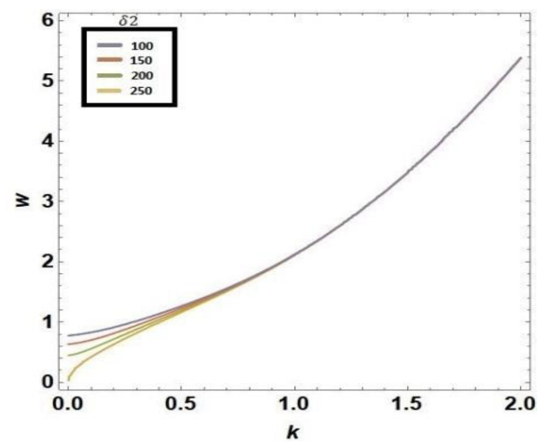

Fig. 1(d): Dispersion Curves for different values of Ion -to-Electron Equilibrium number density ratio (δ_2)

For a particular value of Equilibrium Number Density ratio (δ_2) of Ion to Electrons, ω increases with k. For smaller values of δ_2 (in the order of one-tenths to hundreds), there is no noticeable change in the w v/s k curve with the variation of δ_2. When δ_2 is varied from 1000 to 2500 in the order of 500, significant changes in the graph can be seen. In the long wavelength range (when k<1), it is seen that the value of ω minutely decreases with the increase in δ_2.The 4 curves asymptotically converges near k~ 1.

Now, the charges of ions and electrons are opposite in nature. Therefore the electric fields that are set up by their respective oscillations are also opposite in nature. Therefore the fields get balanced when taken together which ultimately results in the decrease in the frequency of the plasma wave in the low energy region of the dispersion curve. This effect increases with the density of the plasma. So the equilibrium number density ratio of ions to electrons influences the frequency and group velocity of the plasma wave and they decrease with δ_2. In the high energy region of the curve (high wave number), the differences in the ratio of the equilibrium number densities do not bring about any significant change in the wave frequency of the compact dense plasma.

For a definite value of viscous coefficient of a dust (η_d), ω increases with k. With the increase in η_d, the ω curve is found to decrease within a small range of k. The curves for different values of the parameter η_d overlap for small k (<0.3), then diverge slightly in the range 0.3<k≤1 and again asymptotically converge for k>1.

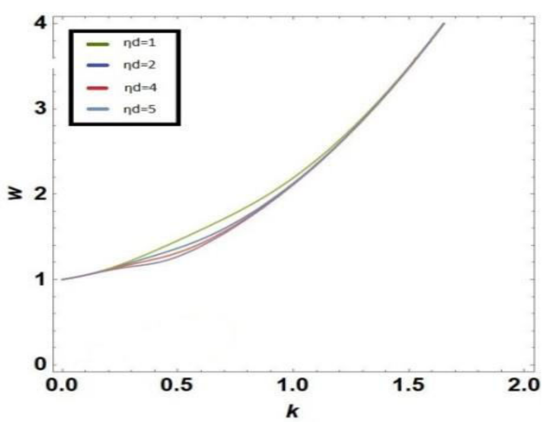

Fig. 1(e): Dispersion Curves for different values of Coefficient of Viscosity of Dust (η_d)

We see that the dispersion curves are almost independent of the viscosity effects except in the small wave number range. In this range of energy due to resonant interactions between the dust particles in the plasma, an exchange of energy happens. This results in the slight divergence of the modes with varying viscosity parameters. The asymptotic convergence of curves for higher values suggests the independence of the plasma propagation on viscosity parameters.

4.2 Imaginary part of the complex linear dispersion relation

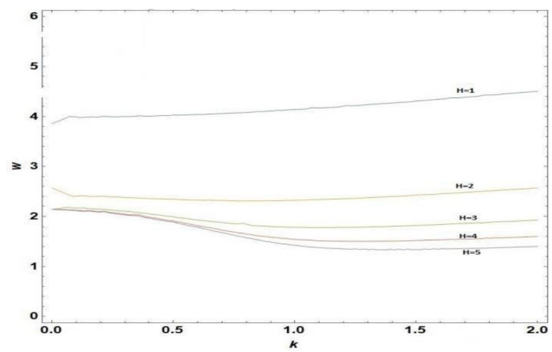

Fig. 2(a): Dispersion curves for different values of quantum diffraction parameter (H)

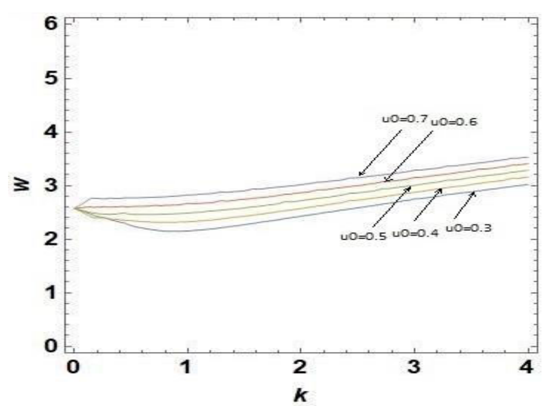

Fig. 2(b): Dispersion Curves for different values of Streaming Velocity (u_0)

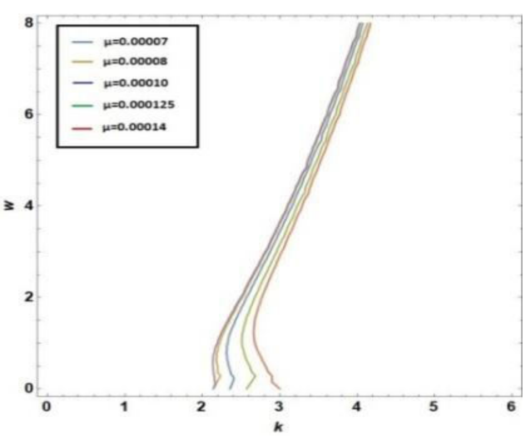

Fig. 2(c): Dispersion Curves for different values of Electron: Dust mass ratio (μ)

When we consider a particular value of electron: dust mass ratio, it is observed that ω increases with

k. The curves show a sudden and a very steep increase of ω in the $2<k\leq 3$ wave number region. Though the curves increase almost parallel, it is quite evident from the graph that due to the increase in μ the curves laterally shift towards the region of low k (long wavelength limit). We know that Bohm potential term, is responsible for typical quantum phenomena such as tunnelling and wave packet spreading. So before the particles in the plasma can lose their respective energies due to dissipative effects, the quantum tunnelling phenomenon occurs and the particles tunnel through the barriers to come to equilibrium with the surrounding environment. Due to this the frequency abruptly increases in the low wave number region and ultimately becomes independent of energy.

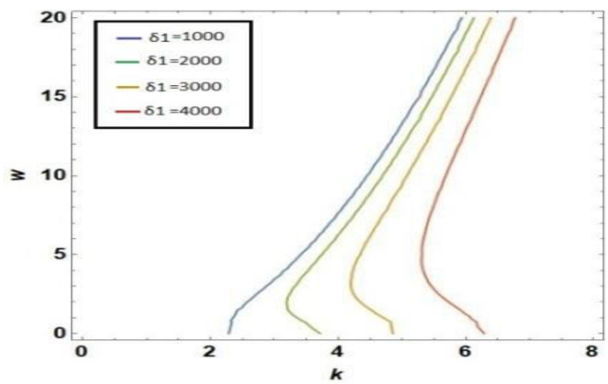

Fig. 2(d): Dispersion Curves for variation of dust to electron equilibrium density ratio (δ_1)

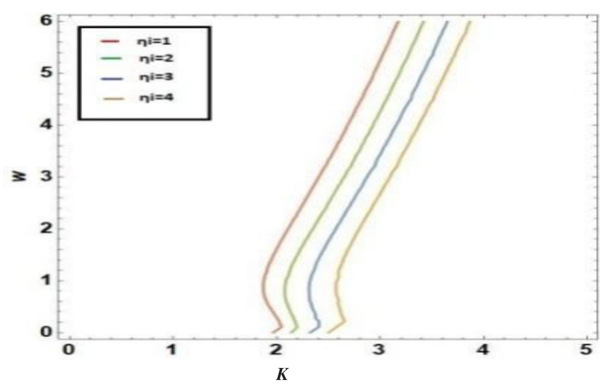

Fig. 2(e): Dispersion Curves for different values of Coefficient of Viscosity of Ions (η_i).

The viscosity effects on plasma results in damping just like any fluid moving in a highly viscous medium experiences damping. And the curves of varying viscosity parameters don't seem to converge with high frequency and k.

4.3 2-D Graphical solution of the evolutionary equation in dispersive, dissipative Medium

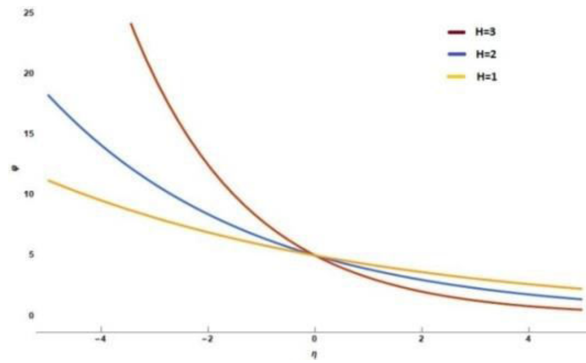

Fig. 3(a): φ v/s η Curves for different values of Quantum Diffraction Parameter (H).

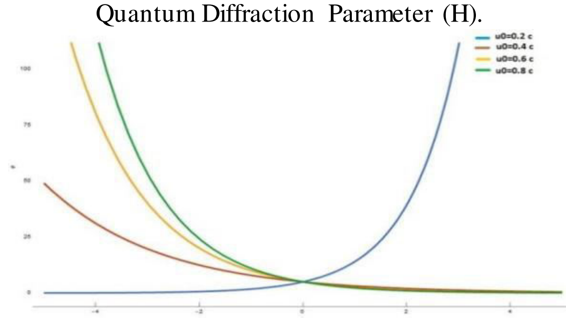

Fig. 3(b): φ v/s η Curves for different values of Streaming Velocity (u_0).

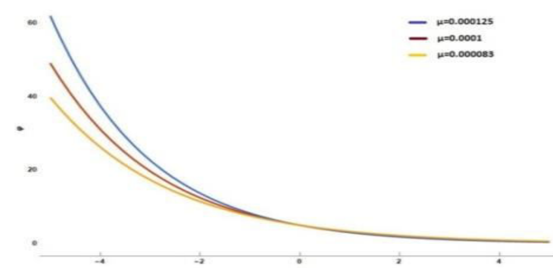

Fig. 3(c): φ v/s η Curves for different values of Electron: Dust mass ratio (μ)

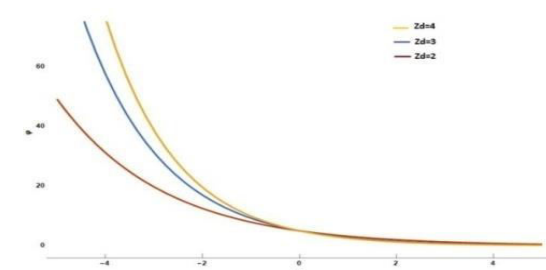

Fig. 3(d): φ v/s η Curves for different values of Dust Charge (Z_d)

After mathematically solving the set of normalized equations, we get an equation for the electrostatic wave potential. This solution is devoid of any nonlinear term. So instead of obtaining the conventional form of KdV-B Equation, we get here a new type of an equation which we term as the "Evolutionary Equation in a Dispersive, Dissipative Medium". On analytically solving the equation, we learn that the solution is an exponentially decaying function with stretched coordinate η (where η is defined in terms of space and time co-ordinates: ξ and τ). We infer that since there is no abrupt change in the potential wave function or no stationary structure formation is observed, this potential do not give rise to shocks and solitary structure. When streaming velocity is decreased below a critical value we observe that this evolutionary formation of potential becomes exponentially increasing. This may be because of the dependence of the phase which may be getting reversed below a certain u_0. .

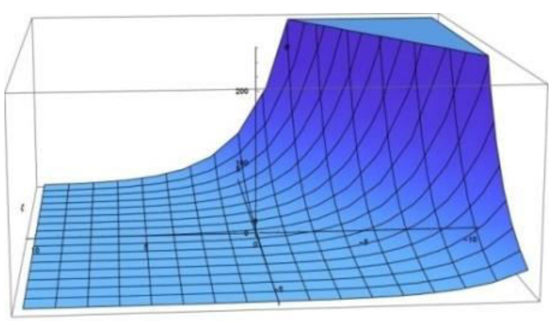

Fig. 4**:** 3-D plot showing the variation of φ with co-ordinates ξ and τ

When we analyse the potential structure with respect to the independent co-ordinates ξ and τ, we observe a Double Layered structure. A double layer is a structure in a plasma and consists of two parallel layers with opposite electrical charge. The sheets of charge cause a strong electric field and a correspondingly sharp change in voltage (electrical potential) across the double layer. Double layers are found in a wide variety of plasmas, from discharge tubes to space plasmas and are especially common in current-carrying plasmas.

5. Concluding Remarks

To summarize, we have investigated the properties of Dust Ion Acoustic waves in plasma in the framework of the Quantum Hydrodynamic Model incorporating ultra-relativistic effects. Numerical and analytical study of the linear dispersion relation (both complex and imaginary) are carried out to examine the Quantum effects due to Bohm Potential, viscosity effects, and many other parameters like Streaming velocity, mass ratios. The existence of Double Layered structure with the odd perturbation approach to our normalized set equations at the critical regime involving stretching of equations with integral powers has been investigated using an analytical approach. It is seen that the amplitude of these potential formations diminish as the negative dust charge Z_d decreases with steep continuous fall near zero charges and becomes almost constant for higher values of η. The Figures [3(a)] and [3(b)] shows how the amplitude decreases significantly with H and u_0. The investigation presented here may be helpful in the understanding of the basic features of dense quantum dusty plasma found in many astrophysical environments including Earth's magnetosphere and Saturn's Rings. We may also get to study different instabilities like the Buneman Instability.

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