The African Review of Physics (2018) **13**: 0004

Lyman-alpha spectroscopy in non-commutative space-time

M. Moumni[1], A. BenSlama[2] and S. Zaim[3]
[1]*Department of Matter Sciences, University of Biskra, Biskra, Algeria*
Laboratory of Photonic Physics and Nano-Materials (LPPNMM)
[2]*Department of Theoretical Physics, University of Constantine1, Constantine, Algeria*
Laboratory of Mathematical and Subatomic Physics (LPMPS)
[3]*Department of Physics, University of Batna1, Batna, Algeria*

We study space-time non-commutativity applied to the hydrogen atom and the corrections induced to transitions frequencies. By writing the Dirac equation for noncommutative Coulomb potential, we compute noncommutative corrections of the energy levels using perturbation methods and by comparing to the Lamb shift accuracy we get a bound on the parameter of non-commutativity. We use this bound to study the effects on the Lyman-α ray and by induction look to possible influence of non-commutativity on some astrophysical and cosmological phenomena.

1. Introduction

Recently, there has been a certain amount of activity around the theme of cosmological and astrophysical applications of non-commutative geometry [1-10]. This work is in the same context, and we are interested in the effects of space-time non-commutativity on the hydrogen spectroscopy and especially on the Lamb shift, the Lyman-α line.

The Lyman-α emission line is produced by the recombination of electrons with ionized hydrogen atoms. The Lyman-α line and related diagnostics are now routinely used in numerous astrophysical studies. For example, it is used to probe massive star formation across the Universe, to study properties of the interstellar and intergalactic medium across cosmic times, and to search for the most distant galaxies [11-13].

This line is also present through one of the largest known individual objects in the Universe: Lyman-Alpha Blob (LAB). A LAB is a huge concentration of a gas emitting the Lyman-α emission line [14-15]. Some of these gaseous structures are more than 400,000 light years across. So far they have only been found in the high-redshift universe because of the ultraviolet nature of the Lyman-α emission line. Lyman-α Blobs may hold valuable clues for scientists to determine how galaxies are formed.

Another characteristic that arises from this line in astronomical spectroscopy is the Lyman-α forest; it is the sum of absorption lines arising from the Lyman-α transition of neutral hydrogen in the spectra of distant galaxies and quasars [16-18]. These absorption lines result from inter-galactic gas through which the galaxy or quasar's light has travelled. The forest is created by the fact that photons that come to us from distant light sources show Hubble redshift that depends on the distance between us and the source of light. Since neutral hydrogen clouds at different positions between Earth and the distant light source see the photons at different wavelengths (due to the redshift), each individual cloud leaves its fingerprint as an absorption line at a different position in the spectrum as observed on Earth.

The Lyman-α forest is an important probe of the intergalactic medium and can be used to determine the frequency and density of clouds containing neutral hydrogen, as well as their temperature. Searching for lines from other elements and the abundance of heavier elements in the clouds can also be studied.

The idea of taking space-time co-ordinates to be non-commutative began in the thirties of the last century. The purpose was to introduce an effective cut off, which regularizes divergences in quantum field theory by introducing a non-commutative structure to space-time at small length scales. However, the theory suffered from several problems such as the violation of unitarity and causality, which led most people to abandon it. However, non-commutative geometry was pursued on the mathematical side and especially in the work of A. Connes in the eighties of the last century [19].

The interest for non-commutative geometry is renewed in 1999 by the work of Seiberg and Witten on string theory [20]. They showed that the dynamics of the endpoints of an open string on a D-brane in the presence of a magnetic background field can be described by a Yang-Mills theory on a non-commutative space-time.

To get a non-commutative space-time, we deform the ordinary space-time by promoting the coordinates to Hermitian operators, which do not commute.

$$\left[x_{nc}^{\mu}, x_{nc}^{\nu}\right] = i\theta^{\mu\nu} = i\,C^{\mu\nu}\big/(\Lambda_{nc})^2\,;\mu,\nu = 0,1,2,3 \qquad (1)$$

Where, $\theta^{\mu\nu}$ is a deformation parameter and *nc* indices denote non-commutative co-ordinates. Ordinary space-time is obtained in the limit $\theta^{\mu\nu} \to 0$. The non-commutative parameter is an anti-symmetric real matrix. Λ_{nc} is the energy scale, where the non-commutative effects of the space-time will be relevant and $C^{\mu\nu}$ are dimensionless parameters. A well-documented review can be found in [21-22].

In this work, we focus on the noncommutative effects on hydrogen atom spectroscopy. We start by writing the Dirac equation for the H-atom in framework of noncommutative space-time. Then, we compute the corrections of the energy levels induced by non-commutativity and get a limit on the non-commutative parameter from the 2P-2S Lamb shift (the 28cm line) theoretical accuracy. We use this bound to estimate the contribution to the Lyman-α line.

2. Noncommutative Dirac spectrum of hydrogen atom

We work on the space-time version of non-commutativity; thus instead of (1), we use

$$\left[x_{st}^{j}, x_{st}^{0}\right] = i\theta^{j0} \tag{2}$$

The *st* subscripts are for noncommutative space-time coordinates. The 0 denotes time and j is for space coordinates.

The solutions to these new commutation relations are chosen as the transformations

$$x_{st}^{j} = x^{j} + i\theta^{j0}\partial_0 \tag{3}$$

Of course, the usual space coordinates x^j satisfy the usual canonical permutation relations.

To simplify writing equations, we use the compact vector notation

$$\vec{r}_{st} = \vec{r} + i\vec{\theta}\partial_0 ; \vec{\theta} \equiv (\theta^{10}, \theta^{20}, \theta^{30}) \tag{4}$$

We have the Dirac equation (where $\alpha_i = \gamma_0\gamma_i$ and the γ_μ are Dirac matrices)

$$i\hbar\partial_0\psi = H\psi = \left[(\vec{\alpha}\cdot\vec{p}) + m\gamma^0 - eA_0\right]\psi \tag{5}$$

The kinetic energy depends on the momentum p, which remains unchanged and so it does not change. This lead us to consider only changes on the potential energy by taking the Coulomb potential and constructing its noncommutative image. To achieve this, we write it using the new coordinates in the same way as the usual one and because of the smallness of the noncommutative parameter, we restrict ourselves to the 1st order in θ [21]:

$$A_0^{nc} = \frac{e}{r_{st}} = \frac{e}{\sqrt{\left(\vec{r} + i\vec{\theta}\partial_0\right)^2}} = \frac{e}{r}\left(1 - i\frac{\vec{r}\cdot\vec{\theta}}{r^2}\partial_0\right) + O\left(\theta^2\right) \tag{6}$$

As we start with the Coulomb potential that is spherically symmetric, an adequate choice of the non-commutative parameter is the rotational invariant form or: $\vec{\theta} = \theta^{0r}\vec{r}/r$. It is equivalent to what was in [6-7] and in [23-26]. This allows us to write the non-commutative Coulomb potential as

$$A_0^{nc} = \frac{e}{r}\left(1 - i\frac{\theta_{st}}{r}\partial_0\right) + O\left(\theta^2\right) = \frac{e}{r} - \frac{eE\theta_{st}}{\hbar}\frac{1}{r^2} \tag{7}$$

We have used the fact that $i\hbar\partial_0\psi = E\psi$. The Hamiltonian can now be expressed as ($\theta^{0r} = \theta_{st}$) and

$$\begin{cases} H = H^{(0)} + H^{(nc)} \\ H^{(0)} = \left[(\vec{\alpha}\cdot\vec{p}) + m\gamma^0 - e^2/r\right] \\ H^{(nc)} = \left[e^2\left(E/\hbar\right)\theta_{st}/r^2\right] \end{cases} \tag{8}$$

$H^{(0)}$ is the Dirac Hamiltonian in the usual relativistic theory and $H^{(nc)}$ is the non-commutative correction to this Hamiltonian. The smallness of the parameter θ allows us to consider NC corrections with perturbation methods; to the 1st order in θ, the corrections of the eigenvalues are

$$\Delta E^{(nc)} = \left\langle H^{(nc)}\right\rangle = e^2 E/\hbar\left\langle r^{-2}\right\rangle\theta_{st} \tag{9}$$

To compute this mean value, one can simply use the expressions of Dirac solutions for hydrogen atom; this has already been done in [14] and the result is

$$\left\langle\frac{1}{r^2}\right\rangle = \frac{2\kappa\left(2\kappa\varepsilon - 1\right)\left(1-\varepsilon^2\right)^{3/2}}{\alpha\sqrt{\kappa^2-\alpha^2}\left[4\left(\kappa^2-\alpha^2\right)-1\right]}\left(\frac{mc}{\hbar}\right)^2\left(\frac{1}{a_0^2}\right) \tag{10}$$

Here we have $a_0 = \hbar^2/me^2$ as the 1st Bohr radius and $\varepsilon = E/mc^2$; E is the Dirac energy:

$$\frac{E_{n,j}}{mc^2} = \left\{1 + \alpha^2\left[\left(n - j - \tfrac{1}{2}\right) + \sqrt{\left(j+\tfrac{1}{2}\right)^2 - \alpha^2}\right]^{-2}\right\}^{-1/2} \tag{11}$$

$\alpha = e^2/\hbar c$ is the fine structure constant and the quantum number associated to the total angular momentum is $j = l \pm 1/2$. The number κ is giving by the two relations: $\kappa = -j - 1/2$ if $j = l + 1/2$ and $\kappa = j + 1/2$ if $j = l - 1/2$. We see from Eqns. (9) and (10) that through κ, the energy depends not only on the value of j but also on the manner to get this value (or on l), unlike the usual Dirac energies in Eqn. (11), which is the same for all the possible ways to obtain j. This implies that non-commutativity remove the degeneracy $j = l + 1/2 = (l+1) - 1/2$ in hydrogen atom (states like $nP_{3/2}$ and $nD_{3/2}$).

3. Lyman lines in non-commutative case:

To study the Lyman lines, we calculate the corrections to the levels $n = 1$,2 and get

$$\begin{aligned} \Delta E^{(nc)}\left(1S_{1/2}\right) &= 1.065084\cdot10^{-4}\left(m^3e^2c^4/\hbar^3\right)\theta \\ \Delta E^{(nc)}\left(2S_{1/2}\right) &= 1.331426\cdot10^{-5}\left(m^3e^2c^4/\hbar^3\right)\theta \\ \Delta E^{(nc)}\left(2P_{1/2}\right) &= 0.443805\cdot10^{-5}\left(m^3e^2c^4/\hbar^3\right)\theta \\ \Delta E^{(nc)}\left(2P_{3/2}\right) &= 0.443765\cdot10^{-5}\left(m^3e^2c^4/\hbar^3\right)\theta \end{aligned} \tag{12}$$

We compute the correction induced for the $2P_{1/2} - 2S_{1/2}$ Lamb shift and we compare this result to the current theoretical accuracy for the Lamb shift 0.08 kHz from [28]; we get the bound $\theta_{st} \lesssim (0.18TeV)^{-2}$. This limit is better than the ones from [22], [29] and [30].

We take this limit as a reference for our computations and use it to study the effects on the Lyman-α lines; we find that

$$\Delta E^{nc}(Ly\alpha_1) \approx \Delta E^{nc}(Ly\alpha_2) = 3.067\times10^{-12}\, eV$$
$$\Rightarrow \Delta\lambda^{nc}(Ly\alpha_1) \approx \Delta\lambda^{nc}(Ly\alpha_2) = 5.82\times10^{-11}\, \overset{0}{\mathrm{A}} \qquad (13)$$

From the National Institute of Standards and Technology (NIST) [31], the accuracy of the Ly-α wavelength is $1.18\cdot10^{-9}$Å, we see that N.C corrections are about 5% smaller. We conclude that, in the current status of experimental limits and taking as reference the bound of the θ coming from the Lamb shift, the effects of non-commutativity on the Lyman-α line are negligible.

For the redshift, we compute the non-commutative corrections as follows

$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}} \Rightarrow \Delta^{(nc)}z = \frac{\Delta^{(nc)}\lambda}{\lambda_{emit}} \approx 5\times10^{-14} \qquad (14)$$

And this can be translated into corrections on cosmological distance (using its definition) as

$$\Delta^{(nc)}d = \left(\Omega_M\,(1+z)^3 + \Omega_\Lambda\,(1+z)^2 + \Omega_k\right)^{-1/2}\frac{c}{H_0}\Delta^{(nc)}z \qquad (15)$$

It is clear that the corrections are inversely proportional to the values of z. The maximum value for these corrections is achieved for $z\to0$. If we take values of the parameters from the ΛCDM model as an example (for a review on ΛCDM, one can see [32]), we get for this limit

$$\Delta^{(nc)}z \approx 5\cdot10^{-14} \Rightarrow \Delta^{(nc)}d \approx 5\cdot10^{-4}\,ly \qquad (16)$$

And we can clearly see that non-commutative corrections are insignificant compared to both galactic and cosmological distances.

4. Conclusion

In this work, we look for non-commutative space-time hydrogen atom and induced phenomenological effects; for this we use the Bopp's shift formulation. We found that applying space-time non-commutativity to the electron in the H-atom modifies the Coulomb potential gives us the potential of Kratzer.

By solving the Dirac equation, we have calculated the corrections induced by space-time noncommutativity to the Dirac energy levels. We find that these corrections act like a Lamb shift and remove the degeneracy of the Dirac energies with respect to the total angular momentum quantum number $j = l + 1/2 = (l+1) - 1/2$ in addition to the degeneracy of the Bohr energies with respect to the orbital quantum number l. This is explained by the fact that Lamb correction can be interpreted as a shift of r in the Coulomb potential due to interactions of the bound electron with the fluctuating vacuum electric field, and non-commutativity is also a shift of r as we can see from the Bopp's shift (3) and (4).

By comparing the theoretical limit of the Lamb shift, we get a bound on the deformation parameter. This bound is not affected if we use the two-particle Dirac equation because we have demonstrated in [19] that this introduces a factor $\left(1+m_e/m_p\right)^{-3/2} = 0.99918$ to our computations. We use this limit to see the non-commutative effects on the Lyman-α line and found that they are about 5% smaller than the actual experimental accuracy. Thus, we see that the non-commutative corrections on the Lyman-α line are negligible up to now.

Similarly, we calculated the corrections induced by non-commutativity on the redshift and, consequently, on astronomical distances. We found that they are smaller for large z and the maximum value is achieved for $z\to0$: $\Delta z \approx 5\times10^{-14}$. We conclude that non-commutative corrections are insignificant compared to both galactic and cosmological distances and they do not affect phenomena associated with the Lyman-Alpha line.

Acknowledgments

This work was done as part of the PNR project (8/P/461) entitled "Theories of Non-Commutative Fields from Matrix Models and Emerging Physics" under the patronage of the DGRSDT of the Ministry of Higher Education and Scientific Research of Algeria.
Authors would like to thank the Laboratory of Mathematical and Subatomic Physics of Constantine University and the Laboratory of Photonic Physics and Nano-Materials of Biskra University for supporting this work.

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Received: 20 April, 2017
Accepted: 06 April, 2018