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# Nonlinear Behaviour of Dust Acoustic Wave Mode in a Dynamic Dusty Plasma Containing Negative Dust Particles and Positrons

S. Ballav[1], S. Kundu[1], A. Das[1], S. Chandra[2*]
[1]*Department of Physics, Indian Institute of Technology, Kharagpur, West Bengal-721302, India*
[2]*Department of Physics, Government General Degree College at Kushmandi, West Bengal-733121, India*
**Institute of Natural Sciences and Applied Technology, Kolkata, India, 700032.*

*E-mail:swastikballav2017@gmail.com (corresponding author)*

In this paper, the main focus of discussion is the evolution of the Korteweg-de Vries(KdV) equation by employing the standard reductive perturbation technique. The evolution of the KdV equation was derived using the conservation laws for DAW(Dust Acoustic Wave) mode in plasma containing dust particles and positrons to ascertain the structure of solitons. In this paper, the KdV equation has been solved both numerically and analytically. Further, setting up the NLSE(Non-Linear Schrodinger Equation) studies about Rogue wave and Dynamical system has been carried out from the derived KdV equation. In addition to that, linear and non-linear analysis of Dispersion relation and group velocity profile for the DAW mode in plasma also has been discussed. All studies are supported by graphical representation to show all analytical results follow the theoretical model.

## 1 Introduction

Investigation of nonlinear phenomena in numerous media is becoming one of the topics of research interests in science and technology in recent years. A dusty plasma is defined as a mixed medium of dust particles, negative and positive ions, electrons. Massive dust particles in a plasma can introduce new modes and instabilities such as dust-ion-acoustic waves and dust-acoustic waves etc. Many fascinating non-linear effects like Solitary waves, Modulation Instability, Double layers, etc. are being continuously analyzed both in complex space plasma and laboratory plasma systems[1][2][3][4][5][6]. The size of a dust grain is in the range of micrometer and sub-micrometer[7][8]. Due to field emission, plasma currents change the properties of plasma waves in space. In many astrophysical environments such as interstellar medium, asteroid zones, cometary tails, planetary rings, Earth's magnetosphere, the neighborhood of stars, radio frequency discharges, dusty plasmas are abundantly found[8][9][20]. It has been found that the presence of statically charged dust grains in plasma can generate extremely low-frequency dust acoustic waves in the absence of a magnetic field[10][11][20]. The vibrations of dust charges cause damping to dust acoustic waves. Many results (with minor corrections) of negative ion plasma can be adapted to dusty plasma for its low-frequency behavior when the wavelength and the inter-particle distance are much larger than the grain size[12][20]. In the case of space plasma with stationary or mobile dust, DA and DIA solitons have been investigated to conclude the structure of solitons with positive or negative potentials based on implicitly occurring dust charges $Z_d$ in some forms. When dust particles are present in the system, the relativistic effects to the small particle as electrons and ions in the space regions, like laser-plasma interaction, Van Allen Radiation belt, Earth's magnetosphere, etc. are not considered[9]. In this project we have considered the plasma model consisting of mobile dust particles, Maxwell-Boltzmann distributed positrons, to derive the Korteweg de-Vries (KdV) equation[13][14] for solitary wave structures. As well as we have extended studies for dispersion relation. There is a certain kind of nonlinear Schrodinger equation(NLSE) derived solutions that give rise to very high amplitude waves[15][16][17]. These waves occur sporadically and vanish within an instant. The resulting wave is known as Rogue Wave(RW). Recently, the rogue waves are theoretically observed in a multicomponent plasma and have been experimentally observed and in the framework of the NLSE. Presently, rogue waves studies are done in many different systems like nonlinear fiber optics, Bose-Einstein condensates, superfluid, optical cavities, plasmonic, narrowband directional ocean waves, and electromagnetic pulse propagation.

## 2 Basic formulations

*A. Governing Equations*

The equations for Dust & Ion-acoustic mode in 3 component plasma are[13][18][19] the following:

1.1. $\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0$

1.2. $(\frac{\partial}{\partial t}+u_d\frac{\partial}{\partial x})u_d=\frac{Q_d}{m_d}\frac{\partial \phi}{\partial x}+\frac{1}{m_d}\eta_d\frac{\partial^2 u_d}{\partial x^2}$

1.3. $\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0$

1.4. $\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x}\right)u_i=\frac{1}{m_i}\left(Q_i\frac{\partial \phi}{\partial x}-\frac{1}{n_i}\frac{\partial p_i}{\partial x}\right)+\frac{1}{m_i}\eta_i\frac{\partial^2 u_i}{\partial x^2}$

1.5. $\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e u_e)=0$

1.6. $\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{\partial x}\right)u_e=\frac{1}{m_e}\left(Q_e\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial p_e}{\partial x}\right)+\frac{\hbar^2}{2m_e\gamma_e}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right)$

1.7. $\frac{\partial^2 \phi}{\partial x^2}=4\pi(Q_e n_e+Q_i n_i+Q_d n_d)$

(1)

Where, $d$, $i$ for dust and ions $n_d$ and $u_d$ and $n_i$ and $u_i$ are the number density and velocity of the respective species. Equation of motion for dust and ions are respectively given as and $\varphi$ is the electrostatic potential, $Q_e = e$, $Q_d = Z_d e$, $Q_i = -Z_d e$, $Q_d = Z_d e$ , $e = 1.6\times10^{-19}$($z_d$ is the number of effective charges of dust it is the average number of electrons accumulated on it), $m_d$ and $m_i$ are the mass of each species respectively, $\eta_d$ and $\eta_i$ are the dissipation coefficient of dust and ions.

*B. Normalization*

We applied the following normalization scheme:

$$x\rightarrow\frac{x\omega_c}{V_{Fh}},t\rightarrow t\omega_c,\Phi\rightarrow\frac{e\Phi}{2k_B T_{Fh}},n_j\rightarrow\frac{n_j}{n_j0},$$

$$u_j\rightarrow\frac{u_j}{V_{Fh}}$$

*Note*:

$$V_{Fh}=\sqrt{\frac{2k_B T_{Fh}}{m_e}},\omega_c=\sqrt{\frac{4\pi n_e e^2}{m_e}}$$

Applying these Normalizing scheme in the governing equation(eq:1) 3 sets of Normalized equations are found which are (a)Dust and Ion-acoustic mode in 3(e, d, i) components; (b) IAW mode in 2 components (d, i); (c) DAW mode in 2 components (positron, negative dust) We have used set-c mode for further study. The normalized governing equations are the following:
The dust-acoustic mode in 2 component plasma [Positron and Negative Dust]

*1) SET C:*

2.1. $\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}(n_d u_d)=0$

2.2. $(\frac{\partial}{\partial t}+u_d\frac{\partial}{\partial x})u_d=\mu_d\frac{\partial \phi}{\partial x}+\eta_d\frac{\partial^2 u_d}{\partial x^2}$

2.3. $\frac{\partial^2 \phi}{\partial x^2}=(n_d-n_p\delta_d p)$

(2)

Note: $\delta_{dp}=\frac{n_{e0}}{z_j n_{j0}}$

Here Positron has been taken as Maxwellian Distributed i.e.

$$n_p=n_{p0}exp(-\varphi)$$

## 3 Dispersion relation

Here we have considered an ideal, homogeneous, unmagnetized, two-component dust- positron plasma. The plasma consists of negatively charged dust particles and positively charged Positrons. We have also considered that the Positrons in the background are Maxwell-Boltzmann distributed, and we assume the dust particles behave as a fluid. The dependent variables (n, u, p, $\varphi$) are expanded as:

$$n_p^q=n_p^q(\xi,\tau)e^{i(kx-\omega t)}$$
$$u_p^q=u_p^q(\xi,\tau)e^{i(kx-\omega t)}$$
$$\phi_p^q=\phi_p^q(\xi,\tau)e^{i(kx-\omega t)}$$

(3)

Where n, u, p, $\varphi$ satisfy the reality condition $A_{-1}^{n}=A_{-1}^{n^*}$ (the asterisk is complex conjugate).
Now, substituting the above expansions[(eq:8),(eq:3)] into the Set-C governing equations we can obtain the nth-order reduced equations.
For the first order (n=1) equation, we obtain following dispersion relation for the DAW:

$$\omega^2-3ku_0\omega+[2k^2u_0^2-\frac{\mu_d Z_d k^2}{(k^2+\delta_{dp}\sigma_e)}]=0$$

(4)

*Here, $\sigma_e^2$ term is neglected.

*Group Velocity*

From, the dispersion relation equation(eq:4) we get the Group Velocity ($V_g$)[19]:

$$V_g = \frac{\partial \omega}{\partial k} = \frac{[3\omega u_0 + \frac{2k\mu_d Z_d \delta_{dp}\sigma_e}{(k^2+\delta_{dp}\sigma_e)^2} - 4ku_0^2]}{[2\omega - 3ku_0]} \quad (5)$$

Now, $\frac{\omega}{\kappa} = V_p$ i.e., phase velocity, so the equation(eq:5) becomes:

$$V_g = \frac{\partial \omega}{\partial k} = \frac{[3V_p u_0 + \frac{2k\mu_d Z_d \delta_{dp}\sigma_e}{(k^2+\delta_{dp}\sigma_e)^2} - 4ku_0^2]}{[2V_p - 3ku_0]} \quad (6)$$

## 4 KdV equation and solitary wave structure:

We employ the standard reductive technique[13][18][19][20][24] on the Set C type equation to obtain the NLSE. The independent variables are stretched as

$$\xi = \varepsilon^{\frac{1}{2}}(x - V_0 t), \tau = \varepsilon^{\frac{3}{2}} t, \eta = \varepsilon \eta_0 \quad (7)$$

Where $\varepsilon$ is a small parameter. For the derivation of KdV equations, we are ignoring the $\eta$ term for the simplification of the calculation. We applied the following perturbation expansion in the governing equation:

$$\begin{bmatrix} n_j \\ u_j \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ u_0 \\ \phi_0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_j^{(1)} \\ u_j^{(1)} \\ \phi_j^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_j^{(2)} \\ u_j^{(2)} \\ \phi_j^{(2)} \end{bmatrix} + \ldots \quad (8)$$

where $j$ is $d$, $i$ for dust, ions, and respectively and $n_j$ and $u_j$ are the normalized number density and normalized velocity of the respective species. Now, substituting the stretching (eq:7) and the perturbation expansion (eq:8) in Set-C normalized equation we get a new set of equations and solving for the lowest order of $\varepsilon$ with boundary conditions i.e.

$$n_d^{(1)}, u_d^{(1)}, \phi \to 0, as \mid \xi \mid \to \infty$$

we obtain the following solutions for the first-order density and velocity:

$$n_d^{(1)} = -\frac{\mu_d Z_d \Phi^{(1)}}{(V_0 - u_0)^2} \quad (9)$$

$$u_d^{(1)} = -\frac{\mu_d Z_d \Phi^{(1)}}{(V_0 - u_0)} \quad (10)$$

Using the next higher-order terms in $\varepsilon$ and after a few algebraic operations [24] , we obtain the KdV equation as follows:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \quad (11)$$

Here,

$$A = -(\frac{1}{2})[\frac{3(\mu_d Z_d)^2 - (\mu_d Z_d)^4 \delta_{dp}\sigma_p^2}{(V_0 - u_0)}]$$

$$B = \frac{(V_0 - u_0)^3}{2\mu_d Z_d}$$

To find the solution to the KdV equation (eq:11) we transform the independent variables $\xi$ and $\tau$ into a single variable $\eta = \xi V_0 \tau$, where $V_0$ is the normalized constant speed of the wave frame. Applying the boundary conditions that as

$$\eta \to \pm\infty, \phi, \frac{\partial \phi}{\partial \eta}, \frac{\partial^2 \phi}{\partial \eta^2} \to \pm\infty$$

we obtain the following possible solution:

$$\phi = \phi_m sech\left(\frac{\eta}{\Delta}\right) \quad (12)$$

Where the amplitude $\varphi_m$ and the width $\Delta$ of the solutions are given by:

So, the balance between the Dispersive term and non-linear term in the KdV equation creates the solitary structure and their magnitude determines the characteristics of such wave.

## 5 Non-linear Schrödinger equation (NLSE)

Using the following Fourier transformation:

$$\phi = \grave{o}^2\phi_0 + \grave{o}\phi_1 e^{i\psi} + \grave{o}\phi_1^* e^{-i\psi} + \grave{o}^2\phi_2 e^{i2\psi} + \grave{o}^2\phi_2^* e^{-i2\psi}$$

$$\psi = k\rho - \omega\theta \quad (13)$$

And the perturbation expansion of $\varphi$(eq-8) on derived KdV equation we get the group velocity expression:

$$V_g = -3Bk^2 \quad (14)$$

Using variable stretching method[15][16][17], we get complex Non-Linear Schrodinger Equations(NLSE):

$$Type - I: i\frac{\delta\phi}{\delta\theta} + P\frac{\delta^2\phi}{\delta\rho^2} = -Q(\phi^*\phi)\phi \quad (15)$$

$$Type - II: i\frac{\delta\phi}{\delta\theta} + P_1[\frac{\delta^2\phi}{\delta\rho^2} + \frac{1}{\phi}\frac{\delta\phi}{\delta\rho}] = -Q_1(\phi^*\phi)\phi \quad (16)$$

*where*,

$$P = -3Bk; Q = -\frac{A^2}{6Bk}; P_1 = -\frac{2A}{k^2}; \; Q_1 = -\frac{A^2}{3Bk}$$

*A. Rouge Wave*

From the above complex NLSE equations we got the following standard solution:

$$\phi(\rho,\theta) = i\sqrt{\frac{2P}{Q}}[\frac{4(1+4iP\theta)}{1+16P^2\theta^2+4\rho^2} - 1]e^{2iP\theta} \quad (17)$$

solving $\varphi$ in terms of $P$, $Q$, $P_1$, $Q_1$ gives rouge wave profile for both types of NLSE equation.

*B. Dynamic System*

We used the NLSE equation to get the Dynamical equation[21][22]:

$$i\frac{\delta\phi}{\delta\theta} + P\frac{\delta^2\phi}{\delta\rho^2} = -Q(\phi^*\phi)\phi$$

$$where, P = -\frac{2A}{k^2}; Q = -\frac{A^2}{6BK}$$

$A$

$$B = \frac{(V_0 - u_0)^3}{2\mu_d Z_d}$$

Using wave transformation $\eta = (l\rho - V\theta)$ from complex NLSE equation we get($l=0.65, V=0.9$):

$$i\frac{\delta\phi}{\delta\eta} + Pl^2\frac{\delta^2\phi}{\delta\eta^2} = -Q(\phi^*\phi)\phi$$

now using complex transformation $\varphi(\eta) = \psi(\eta)e^{i\beta\eta}$ in the above equation we get the equation of motion for Dynamic system

$$\psi'' = M\psi - N\psi^3 \quad (18)$$

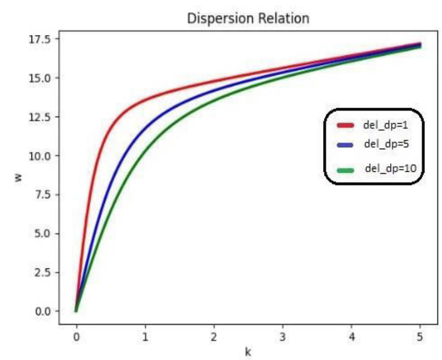
$$here, M = \beta^2 - \frac{\beta V}{Pl^2}; N = \frac{Q}{Pl^2}$$

numerically solving the above equation will give a phase curve and wave propagation curve.

## 6 Results and discussion

*A. Dispersion Relation*

The evolution of dispersion relation has been observed in this non-linear analysis. From the 2D and 3D curve(fig-1), it can be concluded that for higher values of wavenumber(k) the frequency(ω) does not depend on $\delta_{dp} = (\frac{n_{po}}{z_d n_{do}})$



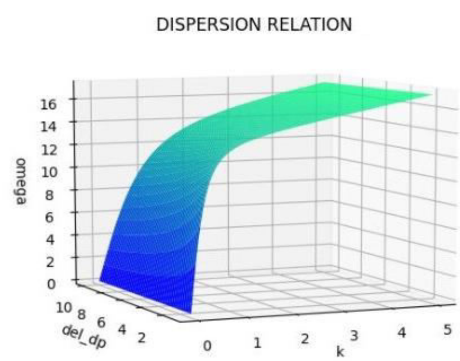




Fig. 1. 2D and 3D plot of $k$ vs $\omega$ for different $\delta_d p$

The first derivative with respect to Wavenumber(k) of the Dispersion relation equation gives the Group Velocity equation. The 2D plot(fig-2) of Group velocity($v_g$) states that when the wave number increases the value of group velocity drastically falls.

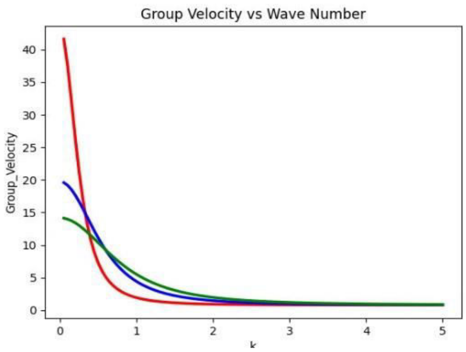



Fig. 2. 2D plot for $k$ vs $V_g$ ($r$ : $\delta_d p$= 1,$b$ : $\delta_d p$= 5,$g$ : $\delta_d p$= 10)

While the volume plot(fig-3) at $\omega$-k space shows a spark, which can be inferred as the classical regime of the system.

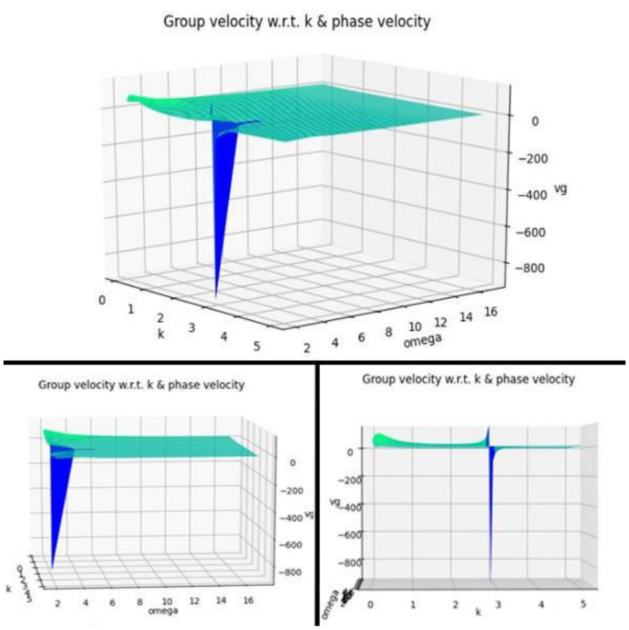



Fig. 3. 3D volume plot for $k$ vs $\omega$ $V_g$ (Here k is in the range 0-5)

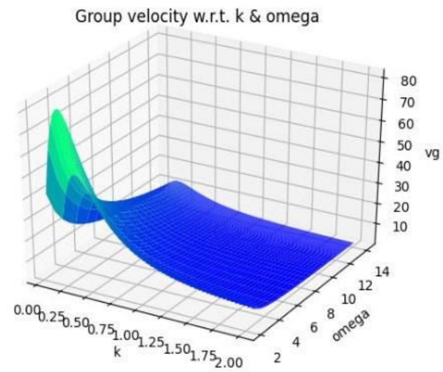



Fig. 4. 3D volume plot for $k$ vs $\omega$ $V_g$ (Here k is in the range 0-2)

Here, (fig-4) the above plot shows magnified part(k range 0-2) of the previous 3D plot(fig-3)

Now, replacing $\omega/k$ values with phase velocity term($V_p$) in group velocity equation gives the relation between phase velocity and group velocity which is inversely proportional(fig-5).

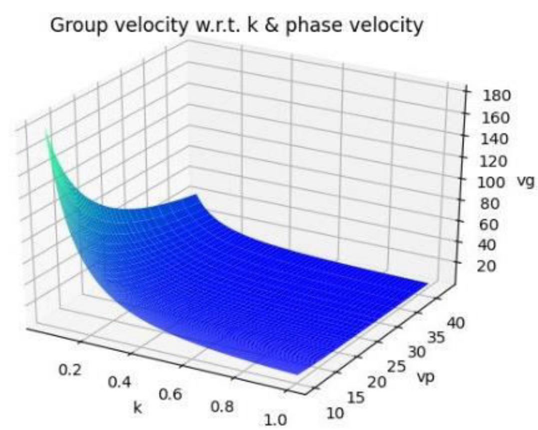



Fig. 5. 3D volume plot for $k$ vs phase velocity($V_p$) vs group velocity($V_g$)

*B. KdV Equation*

Soliton structures can be observed from the analytical solution of the Korteweg–de Vries(KdV) equation. The 2D plot(fig-6) states that $\varphi$ has only values around $\eta$ value |1|. Otherwise, it remains zero.

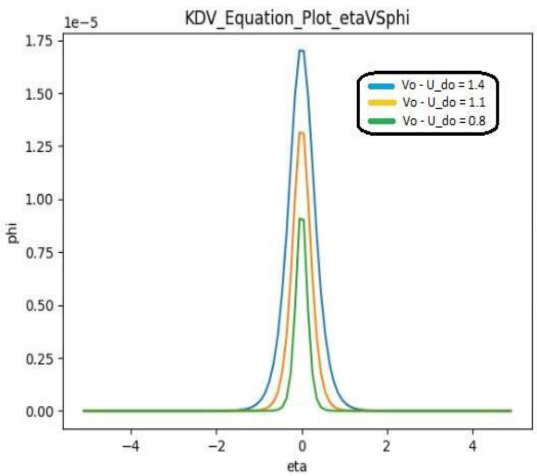
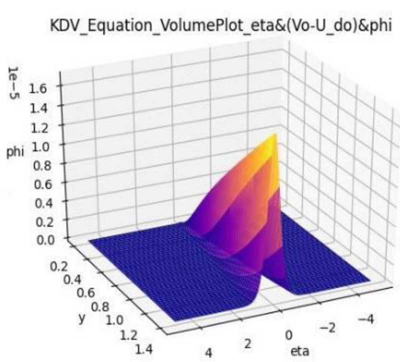



Fig. 6. 2D plot of $\varphi$ vs $\eta$ for different ($V_0$–$u_d o$) denoted as Y

3D plot(fig-7) states the $\varphi$ value increases with the difference of ($V_0$–$U_{d0}$). The 3D plot of $\varphi$ in terms of changing $Z_d$ (fig-8)shows that $\varphi$ value decreases with the increasing value of $Z_d$. Hence, in this case, we get the compressive soliton structure as shown in the plots.



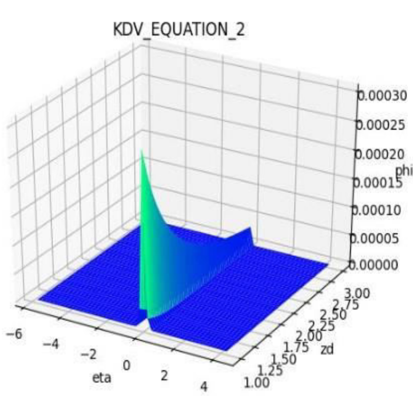




Fig. 7. 3D volume plot of $\varphi$ vs $\eta$ for different

Fig. 8. 3D volume plot of $\varphi$ vs $\eta$ for different $Z_d$

While solving the KdV equation ‘Numerically’ gives a soliton profile(fig-9) that perfectly goes with the analytical solution.

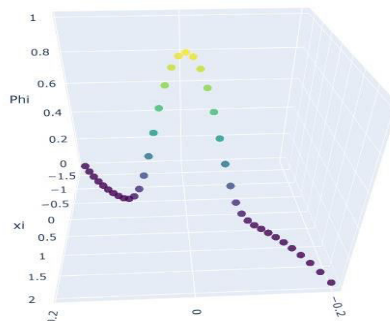



Fig. 9. Numerical solution curve for the KdV equation

*C. Rouge Waves*

Both types of complex Non-Linear Schrodinger Equation(NLSE) give $P/Q > 0$ which indicates the solution is an unstable wave equation. The complex NLSE of the first type(fig-10.a) gives a solitary profile graph. While the complex NLSE of the second type(fig-10.b) gives a short of an instant occur-vanish profile of High amplitude.

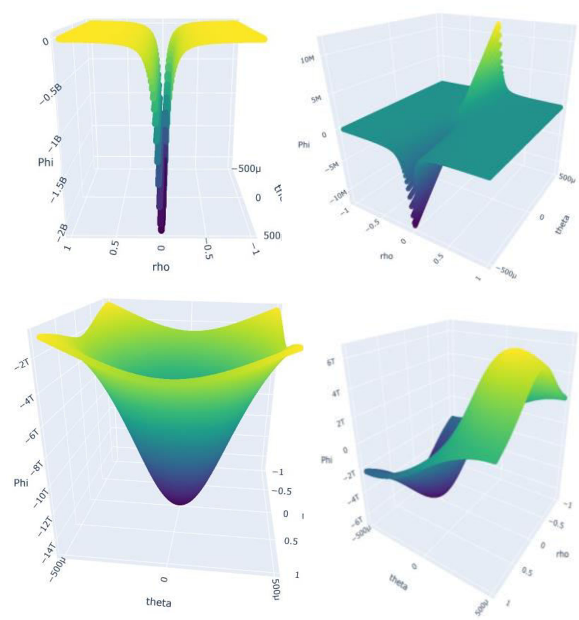


Fig. 10. Rouge Wave solution for a) type-I(real, imaginary) and b) type-II(real, imaginary)

### D. *Dynamic properties*

Dynamic system solution shows the progressive periodical Dust acoustic waves for different lengths. The plots show the wavelength of the periodic wave solution increases ' $l$(length) '. So, the wave period also increases with a wavelength which can be confirmed from the phase curve also.

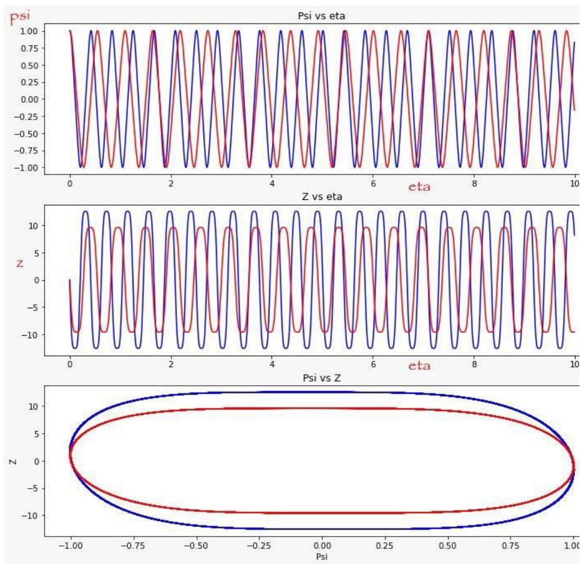



Fig. 11. Progressive waves and phase plot of $k$ vs $\omega$ for different $\delta_d p$

To sum up, we tried to show that the interactions of the individual particles with the potential function generate a compressed solitary wave structure that propagates over space and evaluates with time. We were successful in our objectives. Primarily, we analyzed the formation of and properties of the KdV solitary structure in a two-component plasma both analytically and numerically. We have made use of a standard reductive perturbation technique to derive the KdV equation, and after doing some mathematical analysis and conserving physical quantities like momentum and others, we obtained a solitary profile with a time-dependent amplitude, width, and wave speed. Secondly, we have made use of the same technique in the analysis of the nonlinear Dispersion Relation and observed the evolution of that relationship. Next, we applied partial differentiation in the Dispersion Relation equation to get the Group Velocity equation and we also observed the evolution of the equation in that context. Lastly, we calculated the NLSE equation from the KdV equation and used those complex NLSE equations to get Rouge wave solution and Dynamical systemevolution.

## 7 Conclusion

The Korteweg-de Vries (KdV) equation has been known since 1895[23]. It has two closed-form solutions associated with it. One of them is a well-known solitary wave solution. The relevance of the KdV equation to dusty plasma physics is becoming apparent, mainly to study the shock wave profile of numerous astrophysical phenomenon and laboratory plasma. The typical way in which the KdV equation arises in plasma physics has been illustrated in this work. Now the question comes, how then the KdV equation is related to such a system is. The connection comes about when a special form of perturbation analysis is carried out to examine the behavior of small but finite amplitude waves. The consequence of this is then found to be that the individual perturbations in the components of $u_j$(velocity) and $n_j$(number density) (here j is the respective species) each satisfy a KdV equation so that the behavior of its solutions is of direct relevance to the study of the weak nonlinear dispersive waves in plasmas. Specifically, the solitary wave is remarkably stable to large perturbations. Numerical experiments have shown that a solitary wave will even preserve its identity after numerous interactions with other solitary waves. One surprising consequence of this analysis has been the discovery that after interactions each solitary wave returns to its original amplitude.

Dusty plasmas are mainly consisting of four components i.e., electrons, ions, neutrals, and charged microparticles (dust particles). Because the dust grains are charged, they are electrically coupled to and fully interact with the background plasma. In the laboratory, dusty plasma allows direct visualization of the kinetic behavior of plasma phenomena. In this study, we have discussed the plasma consisting of two components, negative dust particles, and positron.

We have investigated the behavior of the rogue waves from NLSE in the defined plasma system. It is found that at certain parameters like length, dust-acoustic speed, rest mass of sub-particles, density, and temperature ratio. These factors play a significant role in deciding the amount of concentrated energy in the rogue waves.

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