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FRW Cosmological Models with Bulk Viscosity in the Context of Open Thermodynamical System

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Cosmological models with constant deceleration parameter in Einstein's theory of gravitation in the presence of both the creation of matter and bulk viscosity have been discussed by considering the Universe as an open thermo-dynamical system. We have presented here modified field equations in the presence of particle creation and bulk viscosity. Exact solutions have been obtained for a spatially flat FRW model by keeping the deceleration parameter constant for two different choices of the bulk viscosity coefficient viz., (i) $\zeta(t) = \zeta_0 =$ constant and (ii) $\zeta(t) = \zeta_0 \theta$.

1. Introduction

In the standard cosmological model, the Universe is considered as a closed system. In this approach, to account for the observed entropy, it has to be either assumed as an initial condition or accounted through some dissipative mechanism. The classical evolution equations (in General Relativity and scalar tensor theories like the Brans-Dicke theory) are purely adiabatic and reversible; consequently, they cannot explain the origin of cosmological entropy, which might have been generated through irreversible processes during the cosmic expansion. Prigogine et al. [1, 2] investigated the role of irreversible processes in the creation of matter out of gravitational energy in the context of General Relativity. Prigogine and Geheniau [3] and Prigogine and Glansdorff [4] showed that thermodynamics of open systems when applied to cosmology, leads to a reinterpretation in Einstein's equations of the matter-energy stress tensor [1]. This takes into account both the matter creation and entropy production on a macroscopic scale and the second law of thermodynamics is also incorporated into the evolutionary equations in a more meaningful way. Here the Universe starts from a random vacuum fluctuation and the creation of matter particles drives the cosmic expansion. The effect of creation of new particles is equivalent to adding a supplementary negative pressure term p_c to the thermodynamic pressure p which drives the expansion. Johri and Desikan [5, 6] had analyzed the role of irreversible processes, corresponding to the creation of matter out of gravitational energy in the context of Brans-Dicke theory and General theory of Relativity.

It is worthwhile to observe that most of the well-known models of Einstein's theory and Brans-Dicke theory with curvature parameter k = 0, including inflationary models, are models with constant deceleration parameter [7-13]. Models with constant deceleration parameter lead to power law and exponential solutions for the scale factor R(t). They form an interesting class of models in cosmology and are relevant during different epochs of the Universe.

Bulk stress is the only dissipative mechanism that can be incorporated in an isotropic cosmological model. Hence, in this paper we have discussed the effect of bulk stress on the evolution of FRW universe in the context of open systems. Many authors [14-26] have investigated the effect of bulk viscosity on the cosmological evolution of closed systems. The effect of bulk stress on the evolution of FRW universe in the context of open systems for particular choices of particle creation function and constant bulk viscous coefficient was discussed in [27]. The modified field equations in the presence bulk viscosity and particle creation are given in Sec. 2. We note that the number of equations is less than the number of unknowns. Hence, an additional assumption in the form constant deceleration parameter makes the system of equations well defined and enables us to obtain unique solutions for two different choices of the bulk viscosity coefficient. Sec. 3 deals with the solutions of the field equations and their discussion.

2. Field Equations

Consider the Einstein's field equations

$$G_{ab} = -8\pi G T_{ab} \tag{1}$$

with velocity of light $c = 1$, where T_{ab}, the energy momentum tensor in the presence of creation of matter and bulk viscosity, is given by

$$T_{ab} = (\rho + p + p_c - \zeta\theta)u_a u_b - (p + p_c - \zeta\theta)g_{ab} \tag{2}$$

where ρ and p are the energy density and pressure, respectively, p_c is the creation pressure, ζ is the coefficient of bulk viscosity, u_a the fluid-four velocity, and g_{ab} is the metric tensor. We now restrict our attention to the FRW line element, with k=0, given by

$$ds^2 = dt^2 - R^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{3}$$

where $R(t)$ is the scale factor.

The field equation (1) with the above metric and barotropic equation of state

$$p = \gamma\rho, -1 \le \gamma \le 1 \tag{4}$$

now becomes

$$\theta^2 = 24\pi G\rho \tag{5}$$

where $\theta = 3H$. The continuity equation is given by

$$\dot{\rho} + 3(1+\gamma)\rho H = (1+\gamma)\rho\frac{\dot{N}}{N} \tag{6}$$

where $H = \frac{\dot{R}}{R}$ is the Hubble's function and $\frac{\dot{N}}{N} = \psi(t)$ is the source function for particle creation.

Eliminating $\rho(t)$ from (5) and (6) we have

$$\dot{\theta} + \frac{1}{2}(1+\gamma)\theta^2 - \lambda\zeta(t)\theta - \frac{1}{2}(1+\gamma)\frac{\dot{N}}{N}\theta = 0 \tag{7}$$

where $\lambda = 12\pi G$. We assume the deceleration parameter to be constant and obtain solutions for two different choices of the bulk viscosity coefficient.

3. Exact solutions with constant deceleration parameter

Consider a model with constant deceleration parameter, that is,

$$q = \frac{-R\ddot{R}}{(\dot{R})^2} = \beta \tag{8}$$

where β is a constant. Equation (8) can be rewritten as

$$\frac{\ddot{R}}{R} + \beta\left(\frac{\dot{R}}{R}\right)^2 = 0 \tag{9}$$

On integration the above equation yields the exact solution

$$R(t) = \begin{cases} (D+Ct)^{1/(1+\beta)} & \beta \ne -1 \\ R_0 e^{H_0 t} & \beta = -1 \end{cases} \tag{10}$$

where C, D, R_0 and H_0 are constants of integration.

We now consider two different choices for the bulk viscosity coefficient.

Case (i): $\zeta(t) = \zeta_0$ = constant

Equation (7) now becomes

$$\dot{\theta} + \frac{1}{2}(1+\gamma)\theta^2 - \lambda\zeta_0\theta - \frac{1}{2}(1+\gamma)\frac{\dot{N}}{N}\theta = 0 \tag{11}$$

We now obtain solutions for $N(t)$ and $\rho(t)$ for both power law expansion, given by Eqn. (10a) and exponential expansion, given by Eqn. (10b).

For singular models ($R(0) = 0$) and hence, expression in Eqn. (10a) may be written as

$$R(t) = R_0 t^{1/(1+\beta)} \tag{12}$$

Using Eqns. (12) in (11) and on simplifying we get

$$\frac{\dot{N}}{N} = \frac{[3(1+\gamma) - 2(1+\beta)]}{(1+\beta)(1+\gamma)}\frac{1}{t} - 2\lambda\zeta_0 \tag{13}$$

On integrating Eqn. (13) we obtain

$$N(t) = N_0 t^{\frac{[3(1+\gamma)-2(1+\beta)]}{(1+\beta)(1+\gamma)}} \exp(-2\lambda\zeta_0 t) \tag{14}$$

Using Eqns. (12) and (14) in (6) we have

$$\rho(t) = \rho_0 t^{-2} \exp(-2\lambda\zeta_0 t) \tag{15}$$

In the absence of bulk viscosity, Eqns. (14) and (15) reduce to the expressions obtained for $N(t)$ and $\rho(t)$ in [6]. We see from Eqns. (14) and (15) that both $N(t)$ and $\rho(t)$ tend to 0 as $t \to \infty$.

In an open thermodynamical system we have

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} \geq 0$$

where S is the entropy. This imposes a restriction on the bulk viscosity coefficient ζ_0. From Eqn. (13) we see that for the matter dominated era $(\gamma = 0)$ and present time t_p, the bulk viscosity coefficient ζ_0 lies in the range

$$0 \leq \zeta_0 \leq \frac{1-2\beta}{1+\beta}\frac{1}{2\lambda t_p}$$

From the above expression we note that since ζ_0 is positive, for an expanding universe, it restricts the deceleration parameter to lie in the range

. $$-1 < \beta \leq \frac{1}{2}$$

Now considering exponential expansion given by Eqn. (10b), Eqn. (11) on simplifying yields

$$\frac{\dot{N}}{N} = 3H_0 - \frac{2\lambda\zeta_0}{(1+\gamma)} \tag{16}$$

The above equation can be readily integrated to give

$$N(t) = N_0 e^{(3H_0 - \frac{2\lambda\zeta_0}{(1+\gamma)})t} \tag{17}$$

Using Eqns. (10b) and (17) in Eqn. (6) we get the expression for $\rho(t)$ as

$$\rho(t) = \rho_0 \exp(-2\lambda\zeta_0 t) \tag{18}$$

We note that as $t \to \infty$, $\rho(t)$ tends to 0. In the absence of the bulk viscosity, Eqns. (17) and (18) reduce to the expressions obtained for $N(t)$ and $\rho(t)$ in [6].

As in the case of the power law expansion, for the matter dominated era $(\gamma = 0)$, from Eqn. (16) we obtain the range for the bulk viscosity coefficient as

$$0 \leq \zeta_0 \leq \frac{3H_0}{2\lambda}$$

Case (ii): $\zeta(t) = \zeta_0\theta$

Equation (7) now becomes

$$\dot{\theta} + \frac{1}{2}(1+\gamma)\theta^2 - \lambda\zeta_0\theta^2 - \frac{1}{2}(1+\gamma)\frac{\dot{N}}{N}\theta = 0 \tag{19}$$

We now obtain solutions for $N(t)$ and $\rho(t)$ for both power law expansion, given by Eqn. (10a) and exponential expansion, given by eqn. (10b).

As in the case of constant bulk viscosity, considering a singular model (where $R(0) = 0$) we have the expression for the scale factor as

$$R(t) = R_0 t^{1/(1+\beta)} \tag{20}$$

Using Eqn. (20) in Eqn. (19) and on simplifying we get

$$\frac{\dot{N}}{N} = \frac{[3(1+\gamma) - 2(1+\beta) - 6\lambda\zeta_0]}{(1+\beta)(1+\gamma)}\frac{1}{t} \tag{21}$$

On integrating Eqn. (21) we obtain

$$N(t) = N_0 t^{\frac{[3(1+\gamma)-2(1+\beta)-6\lambda\zeta_0]}{(1+\beta)(1+\gamma)}} \tag{22}$$

Using Eqns. (20) and (22) in Eqn. (6) we have

$$\rho(t) = \rho_0 t^{-2-\frac{6\lambda\zeta_0}{(1+\beta)}} \tag{23}$$

As in the previous case, in the absence of the bulk viscosity, Eqns. (22) and (23) reduce to the expressions obtained for $N(t)$ and $\rho(t)$ in [6].

From Eqn. (21) we see that for the matter dominated era $(\gamma = 0)$ and present time t_p, we get the bounds for ζ_0 as

$$0 \leq \zeta_0 \leq \frac{1-2\beta}{6\lambda}$$

From the above expression we note that since ζ_0 is positive, for an expanding universe, it constrains the deceleration parameter to lie in the range $-1 < \beta \leq \frac{1}{2}$.

Now considering exponential expansion given by (10b), (19) on simplifying yields

$$\frac{\dot{N}}{N} = 3H_0 - \frac{6\lambda\zeta_0}{(1+\gamma)} \qquad (24)$$

The above equation can be readily integrated to give

$$N(t) = N_0 e^{(3H_0 - \frac{6\lambda\zeta_0 H_0}{(1+\gamma)})t} \qquad (25)$$

Using Eqns. (10b) and (25) in Eqn. (6) we get the expression for $\rho(t)$ as

$$\rho(t) = \rho_0 \exp(-6\lambda\zeta_0 t) \qquad (26)$$

As in the previous case, we note that as $t \to \infty$, $\rho(t)$ tends to 0. In the absence of the bulk viscosity, Eqns. (25) and (26) reduce to the expressions obtained for $N(t)$ and $\rho(t)$ in [6].

As in the case of the power law expansion, for the matter dominated era $(\gamma = 0)$, from Eqn. (24) we obtain the range for ζ_0 as

$$0 \le \zeta_0 \le \frac{H_0}{2\lambda}$$

4. Conclusion

In this paper, we have discussed cosmological models with constant deceleration parameter in Einstein's theory in presence of the creation of matter and bulk viscosity. We have explored the concept of creation of matter proposed by Prigogine [1] in the context of bulk viscosity, which is the only dissipative mechanism that can be incorporated in an isotropic cosmological model.

We observe that the behavior of the energy density $\rho(t)$ and the number of particles created $N(t)$ differ, based on the choice of the bulk viscosity coefficient. We have considered both the power-law expansion and the exponential expansion of a flat universe and studied their behavior. We have also obtained bounds for the bulk viscosity coefficient for the matter dominated era.

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