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Rogue Waves in a Weakly Relativistic Warm Plasma with Non-extensively Distributed Electrons

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Modulational instability (MI) of ion acoustic waves (IAWs) in a weakly relativistic warm adiabatic unmagnetized plasma whose constitutes are ion fluid and q-non-extensively distributed electrons, using a reductive perturbation technique (multiple scales) is investigated. The domain of the stability and instability is determined. The solution of ion acoustic rogue waves (IARWs) are found. The effect of the physical parameters such as relativistic factor u_0/C and temperature ratio T_i/T_e (T_i is the ion temperature and T_e is the electron temperature) as well as the distribution parameter q on the instability of the system and rogue wave (RW) width and amplitude are studied. Finally, the validity of our results in various regions in astrophysical plasma is briefly discussed.

1. Introduction (10 font bold)

The propagation of ion acoustic waves (IAWs) in a weakly dispersive medium has been studied theoretically and experimentally by many authors [1-5]. The first experimental reported on IAWs is by Ikezi et al. [6]. Watanabe [7] have reported the experimental modulational instability (MI) of the monochromatic IAWs. Also the MI of IAWs in warm non-relativistic plasma has been studied by Jukui and He [8]. The relativistic effect must be considered when the particle velocity approaches to the velocity of light. The MI of IAWs in a weakly relativistic warm plasma for different distribution has been studied by El-Labany [9] and El-Labany et al. [10, 11] The MI has been studied using non-thermal distribution by Zhang et al. [12], non-extensive distribution by Bouzit et al. [13], and superthermal (kappa) distribution by Guo and Mei [14] and Chowdhury et al. [15]. The nonlinear evolutions in plasmas are investigated by different approximation techniques, in which one assumes small deviations for system from the equilibrium state of the linear wave. In fact multiple scales method [10, 11], the derivative expansion method (DEM) [13] and Krylov-Bogoliubov-Mitropolsky method (KBM) [16] lead to nonlinear Schrödinger-type (NST) equation. However, the system of a weakly relativistic warm unmagnetized adiabatic plasma consisting of inertial ions fluid and non-extensively distributed electrons and the rogue wave solution has not been investigated yet; this is our goal. The non-extensive statistic mechanics, depends on the deviations from Boltzmann-Gibbs-Shannon (BGS) statistics. It has been applied in the last few decades. Renyi [17] and afterwards suggested by Tsallis [18] have investigated suitable non-extensive generalization of the BGS entropy for statistical equilibrium. Tsallis extended the standard additivity of the entropies of the nonlinear systems. This nonadditive entropy suggested by Tsallis and the generalized statistics have been employed in different phenomena characterized by non-extensivity [19-27] through the entropic index q which characterizes the degree of nonextensivity of the considered system while the standard extensive BGS statistics is at $q = 1$. The nonextensive statistics are succeeded when applied to many astrophysical scenarios such as solar neutrino problem, stellar polytropes, and peculiar velocity distribution of galaxy clusters [28, 29].

On the other hand, there is a new nonlinear wave phenomenon called rogue wave (RW) or freak wave which is rare, singular, short-lived and high energetic pulse. The first observation of RW was introduced in ocean [30] and later in super fluids [31], optics [32], capillary waves [33], Bose-Einstein condensates [34] and astrophysical objects [35-39]. Therefore, a number of researchers have theoretically investigated the RW properties [35, 36, and 40]. The propagation of ion acoustic rogue wave (IARW) and its properties in an unmagnetized plasma medium with warm ions, electrons and positrons have been reported by Sabry et al. [36].

Their results show that, the IARWs become suddenly high energetic pulse around a critical wave number (k_c) and later decrease with increase of the k_c.
The skeleton of this manuscript is as follow: in section 2 we present the basic system of equations representing our model and we derive the NST equation. In section 3 we obtain the MI and IARWs solution of IAWs and section 4 is devoted to conclusion.

2. Basic equations and derivation of the NST equation

Let us consider a two-component adiabatic unmagnetized collisionless weakly relativistic plasma that contains one warm ion species and q-non-extensively distributed electrons. The dynamics of IAWs in such plasma can be described by the non-dimensional equations

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0\ , \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\gamma u+3\sigma n\frac{\partial n}{\partial x}+\frac{\partial \Phi}{\partial x}=0\ , \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial x^2}=n_e-n, \tag{3}$$

(3)

$$n_e=(1+(q-1)\Phi)^{\frac{(q+1)}{2(q-1)}} \approx 1+\alpha_1\Phi-\alpha_2\Phi^2+\alpha_3\Phi^3+\cdots, \tag{4}$$

,

Where,

$$\left.\begin{aligned}\alpha_1&=\tfrac{q+1}{2},\\ \alpha_2&=\tfrac{(q+1)(q-3)}{8}\\ \alpha_3&=\tfrac{(q+1)(q-3)(3q-5)}{48}\end{aligned}\right\}, \tag{5}$$

n, n_e are the number densities of the ions and electrons respectively. The flow velocity of the ions is u, Φ is the electrostatic potential, $\sigma \ll 1$ is the ratio of ion temperature T_i to electron temperature T_e, x is the space coordinate, t is the time variable and the parameter q stands for the strength of non-extensively, and γ is the relativistic, given by

$$\gamma=(1-\frac{u^2}{c^2})^{\frac{-1}{2}}$$

In the case of a weakly relativistic σ can be approximated by its expansion up to the second term .i.e. [41]

$$\gamma\approx 1+\frac{\mathrm{u}^2}{2\mathrm{c}^2} \tag{6}$$

All physical quantities in Eqns. (1-4) are normalized as follow u by thermal velocity $(k_BT_e/m)^{\frac{1}{2}}$, Φ by thermal potential (k_BT_e/e), n by unperturbed ion density n_0, x and t by Debye length $\lambda_D=(k_BT_e/4\pi e^2n_0)^{\frac{1}{2}}$ and the inverse of the ion plasma frequency $\omega_{pi}^{-1}=(4\pi e^2n_0/m)^{\frac{1}{2}}$ respectively, where m is the ion mass, k_B is the Boltzmann constant and e is the electron charge. To derive the nonlinear Schrödinger-type equation, we employ the general method of a multiple scales. In this method we introduce the independent variables [9]

$$\left.\begin{aligned}\tau_i&=\varepsilon^i t,\xi_0=x,\\ \xi_i&=\varepsilon^i\,(x-\lambda t)\quad (i=1,2,\dots).\end{aligned}\right\} \tag{7a}$$

Then, the time and space derivatives in Eqns. (1-4) are transformed into [11]

$$\left.\begin{aligned}\frac{\partial}{\partial t}&\to\frac{\partial}{\partial \tau_0}+\varepsilon\left(\frac{\partial}{\partial \tau_1}-\lambda\frac{\partial}{\partial \xi_1}\right)+\varepsilon^2\left(\frac{\partial}{\partial \tau_2}-\lambda\frac{\partial}{\partial \xi_2}\right)+\cdots,\\ \frac{\partial}{\partial x}&\to\frac{\partial}{\partial \xi_0}+\varepsilon\frac{\partial}{\partial \xi_1}+\varepsilon^2\frac{\partial}{\partial \xi_2}+\cdots,\end{aligned}\right\} \tag{7b}$$

Where, λ characterizes the group velocity ($\lambda=\frac{\partial\omega}{\partial k}$) and will be determined later. ε is a small dimensionless parameter representing the size of the perturbed amplitude. Now the dependent variables n, u, Φ are expanded in terms of the expansion parameter ε as (EL-Labany 1995 [9]

$$\begin{pmatrix}n\\u\\\Phi\end{pmatrix}=\begin{pmatrix}1\\u_0\\0\end{pmatrix}+\sum_{m=1}^{\infty}\sum_{l=m}^{+m}\varepsilon^m\begin{pmatrix}n_m^{(l)}(\tau_1,,\xi_1,,)\\u_m^{(l)}(\tau_1,,\xi_1,,)\\\Phi_m^{(l)}(\tau_1,,\xi_1,,)\end{pmatrix}\boldsymbol{e^{il(kx-\omega t)}} \tag{7c}$$

n, u and Φ are satisfy the reality condition $A_{-l}^{(m)}=A_l^{(m)*}$ and the asterisk denotes the complex conjugate. Introducing (7b) and (7c) into basic Eqns. (1-4) then, the first order of ε with $l=1$ gives

$$\left.\begin{aligned}u_1^{(1)}&=\tfrac{\tilde{\omega}}{k}n_1^{(1)},\\ &and\\ \Phi_1^{(1)}&=\tfrac{n_1^{(1)}}{(k^2+\alpha_1)}.\end{aligned}\right\} \tag{8}$$

The linear dispersion relation and group velocity λ can be written as

$$\widetilde{\omega}^2\gamma_1 = 3\sigma k^2 + \frac{k^2}{k^2+\alpha_1}, \quad (9)$$

$$\lambda = u_0 + \frac{k}{\gamma_1\widetilde{\omega}}\left(3\sigma + \frac{\alpha_1}{(k^2+\alpha_1)^2}\right), \quad (10)$$

Where, $\gamma_1 = 1 + \frac{3{u_0}^2}{2C^2}$ and $\widetilde{\omega} = \omega - ku_0$.

The second order harmonic terms O (ε^2) of the reduced equations, with $l = 0$ can be written as

$$\left.\begin{aligned} \frac{\partial n_1^{(0)}}{\partial \xi_1} = \frac{\partial u_1^{(0)}}{\partial \xi_1} = \frac{\partial \Phi_1^{(0)}}{\partial \xi_1} = 0, \\ \Phi_1^{(0)} = 0, \\ \Phi_2^{(0)} = \frac{\left(n_2^{(0)} - 2\alpha_2\left|\Phi_1^{(1)}\right|^2\right)}{\alpha_1} \end{aligned}\right\} \quad (11)$$

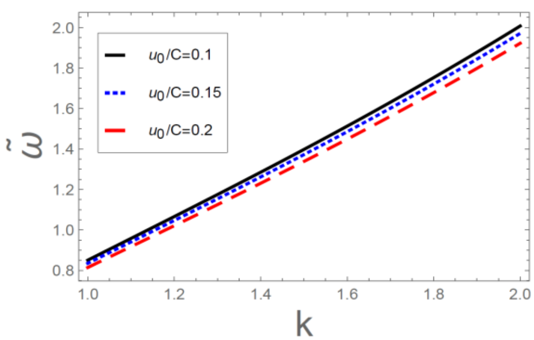
provided that

$$\gamma_1\tilde{\lambda}^2 \neq \frac{1}{\alpha_1} + 3\sigma.$$

For O(ε^2) and $l = 1$ components we have

$$\left.\begin{aligned} \frac{\partial n_1^{(1)}}{\partial \tau_1} = 0, \\ u_2^{(1)} = \frac{\widetilde{\omega}}{k} n_2^{(1)} + \frac{\mathrm{i}}{k}\left(\frac{\widetilde{\omega}}{k} - \check{\lambda}\right)\frac{\partial n_1^{(1)}}{\partial \xi_1}, \\ and \\ \Phi_2^{(1)} = \frac{n_2^{(1)}}{(k^2+\alpha_1)} + \frac{2ik}{(k^2+\alpha_1)^2}\frac{\partial n_1^{(1)}}{\partial \xi_1}; \end{aligned}\right\} \quad (12)$$

i.e. all the physical quantities are independent of τ_1.



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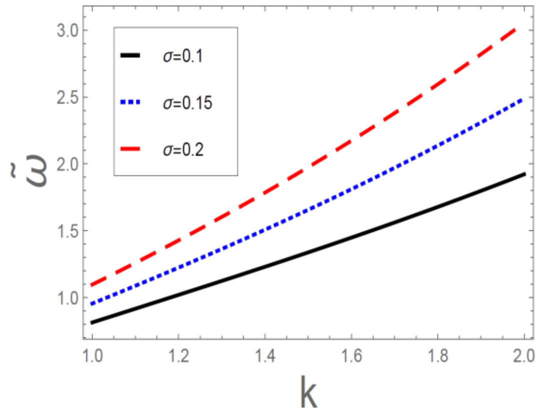


(b)

Fig. 1: The angular frequency ($\widetilde{\omega}$) depicted against wavenumber (k): (a) for different values of u_0/C and $\sigma = 0.1$, (b) for different values of σ and $\frac{u_0}{C} = 0.2$. Here $q = 0.55$.

For $l = 2$ components we have,

$$\left[n_2^{(2)}, u_2^{(2)}, \Phi_2^{(2)}\right]^T = (k^2+\alpha_1)^2[A_n, A_u, A_\Phi]^T \left|\Phi_1^{(1)}\right|^2 \quad (13)$$

where T shows for the transpose and

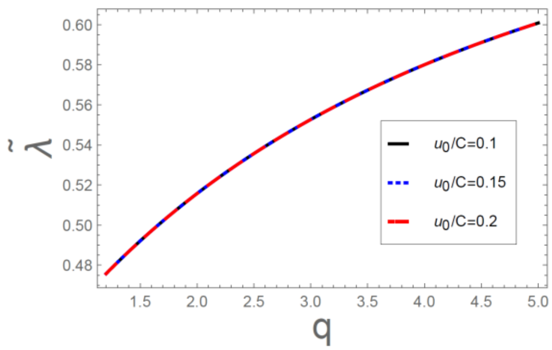
$$A_n = (k^2+\alpha_1)\left[\frac{\widetilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1 - \frac{\widetilde{\omega}}{k}\gamma_2\right) + \frac{3}{2}\sigma + A_\Phi\right]$$

$$A_u = \frac{\widetilde{\omega}}{k}(A_n - 1),$$

$$A_\Phi = \frac{(k^2+\alpha_1)}{3k^2}\left\{\frac{\widetilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1 - \frac{\widetilde{\omega}}{k}\gamma_2\right) + \frac{3}{2}\sigma - \frac{\alpha_2}{(k^2+\alpha_1)^3}\right\},$$

$$\widetilde{\lambda} = \lambda - u_0,$$

$$\gamma_2 = \frac{3u_0}{2C^2}.$$



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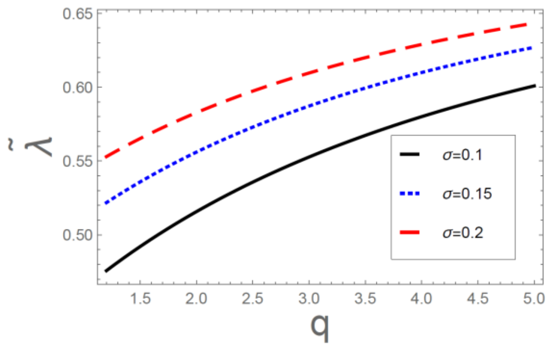


(b)

Fig. 2: The group velocity ($\tilde{\lambda}$) depicted against q: (a) for different values of u_0/C and $\sigma = 0.1$, (b) for different values of σ and $\frac{u_0}{C} = 0.2$. Here $k = 1.4$.

Moreover, the second-order quantities with zeroth harmonic are determined from $l = 0$ components of third order O (ε^3) and are given by,

$$\left[n_2^{(0)}, u_2^{(0)}, \Phi_2^{(0)}\right]^T = (k^2+\alpha_1)^2[B_n, B_u, B_\Phi]^T \left|\Phi_1^{(1)}\right|^2 \qquad (14)$$

$$B_n = \frac{1}{\tilde{\lambda}}\left(\frac{2\tilde{\omega}}{k} + B_u\right),$$

$$B_u = \frac{1}{z}\left\{\frac{\tilde{\omega}^2\tilde{\lambda}^2}{k^2}\left(\frac{\gamma_1}{\check{\lambda}} - 2\gamma_2\right) + 3\sigma\left(\check{\lambda} + \frac{2\tilde{\omega}}{k}\right) + \frac{2\tilde{\omega}}{\alpha_1 k} - \frac{2\alpha_2\check{\lambda}}{\alpha_1(k^2+\alpha_1)^2}\right\},$$

$$B_\Phi = \frac{1}{\alpha_1}\left\{\frac{B_n(k^2+\alpha_1)^2 - 2\alpha_2}{(k^2+\alpha_1)^2}\right\},$$

and

$$z = \gamma_1\tilde{\lambda}^2 - 3\sigma - \frac{1}{\alpha_1}.$$

Finally, from O (ε^3) for $l = 1$ components and using the above obtained quantities we have NST equation,

$$\mathrm{i}\frac{\partial\Phi_1^{(1)}}{\partial\tau} + P\frac{\partial^2\Phi_1^{(1)}}{\partial\xi^2} + Q\,\Phi_1^{(1)}\left|\Phi_1^{(1)}\right|^2 = 0. \qquad (15)$$

where,

$$P = \frac{-k^2}{2\tilde{\omega}\gamma_1(k^2+\alpha_1)^2} \times \left\{ -(k^2-3\alpha_1) + \frac{(k^2+\alpha_1)^3}{k^2}\left(\frac{\tilde{\omega}^2\gamma_1}{k^2} - \frac{2\tilde{\omega}\gamma_1\tilde{\lambda}}{k} + \gamma_1\tilde{\lambda}^2\right)\right\} = \frac{1}{2}\frac{\partial^2\tilde{\omega}}{\partial k^2}$$

and,

$$Q = \frac{-k^2(k^2+\alpha_1)^3}{2\tilde{\omega}\gamma_1}\left\{\left(\frac{\tilde{\omega}^2\gamma_1}{k^2} + 3\sigma\right)(A_n + B_n) + \frac{2\tilde{\omega}}{k}\left(\gamma_1 - \frac{\tilde{\omega}}{k}\gamma_2\right)(A_u + B_u) - \frac{2\alpha_2}{(k^2+\alpha_1)^2}(A_\Phi + B_\Phi) + 2\left(\frac{\tilde{\omega}}{k}\right)^3\gamma_2 - \frac{3}{2C^2}\left(\frac{\tilde{\omega}}{k}\right)^4 - \frac{3\alpha_3}{(k^2+\alpha_1)^4}\right\}$$

Equation (15) represents the evolution of the complex amplitude of the nonlinear IAWs propagating in a weakly relativistic warm with q-non-extensively electrons on the basis of the fluid model in the finite wavenumber region.

3. MI and IARWs

In this section, we study the MI of the IAWs in an unmagnetized adiabatic plasma. Consider a plane wave solution of the NST equation (15) in the form [29]

$$\Phi = (\Phi_0 + \delta\Phi)exp(iQ|\Phi_0|^2\tau) \qquad (16)$$

Where Φ_0 is a real constant amplitude perturbation and the development of a small modulation $\delta\Phi$ satisfies $\delta\Phi \ll \Phi_0$. Substituting Eq. (16) into Eq. (15) we obtain

$$\mathrm{i}\frac{\partial\delta\Phi}{\partial\tau} + P\frac{\partial^2\delta\Phi}{\partial\xi^2} + Q(\Phi_0^2\delta\Phi + \Phi_0^2\delta\Phi^*) = 0. \quad (17)$$

Where, $\Phi = \Phi_1^{(1)}$ and $\delta\Phi^*$ is the complex conjugate of $\delta\Phi$. Let assume that $\delta\Phi = \mathrm{U} + \mathrm{iV}$ and $(\mathrm{U}, \mathrm{V}) = (U_0, V_0)\exp(\mathrm{iKx} - \mathrm{i}\Omega\tau)$, then we get the nonlinear dispersion relation for the amplitude modulation

$$\Omega^2 = P^2K^2(K^2 - K_c^2) \qquad (18)$$

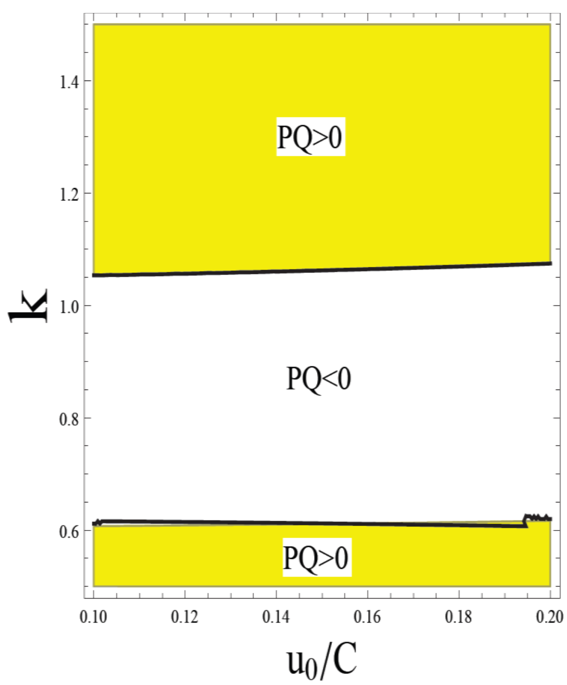
where $K_c = [2Q\Phi_0^2/P]^{1/2}$, which is the critical wavenumber.

The nonlinear dispersion relation (18) shows that the IAWs are modulation stable for $\mathrm{PQ} < 0$ in the presence small perturbation, since Ω is always real for all values of K. On the other hand, when $PQ > 0$ the IAWs are unstable and MI would set in as Ω becomes imaginary. This occurs when $K < K_c$.

Moreover, the NST equation (15) have a variety of solutions from which is the rogue wave solution developed by Darboux Transformation Scheme is localized in both ξ and τ. The first-order rogue (rational) wave solution of the NST equation (15) in the unstable region ($PQ > 0$) are given by [42]

$$\Phi(\xi\ ,\tau) = \sqrt{\frac{2P}{Q}}\left[\frac{4(1+4iP\tau)}{1+16P^2\tau^2+4\xi^2} - 1\right] e^{(2iP\tau)} \qquad (19)$$

The numerical analyses of Eq. (9) to examine the properties of the IAWs for different values of relativistic factor (u_0/C) and temperatures ratio σ (T_i/T_e) at the value of the nonextensive parameter q=0.55 are shown in Figs (1.a) and (1.b). The phase velocity decreases with increasing u_0/C (Fig. (1.a)) and is increases with increasing σ (Fig. (1.b)). Figures (2.a) and (2.b) show the variation of the group velocity with the non-extensive parameter q for different values of u_0/C and σ, given in Eq. (10). These Figures show that the group velocity is independent on u_0/C but increases with the nonextensive parameter (q), and increases with increasing σ.



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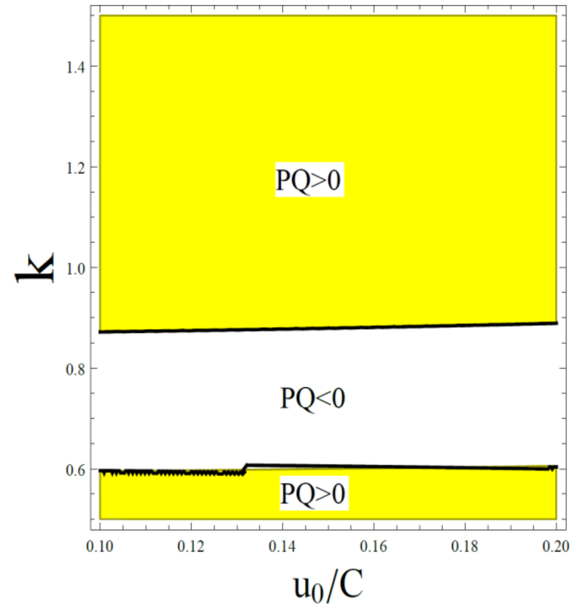
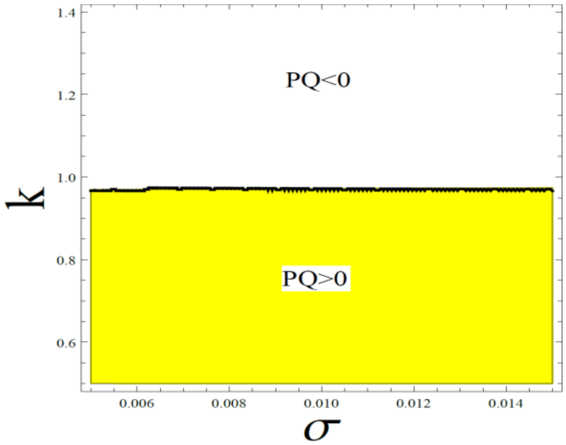


(b)

Fig. 3: Contour plot of the product $PQ = 0$, against k and u_0/C: (a) for $\sigma = 0.1$, (b) for $\sigma = 0.2$. Here $q = 0.55$, where the (white) yellow region represents the (stability) instability region.

On the other hand, we investigated the domains of the stability and instability of the ion acoustic waves on the basis of the NST equation (15). The variation of the critical wavenumbers (higher and lower wavenumbers) with u_0/C for different values of σ is shown in Fig. 3a and Fig. 3b. We noticed that the upper critical wavenumber decreases with increasing σ whereas, the lower wavenumber remains constant. This mean that the increase of σ leads the system to gain more energy, making the system more unstable. The variation of the critical wavenumbers with σ for different values of the non-extensive parameter q is shown in Figs (4.a) and (4.b). It is obvious that, the upper critical wavenumber increases with increasing the non-extensive parameter (q) and we have only one lower wavenumber which decreases with the increase of σ. Also the stability regions increase with the increase of q. This means that increasing the value of q makes the system more stable. The effect of the increase the non-extensive parameter q on the stability and instability of the system is illustrated in Figs (5.a) and Fig (5.b). It is clear that the critical wavenumbers increases with increasing q and the stable region becomes narrower with the increase of σ.



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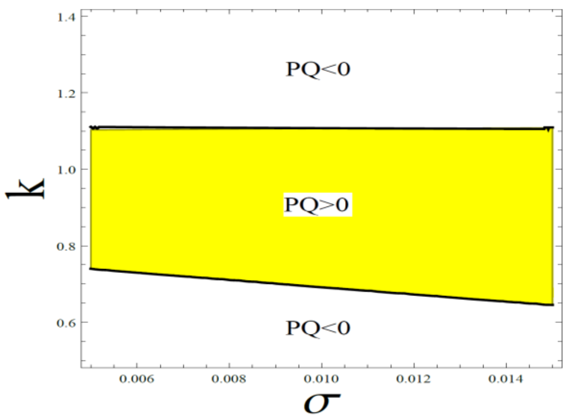
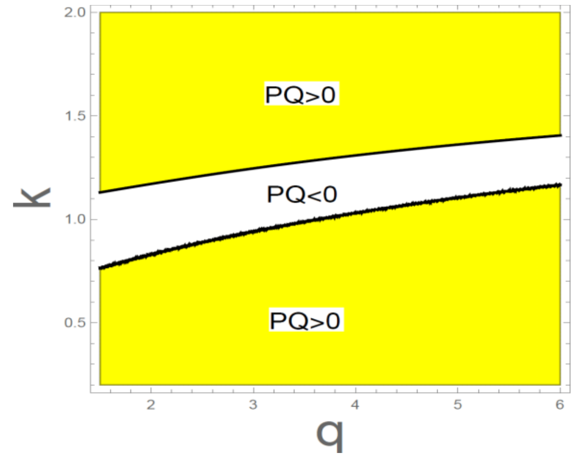


(b)

Fig. 4: Contour plot of the product$PQ = 0$, against k and σ: (a) for $q = 2.87$, (b) for $q = 4$. Here $\frac{u_0}{c} = 0.2$, where the (white) yellow region represents the (stability) instability region.

Now, we study the effect of the ratio u_0/C on the rogue wave amplitude and the width. We found that increasing u_0/C increase the width and the amplitude (as shown in Figs. (6.a) and (6.b)). Figures (7) and (8) show the effect of increasing the non-extensive parameter (q) and σ respectively on the rogue wave amplitude. Figure (7.a) shows a small variation the width and the amplitude of the rogue wave decrease with increase of q for q < 1 but decrease with large variation for $q > 1$ as shown in Fig. (7.b). Finally, for $q < 1$ we notice that the amplitude of the rogue wave decreases with increase of σ as shown in Fig. (8.a) but increases with the increase of σ for $q > 1$ and the system goes to be more unstable as shown in Fig. (8.b).



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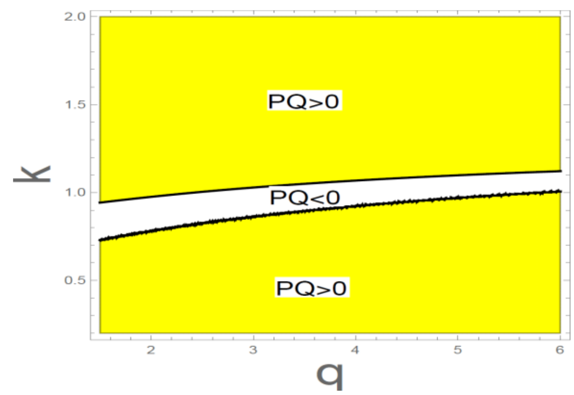
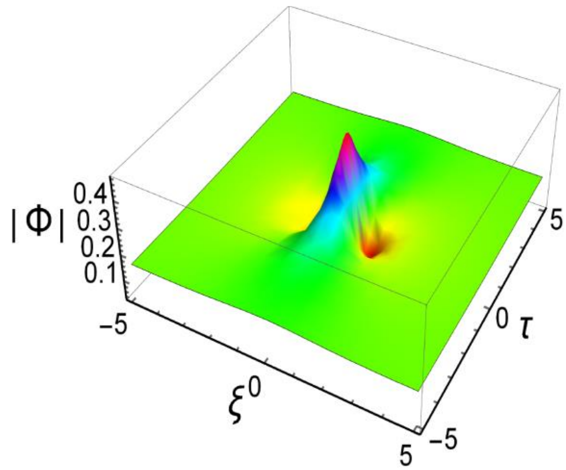


(b)

Fig. 5: Contour plot of the product$PQ = 0$, against k and q (a) at $\sigma = 0.1$ and (b) at$\sigma = 0.2$, where the (white) blue region represents the (stability) instability region. Here$\frac{u_0}{c} = 0.1$.



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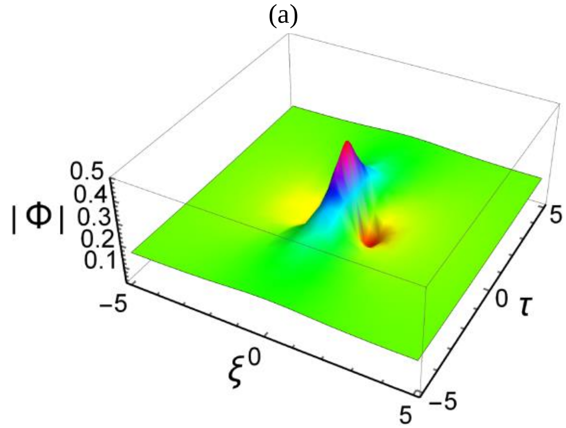
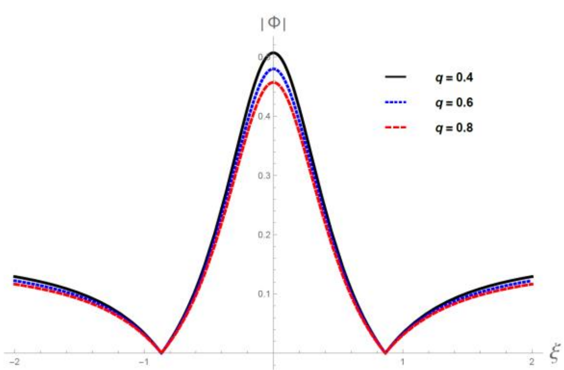


(b)

Fig. 6: The 3D plot of the IARWs amplitude for different values of u_0/C (a) Here $\frac{u_0}{C} = 0.1$ and (b)$\frac{u_0}{C} = 0.15$. Here $\sigma = 0.2$.



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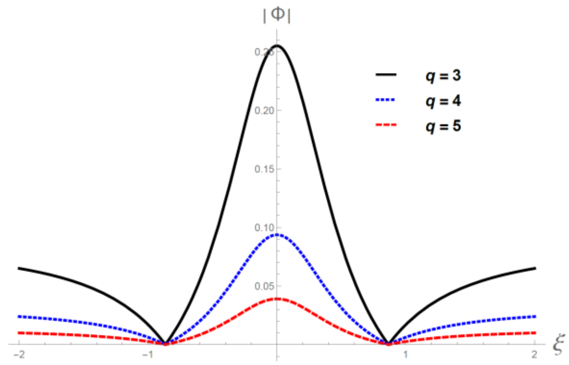
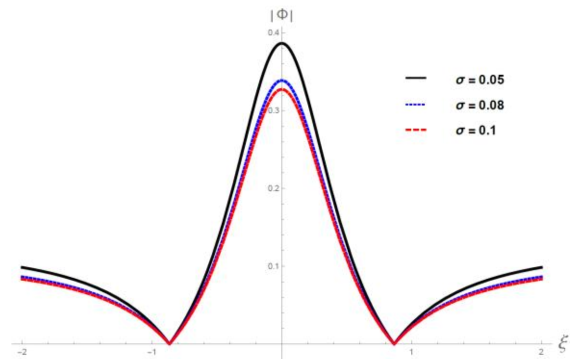


(b)

Fig. 7: The 2D plot of the IARWs amplitude for different values of q where (a) $q < 1$ and (b)$q > 1$.



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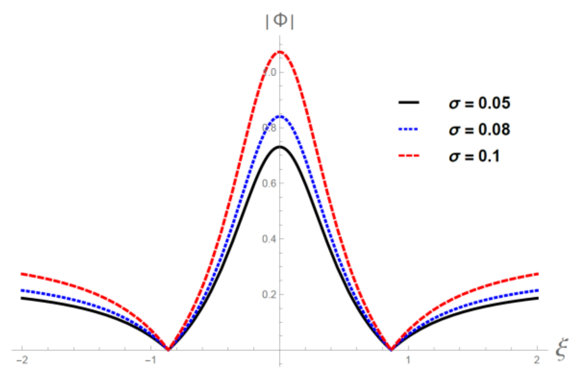

(b)

Fig. 8: The 2D plot of the IARWs amplitude for different values of σ where (a) $q < 1$ and (b) $q > 1$.

4. Conclusion

In this manuscript, we employed the nonlinear hydrodynamic equations of a weakly relativistic unmagnetized adiabatic plasma system including warm ions and non-extensively distributed electrons. Using the multiple scales method a NST equation is investigated. This method is more general than other methods (such as the derivative expansion method) dependents on the elimination of the secular terms. The coefficients of NST equation are found to be strongly dependent on both ion temperature through σ ($= T_i/T_e$), the nonextensive parameter and the ion streaming velocity through (γ_1, γ_2). Moreover we investigated the effect of σ and relativistic factor (u_0/C) on the domain of the stability $(PQ < 0)$ and the unstable region $(PQ > 0)$ by determining the critical wave number at which the sign of PQ changes from positive to negative and vice versa. The width and the amplitude of the IARWs are found to be dependent on the parameters of the system. To show the validity of our results, we considered the cold nonrelativistic limit ($\sigma = 0$ and $\frac{u_0}{c} = 0$) of the present work which is found to agree with the work done previously by Bains et al. [29]. Finally, the existence of weakly relativistic warm plasma and non-extensively distributed electrons in astrophysics environments such as solar neutrino problem, stellar polytropes and galaxy cluster as well as confinement fusion plasma are revealed by a large number of observations [28, 29].

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