LRS Bianchi Type II String Model with Variable Deceleration Parameter in *f(R, T)* Gravity

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We study the LRS Bianchi type-II string cosmological model in (*R*, *T*) theory of gravity. Here, *R* is the Ricci scalar and *T* is the trace of the energy momentum tensor. In this study, we consider a time varying deceleration parameter (DP), which generates an accelerating universe, to obtain the exact solution of the field equations. Geometric string model, massive string model and Takabyasi or p-string model are presented in this theory. Some physical and kinematical properties of the models are studied.

1. Introduction

In the last century, modern cosmology reached a new vision to establish considerable advancements to take into account the current accelerated expanding universe. The two crucial observational groups including supernovae cosmology project and the high-redshift supernovae search team have provided the main evidence for the cosmic acceleration of the Universe [1, 2]. The other cosmic observations like cosmic microwave background (CMB) fluctuations [3, 4], large-scale structure (LSS) [5, 6], cosmic microwave radiation (CMBR) [7, 8] indicate that the present universe is undergoing an accelerated expansion.

It is also believed that the Universe changed with time from early deceleration phase to late-time acceleration phase [9]. Sarkar [10] and Shariff*et al.* [11] have discussed this standard cosmological concept about the acceleration expansion of the Universe. The most promising approach confirmed by the cosmological research community for discussing the cosmic expansion of the Universe is the introduction of the most exotic and mysterious entity dubbed as dark energy, which has positive energy density and negative pressure. Recently, from cosmological and Wilkson microwave anisotropic probe (WAMP) results [12] it was concluded that the Universe embodied with 68.5% dark energy, 26.5% of dark matter and 5% of baryonic matter.

It is believed that the early universe evolved through some phase transitions, thereby yielding a vacuum energy density, which at present, is at least 118 orders of magnitudes smaller than in the Planck time [13]. Such a discrepancy between theoretical expectations and empirical observations constitutes a fundamental problem in the interface uniting astrophysics, particle physics, and cosmology. The recent observational evidence for an accelerated state of the present universe, obtained from distant SNeIa (Perlmutter et al. [14]; Riess et al. [15]), gave strong support to search for alternative cosmologies. Thus, the state of affairs has stimulated the interest in more general models containing an extra component describing dark energy and simultaneously accounting for the present accelerated stage of the Universe. The isotropic models are considered to be the most suitable to study the large scale structure of Universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the Universe with the models having anisotropic background.

In addition, it has been postulated that the standard Einstein–Hilbert action is modified by an arbitrary function *f*(*R*), where *R* is Ricci scalar curvature. The *f*(*R*) gravity becomes an adequate theory to provide the gravitational alternative for dark energy and about the early inflation plus late-time cosmic acceleration of the Universe [16-21]. In 2007, the *f*(*R*) gravity theory was restructured by merging the matter Lagrangian density *Lm* with initial arbitrary function of the Ricci scalar *R* [22]. The unification of dark energy and early time inflation with late-time acceleration from *f(R)* theory to all Lorentz non-invariant theories is discussed by Nojiriand Odintsov [23]. Through continuation of this work of coupling, in 2011, Harko*et al.* [24] proposed a new modified theory named as *f*(*R,T*) theory, where the gravitational part of the action still depend on the Ricci scalar *R* like *f*(*R*) theories and also a function of trace *T* . It is suggested that due to the matter-energy coupling, the leading model of this theory depends on source term representing the variation of energy–momentum tensor. Indefinitely many modified gravitational theories such as *f* (*G*) gravity, *f* (*R, G*) gravity, and *f*(*T*) gravity, etc. were developed to achieve the accelerated expansion of the Universe.

Myrzakulov *et al.* [25] have investigated the inflation in *f*(*R, φ*) theories of gravity where the scalar field is coupled with gravity. Sebastiani and Myrzakulov [26] have briefly reviewed various *f*(*R*) gravity models for inflation, in particular, Starobinsky-like inflation. After that *f*(*R, T*) gravity becomes the most prominent theory for investigating the fate of the late-time accelerating expansion of the universe. A phase transition also occurred from matter dominated era to an accelerated phase during the reconstruction of *f*(*R, T*) gravity theory [27]. In the context of common perfect fluid matter, an axially symmetric cosmological model was constructed in the framework of *f(R, T)* gravity [28]. In *f*(*R, T*) gravity theory, many cosmological models can be constructed by changing choices of the matter source. Recently, Moraes*et al.* [29] derived the Starobinksy model in *f*(*R, T*) gravity.

The presence of strings results in anisotropy in the space-time, though strings are not observable in the present epoch. Unlike domain walls and monopoles, strings cause no harm (to the cosmological models) but rather can lead to very interesting astrophysical consequences. The string gas cosmology will lead to a dynamical evolution of the early universe, very different from what is obtained in standard and inflationary cosmology and can already be seen by combining the basic ingredients from string theory discussed so far.

As the radius of a cloud of strings decreases from an initially large value which maintains thermal equilibrium, the temperature first rises as in standard cosmology since the occupied string states (the momentum modes) get heavier. However, as the temperature approaches the Hagedorn temperature, the energy begins to flow into the oscillatory modes and the increase in temperature levels off. As the radius decreases below the string scale, the temperature begins to decrease as the energy begins to flow into the winding modes whose energy decreases as the radius decreases

Observations have been conducted to obtain the homogeneity and isotropic properties of the Universe. It is believed that at the end of the inflationary era, the geometry of the Universe was homogeneous and isotropic [30], where the FLRW models played an important role in representing both spatially homogeneous and isotropic universe. But the theoretical argument and the anomalies found in CMB provide the evidence for the existence of an anisotropic phase, which is later called isotropic one. After the announcement of Planck probe results [31], it is believed that the early universe may not have been exactly uniform.

Thus, the existence of inhomogeneous and anisotropic properties of the Universe has gained popularity when it comes to constructing cosmological models under the supervision of anisotropic background. Therefore, Bianchi type models are very relevant for describing the early universe with the anisotropic background. Due to some analytical difficulties in studying the inhomogeneous models, many researchers considered the Bianchi type models for investigating the cosmic evolution of the early universe as they are homogeneous and anisotropic. There exist nine types *(I–IX)* of Bianchi space-times in literature. Here, we consider Bianchi type-II space-time, as it is the simplest spatially homogeneous and anisotropic. It is also known as the immediate generalization of the FLRW flat metric with different scale factors in each spatial direction. In some special cases, the Bianchi type-I models include Kasner metric which helps to govern the dynamics near the singularity. The Bianchi type-II cosmological models are more compatible with the simplest mathematical form which attracts various researcher to study different aspects. Bianchi type II space time successfully explains the initial stage of evolution of the Universe.

Asseo and Sol [32] have given the importance of Bianchi type II space-time for the study of the Universe. The string theory is useful to describe an event at the early stage of evolution of the Universe in a lucid way. Cosmic strings play a significant role in the structure formation and evolution of the Universe. The presence of string in the early universe has been explained by Kibble [33], Vilenkin [34], and Zel'dovich [35] using grand unified theories. These strings have stress energy and are classified as massive and geometric strings. The pioneer work in the formation of energy momentum tensor for classical massive strings is due to Letelier [36] who explained that the massive strings are formed by geometric strings (Stachel [37]) with particle attached along its extension. Many authors' namely, Banerjee et al. [38], Tikekar and Patel [39, 40], Wang [41], and Venkateswarlu et al. [42–46], have investigated string cosmological models in different contexts.

In this paper, we study the LRS Bianchi Type-II string models in *f(R, T)* theory of gravity with the help of variable deceleration parameter.

2. Gravitational field equations of *f (R, T)* modified gravity theory

In this theory, the modified gravity action if given by

$$S=\int\sqrt{g}\left(\frac{1}{16\pi G}f(R,T)+L_m\right)d^4x \qquad (1)$$

Where, $f(R,T)$ is an arbitrary function of the Ricci scalar *R* and the trace *T* of the tress energy tensor T_{ij} of the matter, L_m is the matter Lagrangian density. If $f(R,T)$ is replaced by $f(R)$, we get the action for $f(R)$ gravity and replacement of $f(R,T)$ by *R* leads to the action of general relativity. The stress energy tensor of the matter is defined as

$$T_{ij} = \frac{-2\delta\sqrt{-g}\,L_m}{\sqrt{-g}\,\delta g^{ij}} \quad (2)$$

and its trace by $T_{ij} = g^{ij}T_{ij}$.Assuming that the Lagrangian density L_m of matter depends only on the metric tensor g_{ij} and not on its derivative leads to

$$T_{ij} = g_{ij}L_m - \frac{2\delta L_m}{\delta g^{ij}} \quad (3)$$

Varying the action S with respect to metric tensor g_{ij}, the field equation of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij} - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \quad (4)$$

Where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\delta^2 L_m}{\delta g^{ij}\delta g^{lk}} \quad (5)$$

Here $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$ $\Box \equiv \nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m.The contraction of (4) yields

$$f_R(R,T)R_{ij} + 3\Box f_R(R,T)g_{ij} - 2f(R,T) = (8\pi T_{ij} - f_T(R,T)T - f_T(R,T)\Theta_{IJ} \quad (6)$$

Where, $\Theta_{ij} = g^{ij}\Theta_{ij}$.From (4) and (6) we obtain

$$f_R(R,T)\left(R_{ij} - \frac{1}{3}R\,g_{ij}\right) + \frac{1}{6}f(R,T)g_{ij} = (8\pi T_{ij} - f_T(R,T)\left(T_{ij} - \frac{1}{3}Tg_{ij}\right) - f_T(R,T)\left(\Theta_{ij} - \frac{1}{3}\Theta g_{ij}\right) + \nabla_i\nabla_j f_R(R,T) \quad (7)$$

If the matter is regarded as a perfect fluid the stress energy tensor of the matter Lagrangian is given by

$$T_{ij} = (P+\rho)u_iu_j - Pg_{ij} \quad (8)$$

Where, $u^i = (1,0,0,0)$ is the four velocity in co-moving coordinates which satisfies the conditions $u^iu_i = 1$ and $u^i\nabla_j u_i = 0$. Here P and ρ are the pressure and energy density of the fluid, respectively. With the use of Eqn. (5) we obtain

$$\Theta_{ij} = -2T_{ij} - Pg_{ij} \quad (9)$$

It is important to note that the field equations in $f(R,T)$, gravity also depend on the physical nature of matter field through the tensor Θ_{ij}. Hence in this theory depending on the nature of the matter source we can obtain several theoretical models for each choice of $f(R,T)$, Harko et al. [24] considered three explicit forms of f as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (10)$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R, T)$ gravity, depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R, T)$.

Assuming $f(R, T) = R + 2f(T)$ as a first choice where, $f(T)$ is an arbitrary function of the trace of stress energy tensor of matter, we get the gravitational field equations of $f(R, T)$ gravity from Eqn. (9) as

$$R_{ij} - \frac{1}{2}R\,g_{ij} = 8\pi T_{ij} - 2f'(T)\left(T_{ij} - \Theta_{ij}\right) + f(T)g_{ij} \quad (11)$$

Where, the prime denotes differentiation with respect to the argument. The field equations of $f(R, T)$ gravity, in view of Eqn. (9), become

$$R_{ij} - \frac{1}{2}R\,g_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij}[2p\,f'(T) + f(T)]g_{ij} \quad (12)$$

Spatially homogeneous and anisotropic LRS Bianchi type-II space-time is given by the following form

$$ds^2 = -dt^2 + B^2\left(dx + zdy\right)^2 + A^2(dy^2 + dz^2) \quad (13)$$

Where, A and B are functions of cosmic time t only.

We consider the energy momentum tensor for a cosmic string source in the field equations (12) as

$$T_{ij} = \rho u_iu_j - \lambda x_i x_j \quad (14)$$

Where ρ is the energy density of the string cloud, u_i is the four velocity, x_i is the string direction and λ is the string tension density. Also, we have

$$u^i u^j = -x^i x_j = -1, u^i x = 0 \quad (15)$$

and

$$\rho = \rho_p + \lambda \quad (16)$$

Where, ρ_p is the rest energy density particle attached to the string. Letelier [36] has pointed out that λ may be positive or negative. Also we choose (Harko et al. [24])

$$f(T) = \mu T \quad (17)$$

Where, μ is a constant.

Now using commoving coordinate system, the field equations (11) (replacing p by λ in the view of (14), for the metric with the help of (14) to (17), can be explicitly written as

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{1}{4}\frac{B^2}{A^4} = -(\lambda+\rho)\mu \quad (18)$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{3}{4}\frac{B^2}{A^4} = -(8\pi+3\mu)\lambda-\mu\rho \quad (19)$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}^2}{A^2}-\frac{1}{4}\frac{B^2}{A^4} = -(8\pi+3\mu)\rho-\mu\lambda \quad (20)$$

Which reduced to the following two independent equations

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{B^2}{A^4} = (8\pi+2\mu)\lambda \quad (21)$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}-\frac{\dot{A}\dot{B}}{AB}+\frac{1}{2}\frac{B^2}{A^4} = (8\pi+2\mu)\rho \quad (22)$$

Where, an overhead dot indicates differentiation with respect to t.

3. Solutions of the field equations and the corresponding models

We can observe that the field equations (21) and (22) are two independent equations with four unknowns A, B, ρ and λ. Hence to obtain the deterministic solution the following two condition are assumed:

(i) The equations of state

$$\rho = \lambda \quad (23)$$

$$\rho = (1+\omega)\lambda \quad (24)$$

$$\rho + \lambda = 0 . \quad (25)$$

(ii) a special form of deceleration parameter(DP) [47, 48]

$$q = -1 + \frac{\beta}{1+a^{\beta}} \quad (26)$$

The behavior of the Universe is determined by the sign of q. If $q>0$, we have decelerating universe and if $q<0$, we have accelerating universe. If $q = 0$, we have uniform expansion of the Universe. Here $\beta(>0)$ is a constant and a is the scale factor of the metric.

We discuss the solution of the field equations by considering the conditions given by Eqns. (23)-(25).The Hubble parameter $H$is defined as $H=\frac{\dot{a}}{a}$ and the volume is defined as: $V = a^3 = A^2 B$. Using Eqn. (26) in this relation, the values of the metric potentials A ,B are obtained as

$$A = \left(e^{\beta t}-1\right)^{\frac{1}{2\beta}} \quad (27)$$

$$B = \left(e^{\beta t}-1\right)^{\frac{2}{\beta}} \quad (28)$$

Consequently, metric (13) takes the form

$$ds^2 = -dt^2 + \left(e^{\beta t}-1\right)^{\frac{4}{\beta}}(dx+zdy)^2 + \left(e^{\beta t}-1\right)^{\frac{1}{\beta}}(dy^2+dz^2) \quad (29)$$

Which, represents the LRS Bianchi Type-II string model in f(R, T) gravity theory.

From (26) we obtained

$$H = \frac{\dot{a}}{a} = A_1(1+a^{-\beta}) \quad (30)$$

Where, A_1 is a constant of integration and set $A_1=1$.Integrationg (30) and using the initial conditions $a=0$ at $t=0$we have found

$$a = \left(e^{\beta t}-1\right) \quad (31)$$

Setting $a(t) = \frac{1}{(1+z)}$. where z is the redshift.

Now from (31), we obtain the following relation

$$t = \frac{\log\left((\frac{1}{z+1})^{\beta} + 1\right)}{\beta} \quad (32)$$

and the corresponding q(z) is obtained as

$$q = \frac{\beta}{\left(\frac{1}{z+1}\right)^{\beta}+1} - 1 \quad (33)$$

The deceleration parameter describes the evolution of the Universe. The cosmological models of the evolving universe transits from early decelerating phase $(q>0)$ to current accelerating phase $(q<0)$.Whereas, the models can be classified on the basis of the time dependence of DP. Recent observations like SNeIa [49] and CMB anisotropy [50] confirmed that the present universe is undergoing an accelerated phase of expansion and the value lies in between $-1 \le q \le 0$. The deceleration parameter will be negative (i.e., q < 0) for β< 1.325and becomes positive (i.e., q > 0) for β > 2.0. Fig-1 depicts the behavior of deceleration parameter with respect to redshift, in which the value of q lies in specified range of accelerating phase. The values of transition redshift z_{tr} for our model are agreeing with the observational data [51-53]. The transition from deceleration to acceleration phase in *f(R, T)* gravity with polynomial function of T is discussed by Moraes et al. [54]. The model is completely under accelerated phase which is conformity with observational data.

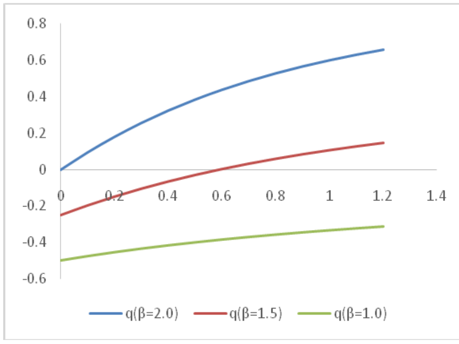

Fig.1. Plot of q verses redshift with different β

3.1. Geometric or Nambu string (ρ = λ)

In this case we assume $\rho-\lambda=0$.This corresponds to the state equation for a cloud of mass less geometric (Nambu) strings, i.e. $\rho_p=0$.Therefore, in this case, from equations (21) and (22),we obtain the energy density and tension density as

$$\lambda=\rho=\frac{3}{4}\frac{e^{2\beta t}}{(e^{\beta t}-1)^2}-\frac{1}{4}(e^{\beta t}-1)^{\frac{2}{\beta}} \qquad (34)$$

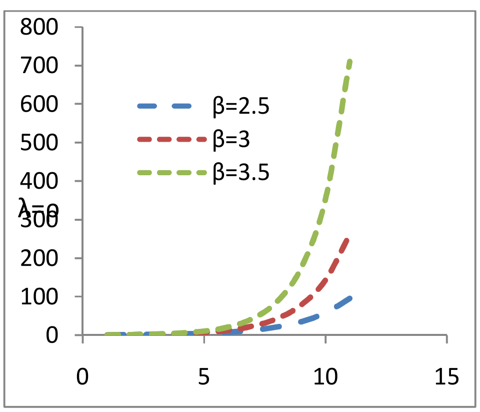

Fig 2. Plot of rest energy density and string density (ρ=λ) versus time t

From equation (34), it is found that tension density λ equals to rest energy density (ρ). Fig. 2 shows the behaviur of geometric strings in *f(R,T)* gravity for different values of β . It is noted that the rest energy density (tension density) is an increasing function of time and it approaches a small positive value at present epoch. This behavior is clearly depicted in Fig. 2.

3.2. Massive string (ρ+λ=0)

The relation between the constant β and time is given by

$$\beta+\frac{3}{2}e^{\beta t}+\frac{1}{2}(e^{\beta t}-1)^{\frac{2}{\beta}+2}e^{-\beta t}=0 \qquad (35)$$

The energy density and tension in string is

$$\rho=-\lambda=\frac{3}{16}\frac{e^{2\beta t}}{(e^{\beta t}-1)^2}-\frac{1}{16}(e^{\beta t}-1)^{\frac{2}{\beta}} \qquad (36)$$

$$\rho_p=\rho-\lambda=\frac{3}{8}\frac{e^{2\beta t}}{(e^{\beta t}-1)^2}-\frac{1}{8}(e^{\beta t}-1)^{\frac{2}{\beta}} \qquad (37)$$

The particle energy density is obtained as

$$\frac{\rho_p}{\lambda}=\frac{\frac{3}{8}\frac{e^{2\beta t}}{(e^{\beta t}-1)^2}-\frac{1}{8}(e^{\beta t}-1)^{\frac{2}{\beta}}}{\left(\frac{3}{4}\frac{e^{2\beta t}}{(e^{\beta t}-1)^2}-\frac{1}{4}(e^{\beta t}-1)^{\frac{2}{\beta}}\right)} \qquad (38)$$

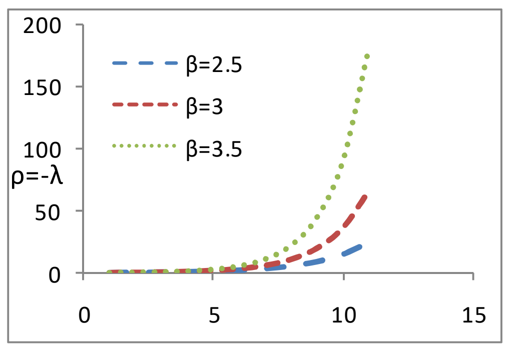

Fig 3. Plot of rest energy density and string density (ρ=-λ) versus time t

From equation (36), it is found that rest energy density (ρ) equals to the negative sign of tension density λ . Fig. 3 is the plot of rest energy density and string density (ρ=-λ) versus time t for $\beta=2.5,3,3.5$ respectively. The behavior of massive string model is quite similar to geometric string model, i.e., the rest energy density and string density (ρ=-λ) is an increasing function of time and it approaches a small positive value at present epoch.

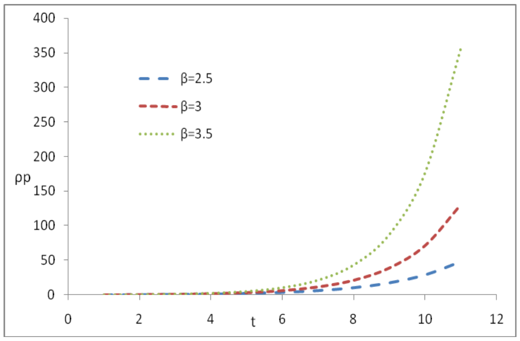

Fig 4: Plot of particle density and string density (ρp) verses time t

The expression for particle density ρ_p is given in equation (37). Fig 4 is the plot of particle density verses time t for $\beta = 2.5, 3, 3.5$ respectively. It is found that the particle density is an increasing function of time and it approaches a small positive value at present epoch. From (37), it is observed that the particle density ρ_p is an increasing function of time and $\rho_p > 0$ for all times. The nature of ρ_p clearly shown in Fig. (4)

The dominant energy conditions implies that $\rho > 0$ and $\rho^2 \geq \lambda^2$. These energy conditions do not restrict the sign of λ, accordingly the expressions given by equation (37) satisfies all these conditions.

According to Refs.(Kibble[33];Krori et al.,[55]), when $\frac{\rho_p}{|\lambda|} > 1$, in the process of evolution ,the universe is dominated by massive strings, and when $\frac{\rho_p}{|\lambda|} < 1$, the universe is dominated by strings .

M From Equation (38), we observe $\frac{\rho_p}{|\lambda|} < 1$ which shows that the universe is dominated by strings in the beginning of evolution of the universe.

3.3. p-strings or Takabayasi strings

Each massive string is formed by a geometric string with particles attached along its extension. Hence, the string that form the cloud are a generalization of Takabayasi's relativistic model of strings (called p-string). This is simplest model wherein we have particles and strings together.

The p-strings or Takabayasi strings are represented by $\rho = (1+\omega)\lambda$, $\omega > 0$.

The string energy density ρ, tension density λ are given by

$$\rho = \frac{12e^{2\beta t} + 6\beta e^{\beta t}}{4(e^{\beta t}-1)^2} + \frac{1}{2}(e^{\beta t}-1)^{\frac{2}{\beta}} \tag{39}$$

$$\lambda = \frac{1}{(1+\omega)}\left(\frac{18e^{2\beta t} + 10\beta e^{\beta t}}{4(e^{\beta t}-1)^2} + (e^{\beta t}-1)^{\frac{2}{\beta}}\right) \tag{40}$$

Where,

$$\omega = \frac{-\frac{1}{2}(e^{\beta t}-1)^{\frac{2}{\beta}} - \frac{6e^{2\beta t} + 4\beta e^{\beta t}}{4(e^{\beta t}-1)^2}}{\left(\frac{18e^{2\beta t} + 10\beta e^{\beta t}}{4(e^{\beta t}-1)^2} + (e^{\beta t}-1)^{\frac{2}{\beta}}\right)} \tag{41}$$

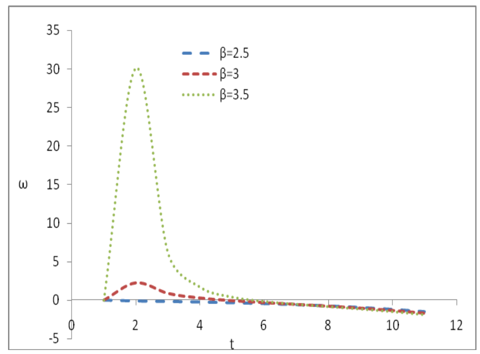

Fig 5: Plot of *ω= [(ρ/λ)-1]* verses time *t*

The expression of *ω= [(ρ/λ)-1]* is given in equation (41). Fig. 5 is Plot of *ω* verses time t for $\beta = 3.5, 4, 4.5$ respectively. It is found that *ω* is a decreasing function of time and it approaches a negative value at large value of *t.* As *ω>0*, we find that *ρ, λ* remain positive throughout the evolution of the Universe.

The rate of expansion of the Universe with respect to time is defined by Hubble's parameter as well as DP. Detailed kinematical descriptions of the cosmological expansions can be obtained by taking in to account some extended set of parameters having higher order time derivatives of the scale factor.

The spatial volume turns out to be

$$V = a^3 = A^2B = \left(e^{\beta t}-1\right)^{\frac{2}{\beta}} \tag{42}$$

The above equation indicates that in both the models the spatial volume is zero at initial time $t = 0$. It shows that the evolution of our universe starts with big bang scenario. It is further noted that from Eqn. (31) the average scale factor becomes zero at the initial epoch. Hence, both models have a point-type singularity [56]. The spatial volume increases with time.

The Hubble's parameter *H*, expansion scalar *θ* and shear scalar σ^2become

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{e^{\beta t}}{\left(e^{\beta t} - 1\right)} \quad (43)$$

The expansion scalar θ, shear scalar σ^2 are given by

$$\theta = 3H = 3\frac{e^{\beta t}}{\left(e^{\beta t} - 1\right)} \quad (44)$$

$$\sigma^2 = \frac{1}{2}\left(2H_1^{\,2} + H_2^{\,2} - \frac{\theta^2}{2}\right) = \frac{3}{4}\frac{e^{2\beta t}}{\left(e^{\beta t} - 1\right)^2} \quad (45)$$

From the above equations, we can observe that the Hubble factor, scalar expansion and shear scalar diverge at $t = 0$ and they become finite as $t \to \infty$. It is noted here that the isotropic condition $\frac{\sigma^2}{\theta^2}$ becomes constant (from early to late-time), which shows that the model does not approach isotropy throughout the evolution of the universe.

The mean anisotropic parameter is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{1}{2} \quad (46)$$

The anisotropic parameter becomes constant for our model. From the above mentioned equation, it can be observed that our models are expanding and accelerating the Universe, which starts at a big bang singularity.

4. Jerk parameter

The jerk parameter is considered as one of the important quantities for describing the dynamics of the universe. The models close to ΛCDM (gamma cold dark matter) can be described by the cosmic jerk parameter j [57, 58]. For flat ΛCDM model the value of jerk is $j = 1$ [59]. Jerk parameter is a dimensionless third derivative of scale factor a with respect to cosmic time t and is given as

$$j = \frac{a^2}{\dot{a}^3}\frac{d^3 a}{dt^3} \quad (47)$$

The above expression can be written in terms of deceleration parameter as

$$j = q + 2q^2 - \frac{\dot{q}}{H} \quad (48)$$

Thus, the jerk parameter for our models is

$$j = 1 - 3\beta e^{-\beta t} + 2\beta^2 e^{-2\beta t} - \beta^2 e^{-2\beta t}\left(e^{\beta t} - 1\right) \quad (49)$$

From Fig. 6, it is clear that our value does not overlap with the value $j = 2.16^{+0.81}_{-0.75}$ obtained from a combination of three kinematical data sets: the gold sample data of type Ia Supernovae [60], the SNIa data are obtained from the SNLS project [61], and the X-ray galaxy cluster distance measurements [62]. We have plotted the jerk parameter for different values of β in Fig.6. One can observe that the jerk parameter remains positive throughout the universe and is equal to the ΛCDM model at $t \geq 5.5$ for the considered values of β.

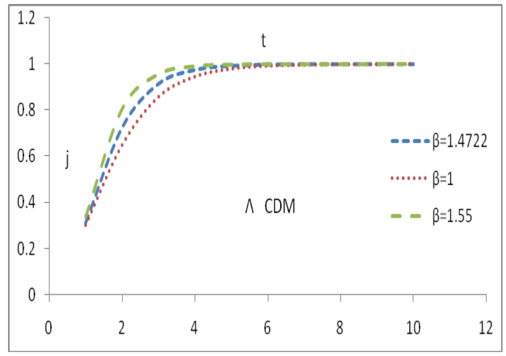

Fig.6 Behaviour of Jerk parameter versus t with different values of β

5. r − s parameter

The state-finder pair {r, s} is defined as [59]

$$r = \frac{\dddot{a}}{a H^3}$$

$$s = \frac{r - 1}{3(q - \frac{1}{2})} \quad (50)$$

The state-finder pair is a geometrical diagnostic parameter, which is constructed from a space-time metric directly. The state finder pair *(r, s)* is more universal compared to physical variables, which depend on the properties of physical fields describing DE, astrophysical variables are model dependent. For the flat ΛCDM model the state-finder pair is *{r, s}* = *{*1, 0*}* [63]. The values of the state-finder parameter for our model are

$$r = 1 + 3\beta e^{-\beta t} + \beta^2 e^{-2\beta t} \quad (51)$$

$$s = \frac{3\beta e^{-\beta t} + \beta^2 e^{-2\beta t}}{3\left(\beta e^{-\beta t} - \frac{3}{2}\right)} \quad (52)$$

Clearly the state-finder pair {r, s}→{1, 0} as t→∞.

6. Conclusions

String cosmological models play a vital role in the discussion of early stages of evolution of the Universe. Hence, in this paper, we have investigated LRS Bianchi type-II cosmological models in the presence of massive string source in *f (R, T)* gravity proposed by Harko et al. [24]. Our work in this paper will be helpful to study the structure formation of the universe in *f (R, T)* gravity, which is a viable alternative to general relativity. We have used the three equations of state for strings which correspond to (i) geometric strings, (ii) massive strings and (iii) Takabayasi strings. In geometrical string model we have observed that proper energy density remains positive throughout the evolution. In massive string that proper energy density and particle density remains positive throughout the evolution. It is interesting to note that the LRS Bianchi type-II string in this theory, do survive. However, we have presented the Takabyasi strings and massive strings in this particular space-time in this modified theory.The metric potentials *A, B* do not vanish for this model. The volume scale factor '*a'* increases exponentially with time which indicates that the Universe starts expansion with zero volume for$t=0$.Jerk parameter and state-finder trajectory in the r −s plane are close to ΛCDM model. It is also noted that at $t=0$, the model represents a flat model in *f(R,T)* gravity in presence of massive strings.

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