# New Exact Energy Eigen-values for (*MIQYH*) and (*MIQHM*) Central Potentials: Non-relativistic Solutions

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In this paper, we solved modified Schrödinger equation (*MSE*) for two potentials namely: modified inversely quadratic Yukawa potential plus inversely quadratic Hellmann potential (*MIQYH*) and modified inversely quadratic Hellmann plus Mie-type potential (*MIQHM*), which are equal to the sum of inversely quadratic Yukawa potential plus inversely quadratic Hellmann and inversely quadratic Hellmann plus Mie-type potential, respectively, using a generalization of Boopp's shift method and standard perturbation theory instead of using directly star product method. We then obtained modified energy eigenvalues and corresponding modified Hamiltonians in both three dimensional non-commutative space and phase (*NC-3D: RSP*).

## 1. Introduction

In the last few decades, central potentials in two-, three- and D-dimensional spaces have been studied with various methods in different fields of nuclear physics, spectroscopy, quantum chemistry, and many other fields of sciences using three fundamental equations: Schrödinger, Klein-Gordon and Dirac equations. The first one is undoubtedly the most widely studied equation of modern physics [1-26]. Over the past few years, a considerable effort has been made to solve Schrödinger equation with previously central potentials in two as well as in three dimensions in the case of non-commutative space and phase to give profound physical and chemical interpretation of different fields at nano and Plank's scales [27-53]. The algebraic physical structure of ordinary quantum mechanics based on the following fundamental three canonical commutations relations (CCRs), which plays as fundamental postulates of quantum mechanics, $\lfloor x_i, p_j \rfloor$, $\lfloor x_i, x_j \rfloor$ and $\lfloor p_i, p_j \rfloor$, in both Schrödinger and Heisenberg pictures, respectively, as:

$$\lfloor x_i, p_j \rfloor = i\delta_{ij} \quad \text{and} \quad \lfloor x_i, x_j \rfloor = \lfloor p_i, p_j \rfloor = 0 \qquad (1a)$$

And

$$\lfloor x_i(t), p_j(t) \rfloor = i\delta_{ij} \quad \text{and} \quad \lfloor x_i(t), x_j(t) \rfloor = \lfloor p_i(t), p_j(t) \rfloor = 0 \qquad (1b)$$

Where the usual canonical coordinates and new momentum $x_i(t)$ and $p_i(t)$ are determined from the projection relations:

$$\begin{aligned} x_i(t) &= \exp(iH(t-t_0))x_i \exp(-iH(t-t_0)) \\ p_i(t) &= \exp(iH(t-t_0))p_i \exp(-iH(t-t_0)) \end{aligned} \qquad (1c)$$

Here $\{x_i(t)\}$ , $\{p_i(t)\}$ and $H$ are Hermitian operators on a Hilbert space of physical states, which satisfy the Heisenberg equation of motions, respectively, as

$$\frac{dx_i(t)}{dt} = i[H, x_i(t)] \quad \text{and} \quad \frac{dp_i(t)}{dt} = i[H, p_i(t)] \qquad (1d)$$

There will be changes in noncommutative three dimensional spaces and phases to the new canonical commutations relations (NCCRs), in both Schrödinger and Heisenberg pictures, as follows

$$\left[\hat{x}_i \overset{*}{,} \hat{p}_j\right] = i\delta_{ij}, \left[\hat{x}_i \overset{*}{,} \hat{x}_j\right] = i\theta_{ij} \quad \text{and} \quad \left[\hat{p}_i \overset{*}{,} \hat{p}_j\right] = i\bar{\theta}_{ij} \qquad (2a)$$

And

$$\left[\hat{x}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\delta_{ij}, \left[\hat{x}_i(t) \overset{*}{,} \hat{x}_j(t)\right] = i\theta_{ij} \text{ and} \left[\hat{p}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\bar{\theta}_{ij} \qquad (2b)$$

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The two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$, very small as compared to the energy, are elements of two antisymmetric real matrixes and $(*)$ denotes the new star product, which is generalized between two arbitrary functions $f(x,p)$ and $g(x,p)$ to $(f*g)(x,p)$ instead of the old product $(fg)(x,p)$ [29,30,32,51,52,53]:

$$(f*g)(x,p)=(fg)(x,p)-\frac{i}{2}\theta^{\mu\nu}\frac{\partial f(x,p)}{\partial x^{\mu}}\frac{\partial g(x,p)}{\partial x^{\nu}}-\frac{i}{2}\bar{\theta}^{\mu\nu}\frac{\partial f(x,p)}{\partial p^{\mu}}\frac{\partial g(x,p)}{\partial p^{\nu}} \quad (3a)$$

The new canonical coordinates and new momentum $\hat{x}_i(t)$ and $\hat{p}_i(t)$ are determined from two projection relations, respectively, as

$$\hat{x}_i(t)=\exp(iH_{nc}(t-t_0))*\hat{x}_i*\exp(-iH_{nc}(t-t_0))$$
$$\hat{p}_i(t)=\exp(iH_{nc}(t-t_0))*\hat{p}_i*\exp(-iH_{nc}(t-t_0)) \quad (3b)$$

Which satisfy the new Heisenberg equation of motion given as

$$\frac{d\hat{x}_i(t)}{dt}=i\left[H_{nc}\overset{*}{,}\hat{x}_i(t)\right] \quad \text{and} \quad \frac{d\hat{p}_i(t)}{dt}=i\left[H_{nc}\overset{*}{,}\hat{p}_i(t)\right] \quad (3c)$$

The formalism of star product, Boopp's shift method and the Seiberg-Witten map played crucial role in this new theory. The Boopp's shift method will be applied in this paper. Instead of solving the Schrödinger equation in (NC-3D:RSP) with star product, the equation will be treated by using directly the two new commutators, in addition to usual commutator on quantum mechanics, in the both Schrödinger and Heisenberg representations [28-34,43-53] given as

$$\left[\hat{x}_i,\hat{x}_j\right]=i\theta_{ij} \text{ and } \left[\hat{p}_i,\hat{p}_j\right]=i\bar{\theta}_{ij} \quad (4a)$$

And

$$\left[\hat{x}_i(t),\hat{x}_j(t)\right]=i\theta_{ij} \text{ and } \left[\hat{p}_i(t),\hat{p}_j(t)\right]=i\bar{\theta}_{ij} \quad (4b)$$

Where, the new operators $\hat{x}_i$ and $\hat{p}_i$ in (*NC-3D: RSP*) are depended on ordinary operator $x_i$ and $p_i$, and from the projection relations for $\left(c=\hbar=1 \text{ and } i,j=\overline{1,3}\right)$ we obtained

$$\hat{x}=x-\frac{\theta_{12}}{2}p_y-\frac{\theta_{13}}{2}p_z,\ \hat{y}=y-\frac{\theta_{21}}{2}p_x-\frac{\theta_{23}}{2}p_z$$
$$\text{and} \quad \hat{z}=z-\frac{\theta_{31}}{2}p_x-\frac{\theta_{32}}{2}p_y \quad (5)$$

And

$$\hat{p}_x=p_x-\frac{\bar{\theta}_{12}}{2}y-\frac{\bar{\theta}_{13}}{2}z,\ \hat{p}_y=p_y-\frac{\bar{\theta}_{21}}{2}x-\frac{\bar{\theta}_{23}}{2}z$$
$$\text{and} \quad \hat{p}_z=p_z-\frac{\bar{\theta}_{31}}{2}x-\frac{\bar{\theta}_{32}}{2}y \quad (6)$$

The non-vanish 9-commutators in (*NC-3D: RSP*) can be determined and given as

$$\begin{aligned}&\left[\hat{x},\hat{p}_x\right]=\left[\hat{y},\hat{p}_y\right]=\left[\hat{z},\hat{p}_z\right]=i,\\&\left[\hat{x},\hat{y}\right]=i\theta_{12},\left[\hat{x},\hat{z}\right]=i\theta_{13},\left[\hat{y},\hat{z}\right]=i\theta_{23}\\&\left[\hat{p}_x,\hat{p}_y\right]=i\bar{\theta}_{12},\left[\hat{p}_y,\hat{p}_z\right]=i\bar{\theta}_{23},\left[\hat{p}_x,\hat{p}_z\right]=i\bar{\theta}_{13}\end{aligned} \quad (7)$$

The aim of this work is to study the two (*IQYH*) and (*IQHM*) potentials in non-commutative three-dimensional space and phase to discover the global spectrum in this new symmetry, which plays an important role in many fields of physics such as molecular physics, solid state and chemical physics based on two principal references [25,26]. This work is based essentially on our previous works [28-34,43-53]. The rest of this paper is organized as follows. In the next section, we briefly present and review the basics of eigenvalues and eigenfunctions for: (*IQYH*) and (*IQHM*) potentials in ordinary three-dimensional spaces. In Sec. 3, we give a brief review of Boopp's' shift method, then, we derive the spin-orbital non-commutative Hamiltonians for (*MIQYH*) and (*MIQHM*) potentials in (NC-3D: RSP), we find the exact spectrum produced by non-commutative spin-orbital Hamiltonians $\hat{H}_{so-yh}$ and $\hat{H}_{so-hm}$ for (*MIQYH*) and (*MIQHM*) potentials by applying ordinary perturbation theory. Then we deduce the exact spectrum produced by non-commutative magnetic Hamiltonians $\hat{H}_{m-yh}$ and $\hat{H}_{m-hm}$ for (*MIQYH*) and (*MIQHM*) potentials in (NC: 3D-RSP). In Sec. 4 we summarize the global spectrums for (*MIQYH*) and (*MIQHM*) potentials. Finally, in Sec. 5, we present our concluding remarks.

## 2. The (*IQYH*) and (*IQHM*) Potentials in Ordinary Two Dimensional Spaces

The purpose of this section is to give a brief review of eigen-values and eigen-functions for two ordinary (*IQYH*) and (*IQHM*) potentials, $[V_{yh}(r), V_{hm}(r)]$, based on two references [25,26], respectively, and given as

$$V_{yh}(r)=\frac{1}{r^2}(b-V_0)+\frac{1}{r}(2V_0-a-b\delta)+(b-2V_0)\delta^2 \tag{8a}$$

$$V_{hm}(r)=\frac{B+b}{r^2}-\frac{a-A+b\delta}{r}+\left(C+b\delta^2\right) \tag{8b}$$

Where, $r$ , $(a,b)$ , $\delta$ , $(A,B,C)$ are inter nuclear distance, strengths of the Coulomb and Yukawa potentials, the screening parameter, respectively, and $V_0$ is the dissociation energy. The ordinary two Schrödinger equations (SE) with two potentials $V_{yh}(r)$ and $V_{hm}(r)$ can be written in spherical coordinates $(r,\theta,\varphi)$ [25,26] as

$$\begin{aligned}&\frac{1}{2\mu}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial\varphi^2}\right)\Psi(r,\theta,\varphi)\\&+V_{yh}(r)\Psi(r,\theta,\varphi)=E_{yh}\Psi(r,\theta,\varphi)\end{aligned} \tag{9a}$$

$$\begin{aligned}&\frac{1}{2\mu}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial\varphi^2}\right)\Psi(r,\theta,\varphi)\\&+V_{hm}(r)\Psi(r,\theta,\varphi)=E_{hm}\Psi(r,\theta,\varphi)\end{aligned} \tag{9b}$$

Where, $E_{yh}$ and $E_{hm}$ are ordinary energies corresponding $V_{yh}(r)$ and $V_{hm}(r)$ in ordinary three-dimensional spaces, the method of separation of variable has been applied as given in two references [25,26]:

$$\Psi(\vec{r})=\frac{R_{nl}(r)}{r}\phi_{ml}(\theta)\Phi_m(\varphi) \tag{10}$$

The two radial functions $R_{yh-nl}(r)$ and $R_{hm-nl}(r)$ and the two spherical functions $\phi_{ml}(\theta)$ and $\Phi_m(\varphi)$ for two ordinary (*IQYH*) and (*IQHM*) potentials satisfying the following four differential equations, respectively [25,26], as

$$\begin{aligned}&\frac{d^2R_{yh-nl}(r)}{dr^2}+2\mu\left(E-V_{yh}(r)-\frac{\lambda}{2\mu r^2}\right)R_{yh-nl}(r)=0\\&\frac{d^2R_{hm-nl}(r)}{dr^2}+2\mu\left(E-V_{hm}(r)-\frac{\lambda}{2\mu r^2}\right)R_{hm-nl}(r)=0\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{d^2\phi_{ml}(\theta)}{d\theta^2}+\cot(\theta)\frac{d\phi_{ml}(\theta)}{d\theta}\left(\lambda-\frac{m^2}{\sin^2(\theta)}\right)\phi_{ml}(\theta)=0\\&\frac{d^2\Phi_m(\varphi)}{d\varphi^2}+m^2\frac{d\Phi_m(\varphi)}{d\varphi}=0\end{aligned} \tag{12}$$

Here, $\lambda=l(l+1)$ , according Nikiforov-Uvarov method, the normalized energy eigen-functions $\Psi_{yh}(\vec{r})$ and corresponding eigen-values $E_{yh}$ for ordinary (*IQYH*) potential [25]

$$\Psi_{yh}(\vec{r})=N_n z^{\frac{1}{2}+\left(-1+\sqrt{1+4\gamma_1}\right)/2}e^{-\sqrt{\alpha}z}L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\phi_{ml}(\theta)\Phi_m(\varphi) \tag{13a}$$

and

$$E_{yh}=(2V_0-b)\delta^2-\frac{\mu(2V_0-a-b\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(b-V_0)+(l+1/2)^2}\right)^2} \tag{13b}$$

The normalized energy eignenfunctions $\Psi_{hm}(\vec{r})$ and corresponding eigenvalues $E_{hm}$ for ordinary *(IQHM)* potential [26]

$$\Psi_{yh}(\vec{r})=N_n z^{\frac{1}{2}+\left(-1+\sqrt{1+4\gamma_2}\right)/2}e^{-\sqrt{\alpha}z}L_n^{\sqrt{1+4\gamma_2}}\left(-2\sqrt{\alpha}z\right)\phi_{ml}(\theta)\Phi_m(\varphi) \tag{14a}$$

and

$$E_{hm}=C+\delta b^2-\frac{\mu(a-A\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(B+b)+(l+1/2)^2}\right)^2} \tag{14b}$$

Where, $z=r^2$ , $N_n$ is the normalization constant and the two factors ( $\gamma_1$ , $\gamma_2$ ) are given by equations

$$\begin{cases}\gamma_1 = 2\mu(b - V_0) + l(l+1) & for \quad V_{yh}(r) \\ \gamma_2 = 2\mu(B + b) + l(l+1) & for \quad V_{hm}(r)\end{cases} \tag{15}$$

## 3. Non-commutative Three Dimensional Phase-spaces (NC-3D: RSP) Hamiltonians for (*MIQYH*) and (*MIQHM*) Potentials

### 3.1. Formalism of Boopp's shift method

The main goal to this sub-section is to present fundamental principles of modified Schrödinger equation in (NC-3D: RSP) based on our previous work [29,30,32,51-53]. To achieve this goal, we shall apply four important steps to ordinary Schrödinger equation described below.

1- Ordinary three-dimensional two Hamiltonian operators $\left[\hat{H}_{yh}(p_i, x_i), \hat{H}_{hm}(p_i, x_i)\right]$ will be replaced by new two Hamiltonian operator s $\left[\hat{H}_{ncyh}(\hat{p}_i, \hat{x}_i), \hat{H}_{nchm}(\hat{p}_i, \hat{x}_i)\right]$, respectively.

2- Ordinary two complex wave functions $\left[\Psi_{yh}(\vec{r}), \Psi_{hm}(\vec{r})\right]$ will be replaced by new two complex wave functions $\left[\hat{\Psi}_{ncyh}(\vec{\hat{r}}), \hat{\Psi}_{nchm}(\vec{\hat{r}})\right]$ respectively.

3-Ordinary two energies $(E_{yh}, E_{hm})$ will be replaced by new two values $(E_{ncyh}, E_{nchm})$, respectively.

And the last step corresponds to replacing the ordinary old product by new star product $(*)$, which allows us to construct the two modified Schrödinger equations in both (*NC-3D: RSP*) as

$$\hat{H}_{ncyh}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}_{ncyh}(\vec{\hat{r}}) = E_{ncyh}\hat{\Psi}_{ncyh}(\vec{\hat{r}}) \tag{16a}$$

$$\hat{H}_{nchm}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}_{nchm}(\vec{\hat{r}}) = E_{nchm}\hat{\Psi}_{nchm}(\vec{\hat{r}}) \tag{16b}$$

The Boopp's shift method allows us to find the two reduced and modified Schrödinger equations without star product as

$$H_{yh}(\hat{p}_i, \hat{x}_i)\Psi_{yh}(\vec{r}) = E_{ncyh}\Psi_{yh}(\vec{r}) \tag{17a}$$

$$H_{hm}(\hat{p}_i, \hat{x}_i)\Psi_{hm}(\vec{r}) = E_{nchm}\Psi_{hm}(\vec{r}) \tag{17b}$$

Where the two modified Hamiltonians $H_{yh}(\hat{p}_i, \hat{x}_i)$ and $H_{hm}(\hat{p}_i, \hat{x}_i)$ for two (*MIQYH*) and (*MIQHM*) potentials defined as a function of the two operators $\hat{x}_i$ and $\hat{p}_i$, which can be expressed as a function of generalized coordinates $x_i(x, y, z)$ and generalized momentums $p_i(p_x, p_y, p_z)$ in the usual quantum mechanics as

$$H_{yh}(\hat{p}_i, \hat{x}_i) = \frac{\hat{p}_i^{\,2}}{2\mu} + V_{yh}(\hat{r}) \tag{18a}$$

$$H_{hm}(\hat{p}_i, \hat{x}_i) = \frac{\hat{p}_i^{\,2}}{2\mu} + V_{hm}(\hat{r}) \tag{18b}$$

Here the two modified $V_{yh}(\hat{r})$ and $V_{hm}(\hat{r})$ potentials are obtained by the following procedure, respectively, as

$$V_{yh}(\hat{r}) = \frac{(b - V_0)}{\hat{r}^2} + \frac{(2V_0 - a - b\delta)}{\hat{r}} + (b - 2V_0)\delta^2 \tag{19a}$$

$$V_{hm}(\hat{r}) = \frac{B + b}{\hat{r}^2} - \frac{a - A + b\delta}{\hat{r}} + \left(C + b\delta^2\right) \tag{19b}$$

Based on our references [29,30,32,51-53], we can write the two operators $\hat{r}^2$ and $\hat{p}^2$ in (*NC-3D: RSP*) as

$$\begin{aligned} \hat{r}^2 &= r^2 - \vec{\mathbf{L}}\vec{\Theta} \\ \frac{\hat{p}^2}{2\mu} &= \frac{p^2}{2\mu} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2\mu} \end{aligned} \tag{20}$$

Where, the two couplings $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}$ are given respectively by

$$\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13} \tag{21}$$

And

$$\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}} \equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13} \tag{22}$$

With, $\Theta \equiv \frac{\boldsymbol{\theta}}{\mathbf{2}}$, after straightforward calculations one can obtains the different terms for (*MIQYH*) and (*MIQHM*) potentials in (*NC-3D: RSP*) as

$$\frac{b-V_0}{\hat{r}^2}=\frac{b-V_0}{r^2}+\frac{(b-V_0)}{r^4}\vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{B+b}{\hat{r}^2}=\frac{B+b}{r^2}+\frac{(B+b)}{r^4}\vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{2V_0-a-b\delta}{\hat{r}}=\frac{2V_0-a-b\delta}{r}+\frac{(2V_0-a-b\delta)}{2r^3}\vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{a-A+b\delta}{\hat{r}}=\frac{a-A+b\delta}{r}+\frac{(a-A+b\delta)}{2r^3}\vec{\mathbf{L}}\vec{\Theta} \quad (23)$$

Which allow us to write the two (*MIQYH*) and (*MIQHM*) global potentials $(V_{myh}(\hat{r}),V_{mhm}(\hat{r}))$ in (NC-3D: RSP) as

$$V_{myh}(\hat{r})=\frac{1}{r^2}(b-V_0)+\frac{1}{r}(2V_0-a-b\delta)+(b-2V_0)\delta^2+V_{pert-yh}(r,\Theta,\overline{\theta}) \quad (24a)$$

$$V_{mhm}(\hat{r})=\frac{B+b}{r^2}-\frac{a-A+b\delta}{r}+(C+b\delta^2)+V_{pert-hm}(r,\Theta,\overline{\theta}) \quad (24b)$$

Where the two additive operators $V_{pert-yh}(r,\Theta,\overline{\theta})$ and $V_{pert-hm}(r,\Theta,\overline{\theta})$ are given by

$$V_{pert-yh}(r,\Theta,\overline{\theta})=\left[\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right]\vec{\mathbf{L}}\vec{\Theta}+\frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2\mu} \quad (25a)$$

$$V_{pert-hm}(r,\Theta,\overline{\theta})=\left[\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right]\vec{\mathbf{L}}\vec{\Theta}+\frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2\mu} \quad (25b)$$

We can observe that the above two operators are proportional to two infinitesimals parameters $\Theta$ and $\overline{\theta}$, which allows us to consider them as a two perturbative terms.

### 3.2. The spin-orbital non-commutative Hamiltonian for modified (*IQYH*) and (*IQHM*) potentials in (NC- 3D: RSP)

In order to discover the new contribution of two perturbative terms $V_{pert-yh}(r,\Theta,\overline{\theta})$ and $V_{pert-hm}(r,\Theta,\overline{\theta})$ for two (*MIQYH*) and (*MIQHM*) potentials, we turn to the case of spin ½ particles described by the (MSE), we make the two simultaneous transformations

$$\vec{\mathbf{L}}\vec{\Theta}\to 2\Theta\overleftrightarrow{S}\overleftrightarrow{L} \quad \text{and} \quad \vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}\to 2\overline{\theta}\overleftrightarrow{S}\overleftrightarrow{L} \quad (26)$$

Then the above two perturbed operators $V_{pert-yh}(r,\Theta,\overline{\theta})$ and $V_{pert-hm}(r,\Theta,\overline{\theta})$ becomes, respectively, as

$$V_{pert-yh}(r,\Theta,\overline{\theta})=2\left[\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}+\frac{1}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}} \quad (27a)$$

$$V_{pert-hm}(r,\Theta,\overline{\theta})=2\left[\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}+\frac{1}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}} \quad (27b)$$

Here $\overleftrightarrow{S}$ denotes the spin of a fermionic particle (like electron). It is possible to replace the spin-orbital interaction $\vec{\mathbf{L}}\vec{S}$ by $G^2=\frac{1}{2}\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$ to obtain directly the corresponding eigenvalues, and then new physical form of Eqns. (27a) and (27b) can be expressed as

$$V_{pert-yh}(r,\Theta,\overline{\theta})=\left[\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}+\frac{1}{2\mu}\right]\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right) \quad (28a)$$

$$V_{pert-hm}(r,\Theta,\overline{\theta})=\left[\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}+\frac{1}{2\mu}\right]\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right) \quad (28b)$$

It is well known that four operators ($\overleftrightarrow{J}^2$, $\overleftrightarrow{L}^2$, $\overleftrightarrow{S}^2$ and $J_z$) form a complete basis in ordinary quantum mechanics, then the operator $\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$ will give two eigen-values $k_{\pm}\equiv\frac{1}{2}\left\{\left(l\pm\frac{1}{2}\right)(l+\frac{1}{2}\pm 1)+l(l+1)-\frac{3}{4}\right\}$, corresponding $j=l\pm\frac{1}{2}$, respectively [29-30,32,51-53]. Then, one can form a two diagonal matrixes $\hat{H}_{so-yh}$ and $\hat{H}_{so-hm}$ of the order $(3\times 3)$, with non null elements: [ $(\hat{H}_{so-yh})_{11}$, $(\hat{H}_{so-yh})_{22}$, $(\hat{H}_{so-yh})_{33}$ ] and [ $(\hat{H}_{so-hm})_{11}$, $(\hat{H}_{so-hm})_{22}$, $(\hat{H}_{so-hm})_{33}$ ] for (*MIQYH*) and (*MIQHM*) potentials in both (NC-3D: RSP) given by

$$\left(\hat{H}_{so-yh}\right)_{11}=k_{+}\alpha\left\{\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}\text{if } j=l+\tfrac{1}{2}\Rightarrow\text{spin up}$$
$$\left(\hat{H}_{so-yh}\right)_{22}=k_{-}\alpha\left\{\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}\text{if } j=l-\tfrac{1}{2}\Rightarrow\text{spin down}$$
$$\left(\hat{H}_{so-yh}\right)_{33}=0 \tag{29a}$$

And

$$\left(\hat{H}_{so-hm}\right)_{11}=k_{+}\alpha\left\{\Theta\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}\text{if } j=l+\tfrac{1}{2}\Rightarrow\text{spin up}$$
$$\left(\hat{H}_{so-hm}\right)_{22}=k_{-}\alpha\left\{\Theta\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}\text{if } j=l-\tfrac{1}{2}\Rightarrow\text{spin down}$$
$$\left(\hat{H}_{so-hm}\right)_{33}=0 \tag{29b}$$

After some calculation, one can show that the two radial functions $R_{yh-nl}(r)$ and $R_{hm-nl}(r)$ for (*MIQYH*) and (*MIQHM*) potentials and two spherical functions $\phi_{ml}(\theta)$ and $\Phi_m(\varphi)$ satisfy the following differential equations, in the new structure (*NC-3D: RSP*), as

$$\frac{d^2R_{yh-nl}(r)}{dr^2}+2\mu\left(\begin{array}{l}E-V_{yh}(r)-\dfrac{\lambda}{2\mu r^2}\\ -2\left[\dfrac{(b-V_0)}{r^4}+\dfrac{(2V_0-a-b\delta)}{2r^3}+\dfrac{1}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}}\end{array}\right)R_{yh-nl}(r)=0 \tag{30a}$$

$$\frac{d^2R_{hm-nl}(r)}{dr^2}+2\mu\left(\begin{array}{l}E-V_{hm}(r)-\dfrac{\lambda}{2\mu r^2}\\ -2\left[\dfrac{(B+b)}{r^4}-\dfrac{(a-A+b\delta)}{2r^3}+\dfrac{1}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}}\end{array}\right)R_{hm-nl}(r)=0 \tag{30b}$$

And

$$\frac{d^2\phi_{ml}(\theta)}{d\theta^2}+\cot(\theta)\frac{d\phi_{ml}(\theta)}{d\theta}\left(\lambda-\frac{m^2}{\sin^2(\theta)}\right)\phi_{ml}(\theta)=0$$
$$\frac{d^2\Phi_m(\varphi)}{d\varphi^2}+m^2\,\frac{d\Phi_m(\varphi)}{d\varphi}=0 \tag{30c}$$

### 3.3. The exact spectrum produced by non-commutative spin-orbit Hamiltonians $\hat{H}_{so-yh}$ and $\hat{H}_{so-hm}$ for (*MIQYH*) and (*MIQHM*) potentials by using the standard perturbation method in (*NC- 3D: RSP*)

The aim of this sub-section is to obtain the modifications of the energy levels for $n^{th}$ excited states $E_{\text{u-yh}}$ and $E_{\text{d-yh}}$ corresponding to a fermionic particle with two polarizations, spin up and spin down, respectively, in the first order of two infinitesimal parameters $\Theta$ and $\bar{\theta}$. In order to achieve this goal, we apply the standard perturbation theory using Eqns. (13a) and (27a) for (*MIQYH*) potential

$$E_{\text{u-yh}}=\frac{1}{2}|N_n|^2k_{+}\int_0^{+\infty}z^{\frac{3}{2}+\left(-1+\sqrt{1+4\gamma_1}\right)}e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\right]^2\left(\Theta\left(\frac{(b-V_0)}{z^2}+\frac{(2V_0-a-b\delta)}{2z^{\frac{3}{2}}}\right)+\frac{\bar{\theta}}{2\mu}\right)dz \tag{31}$$

$$E_{\text{d-yh}}=\frac{1}{2}|N_n|^2k_{-}\int_0^{+\infty}z^{\frac{3}{2}+\left(-1+\sqrt{1+4\gamma_1}\right)}e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\right]^2\left(\Theta\left(\frac{(b-V_0)}{z^2}+\frac{(2V_0-a-b\delta)}{2z^{\frac{3}{2}}}\right)+\frac{\bar{\theta}}{2\mu}\right)dz \tag{32}$$

It is possible to write both $E_{\text{u-yh}}$ and $E_{\text{d-yh}}$ as functions of three terms $T_{yh}^1$, $T_{yh}^2$ and $\bar{T}_{yh}$ as

$$E_{\text{u-yh}}=\frac{1}{2}|N_n|^2k_{+}\left\{\Theta\left(T_{yh}^1+T_{yh}^2\right)+\frac{\bar{\theta}}{2\mu}\bar{T}_{yh}\right\} \tag{33}$$

And

$$E_{\text{d-yh}}=\frac{1}{2}|N_n|^2k_{-}\left\{\Theta\left(T_{yh}^1+T_{yh}^2\right)+\frac{\bar{\theta}}{2\mu}\bar{T}_{yh}\right\} \tag{34}$$

The explicit mathematical forms of three terms $T_{yh}^1$, $T_{yh}^2$ and $\bar{T}_{yh}$ are given by

$$T_{yh}^1=(b-V_0)\int_0^{+\infty}z^{\left[\frac{1}{2}+\left(-1+\sqrt{1+4\gamma_1}\right)\right]-1}e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\right]^2dz,$$
$$T_{yh}^2=\frac{(2V_0-a-b\delta)}{2}\int_0^{+\infty}z^{\left[\left(-1+\sqrt{1+4\gamma_1}\right)+1\right]-1}e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\right]^2dz,$$
$$\bar{T}_{yh}=\int_0^{+\infty}z^{\left[\frac{5}{2}+\left(-1+\sqrt{1+4\gamma_1}\right)\right]-1}e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma_1}}\left(-2\sqrt{\alpha}z\right)\right]^2dz \tag{35}$$

Applying the following special integration [54]

$$\int_0^{+\infty} t^{\alpha-1}\exp(-\delta t)\Gamma_m^{\lambda}(\delta t)\Gamma_n^{\beta}(\delta t)dt=\frac{\delta^{-\alpha}\Gamma(n-\alpha+\beta+1)\Gamma(m+\lambda+1)}{m!n!\Gamma(1-\alpha+\beta)\Gamma(1+\lambda)}$$
$$_3F_2(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1) \tag{36}$$

To obtain the modifications to the energy levels for $n^{th}$ excited states, where $_3F_2(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1)$ denotes the hypergeometric function obtained from $_pF_q(\alpha_1,...,\alpha_p,\beta_1,....,\beta_q,z)$ for $p=3$ and $q=2$. After straightforward calculations, we can obtain explicitly the results as

$$T_{yh}^1=(b-V_0)\frac{\left(2\sqrt{\alpha}\right)^{\sqrt{1+4\gamma_1}-\frac{1}{2}}\Gamma\left(n+\frac{3}{2}\right)\Gamma\left(n+\sqrt{1+4\gamma_1}+1\right)}{(n!)^2\Gamma\left(\frac{3}{2}\right)\Gamma\left(1+\sqrt{1+4\gamma_1}\right)}$$
$$_3F_2\left(-n,\sqrt{1+4\gamma_1}-\frac{1}{2},-\frac{1}{2};\sqrt{1+4\gamma_1}-n-\frac{1}{2},\sqrt{1+4\gamma_1}+1;1\right) \tag{37}$$

And

$$T_{yh}^2=\frac{(2V_0-a-b\delta)}{2}\frac{\left(2\sqrt{\alpha}\right)^{-\sqrt{1+4\gamma_1}}\Gamma(n+2)\Gamma\left(n+\sqrt{1+4\gamma_1}+1\right)}{(n!)^2\Gamma\left(3+2\sqrt{1+4\gamma_1}\right)\Gamma\left(1+\sqrt{1+4\gamma_1}\right)}$$
$$_3F_2\left(-n,\sqrt{1+4\gamma_1},0;\sqrt{1+4\gamma_1}-n,\sqrt{1+4\gamma_1}+1;1\right) \tag{38}$$

$$\overline{T}_{yh}=\frac{\left(2\sqrt{\alpha}\right)^{\sqrt{1+4\gamma_1}+\frac{3}{2}}\Gamma\left(n-\frac{1}{2}\right)\Gamma\left(n+\sqrt{1+4\gamma_1}+1\right)}{(n!)^2\Gamma\left(\frac{1}{2}\right)\Gamma\left(1+\sqrt{1+4\gamma_1}\right)}$$
$$_3F_2\left(-n,\frac{3}{2}+\sqrt{1+4\gamma_1},\frac{3}{2};\frac{3}{2}+\sqrt{1+4\gamma_1}-n,\sqrt{1+4\gamma_1}+1;1\right) \tag{39}$$

Inserting the above obtained expressions (37), (38) and (39) into Eqns. (33) and (34), gives the results for exact modifications of $E_{\text{u-yh}}$ and $E_{\text{d-yh}}$ produced by new spin-orbital effect for (MIQYH) potential as

$$E_{\text{u-yh}}=\frac{1}{2}\left|N_n\right|^2 k_+\left\{\Theta T_{yh}+\frac{\overline{\theta}}{2\mu}\overline{T}_{yh}\right\} \tag{40}$$

$$E_{\text{d-yh}}=\frac{1}{2}\left|N_n\right|^2 k_-\left\{\Theta T_{yh}+\frac{\overline{\theta}}{2\mu}\overline{T}_{yh}\right\} \tag{41}$$

Where the new factor $T_{yh}$ is given by

$$T_{yh}=T_{yh}^1+T_{yh}^2 \tag{42}$$

To obtain the modifications to energy levels for $n^{th}$ excited states $E_{\text{u-hm}}$ and $E_{\text{d-hm}}$ for spin up and spin down, respectively, in the first order of two parameters $\Theta$ and $\overline{\theta}$ produced by spin-orbital influence for (*MIQHM*) potential, it is sufficient to apply the following simultaneous transformations:

$$\begin{aligned}&\gamma_1\rightarrow\gamma_2\\&b-V_0\rightarrow B+b\\&2V_0-a-b\delta\rightarrow-(a-A+b\delta)\end{aligned} \tag{43}$$

Thus, the three previously obtained factors $T_{yh}^1$, $T_{yh}^2$ and $\overline{T}_{yh}$ will be replaced by three new factors $T_{hm}^1$, $T_{hm}^2$ and $\overline{T}_{hm}$, respectively, as

$$T_{hm}^1=(B+b_0)\frac{\left(2\sqrt{\alpha}\right)^{\sqrt{1+4\gamma_2}-\frac{1}{2}}\Gamma\left(n+\frac{3}{2}\right)\Gamma\left(n+\sqrt{1+4\gamma_2}+1\right)}{(n!)^2\Gamma\left(\frac{3}{2}\right)\Gamma\left(1+\sqrt{1+4\gamma_2}\right)}$$
$$_3F_2\left(-n,\sqrt{1+4\gamma_2}-\frac{1}{2},-\frac{1}{2};\sqrt{1+4\gamma_2}-n-\frac{1}{2},\sqrt{1+4\gamma_2}+1;1\right) \tag{44}$$

$$T_{hm}^2=-\frac{(a-A+b\delta)}{2}\frac{\left(2\sqrt{\alpha}\right)^{-\sqrt{1+4\gamma_2}}\Gamma(n+2)\Gamma\left(n+\sqrt{1+4\gamma_2}+1\right)}{(n!)^2\Gamma\left(3+2\sqrt{1+4\gamma_2}\right)\Gamma\left(1+\sqrt{1+4\gamma_2}\right)}$$
$$_3F_2\left(-n,\sqrt{1+4\gamma_2},0;\sqrt{1+4\gamma_1}-n,\sqrt{1+4\gamma_2}+1;1\right) \tag{45}$$

$$\overline{T}_{hm}=\frac{\left(2\sqrt{\alpha}\right)^{\sqrt{1+4\gamma_2}+\frac{3}{2}}\Gamma\left(n-\frac{1}{2}\right)\Gamma\left(n+\sqrt{1+4\gamma_1}+1\right)}{(n!)^2\Gamma\left(\frac{1}{2}\right)\Gamma\left(1+\sqrt{1+4\gamma_2}\right)}$$
$$_3F_2\left(-n,\frac{3}{2}+\sqrt{1+4\gamma_2},\frac{3}{2};\frac{3}{2}+\sqrt{1+4\gamma_2}-n,\sqrt{1+4\gamma_2}+1;1\right) \tag{46}$$

Which allows us to gett the following results for exact modifications of $E_{\text{u-hm}}$ and $E_{\text{d-hm}}$ for a polarized fermionic particle with spin up and spin down, respectively, in the first order of two parameters $\Theta$ and $\overline{\theta}$ for (*MIQHM*) potential, as

$$E_{\text{u-hm}} = \frac{1}{2}\left|N_n\right|^2 k_+ \left\{\Theta T_{hm} + \frac{\overline{\theta}}{2\mu}\overline{T}_{hm}\right\} \qquad (47)$$

$$E_{\text{d-hm}} = \frac{1}{2}\left|N_n\right|^2 k_+ \left\{\Theta T_{km} + \frac{\overline{\theta}}{2\mu}\overline{T}_{hm}\right\} \qquad (48)$$

Where, the factor $T_{hm}$ is given by:

$$T_{hm} = T_{hm}^1 + T_{hm}^2 \qquad (49)$$

### 3.4. The exact spectrum produced by noncommutative magnetic Hamiltonians $\hat{H}_{m-yh}$ and $\hat{H}_{m-hm}$ for modified (*IQYH*) and (*IQHM*) potentials in (*NC-3D: RSP*)

Having found out how to calculate the corrections of energies for the automatically produced spin-orbital, we can discover a second symmetry produced by the effect and the influence of the non-commutativity of phase space, known by modified Zeeman effect for (*MIQYH*) and (*MIQHM*) potentials. To find this physical symmetry, we apply the same strategy as in our previous works [29,30,32,51-53] given as

$$\Theta \to \chi B \quad \text{and} \quad \overline{\theta} \to \overline{\sigma} B \qquad (50)$$

The two parameters $\chi$ and $\overline{\sigma}$ are just only infinitesimal real proportionality constants and $B$ is a uniform external magnetic field, which we orient along $(Oz)$ axis and then we can make the following two translations for (MIQYH) and (MIQHM) potentials

$$\left\{\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\overline{\theta}}{2\mu}\right\}\vec{B}\vec{L} \to B\left(\chi\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\overline{\sigma}}{2\mu}\right)L_z$$
$$\left\{\Theta\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\overline{\theta}}{2\mu}\right\}\vec{B}\vec{L} \to B\left(\chi\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\overline{\sigma}}{2\mu}\right)L_z \qquad (51)$$

Which, allow us to introduce the two modified new magnetic Hamiltonians $\hat{H}_{m-yh}$ and $\hat{H}_{m-hm}$ in (*NC-3D: RSP*) for (*MIQYH*) and (*MIQHM*) potentials, respectively, as

$$\hat{H}_{m-yh} = \left(\chi\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\overline{\sigma}}{2\mu}\right)\left(\vec{B}\vec{J}+\hat{H}_z\right)$$
$$\hat{H}_{m-hm} = \left(\chi\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\overline{\sigma}}{2\mu}\right)\left(\vec{B}\vec{J}+\hat{H}_z\right) \qquad (52)$$

Where, $\hat{H}_z \equiv -\overleftrightarrow{S}\overleftrightarrow{B}$ denote to the ordinary Hamiltonian operator for Zeeman effect in ordinary quantum mechanics. To obtain the exact non-commutative magnetic modifications of energy ($E_{\text{mag-yh}}$, $E_{\text{mag-hm}}$) for modified (*IQYH*) and (*IQHM*) potentials, it is sufficient to replace the 3-parameters: $k_+$, $\Theta$ and $\overline{\theta}$ in Eqns. (33) and (40) by the following new parameters: $m$, $\chi$ and $\overline{\sigma}$, respectively, as

$$E_{\text{mag-yh}} = \frac{1}{2}\left|N_n\right|^2 B\left\{\chi T_{yh} + \frac{\overline{\sigma}}{2\mu}\overline{T}_{yh}\right\}m \qquad (53)$$

$$E_{\text{mag-hm}} = \frac{1}{2}\left|N_n\right|^2 B\left\{\chi T_{hm} + \frac{\overline{\sigma}}{2\mu}\overline{T}_{hm}\right\}m \qquad (54)$$

Where, $m$ denote to the eigen-values of the operator $L_z$, which can take values $-l,-l+1,...,0,...,l$.

## 4. Results

Let us now summerise the global exact spectrum of $n^{th}$ excited states ($E_{\text{ncu-hy}}$, $E_{\text{ncd-yh}}$ and $E_{\text{com-yh}}$) and ($E_{\text{ncu-hm}}$, $E_{\text{ncd-hm}}$ and $E_{\text{com-hm}}$) for (*MIQYH*) and (*MIQHM)* potentials in (*NC-3D: RSP*) produced by the diagonal elements [ $\left(\hat{H}_{nc-yh}\right)_{11}$, $\left(\hat{H}_{nc-yh}\right)_{22}$ and $\left(\hat{H}_{nc-yh}\right)_{33}$ ] and [ $\left(\hat{H}_{nc-hm}\right)_{11}$, $\left(\hat{H}_{nc-hm}\right)_{22}$ and $\left(\hat{H}_{nc-hm}\right)_{33}$ ] of non-commutative Hamiltonians operator $\hat{H}_{nc-yh}$ and $\hat{H}_{nc-hm}$. The original two eigen-values ($E_{yh}$, $E_{hm}$) in ordinary three-dimensional spaces for (*IQYH*) and (*IQHM*) potentials and the obtained results (34), (35), (40), (41), (47), (48), (53), (54) allow us to getting the following global results as

$$E_{\text{ncu-hy}} = (2V_0-b)\delta^2 - \frac{\mu(2V_0-a-b\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(b-V_0)+(l+1/2)^2}\right)^2}$$
$$+\frac{\left|N_n\right|^2}{2}k_+\left\{\Theta T_{yh}+\frac{\overline{\theta}}{2\mu}\overline{T}_{yh}\right\}+\frac{1}{2}\left|N_n\right|^2 B\left\{\chi T_{yh}+\frac{\overline{\sigma}}{2\mu}\overline{T}_{yh}\right\}m \qquad (55)$$

$$E_{\text{ncd-yh}}=(2V_0-b)\delta^2-\frac{\mu(2V_0-a-b\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(b-V_0)+(l+1/2)^2}\right)^2}$$
$$+\frac{|N_n|^2}{2}k_-\left\{\Theta T_{yh}+\frac{\bar{\theta}}{2\mu}\bar{T}_{yh}\right\}+\frac{1}{2}|N_n|^2B\left\{\chi T_{yh}+\frac{\bar{\sigma}}{2\mu}\bar{T}_{yh}\right\}m \tag{56}$$

$$E_{\text{com-yh}}=(2V_0-b)\delta^2-\frac{\mu(2V_0-a-b\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(b-V_0)+(l+1/2)^2}\right)^2} \tag{57}$$

And

$$E_{\text{ncu-hm}}=C+\delta b^2-\frac{\mu(a-A\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(B+b)+(l+1/2)^2}\right)^2}$$
$$+\frac{|N_n|^2}{2}k_+\left\{\Theta T_{hm}+\frac{\bar{\theta}}{2\mu}\bar{T}_{hm}\right\}+\frac{1}{2}|N_n|^2B\left\{\chi T_{hm}+\frac{\bar{\sigma}}{2\mu}\bar{T}_{hm}\right\}m \tag{58}$$

$$E_{\text{ncd-hm}}=C+\delta b^2-\frac{\mu(a-A\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(B+b)+(l+1/2)^2}\right)^2}$$
$$+\frac{|N_n|^2}{2}B\left\{\chi T_{hm}+\frac{\bar{\sigma}}{2\mu}\bar{T}_{hm}\right\}m\frac{1}{2}|N_n|^2k_-\left\{\Theta T_{hm}+\frac{\bar{\theta}}{2\mu}\bar{T}_{hm}\right\} \tag{59}$$

$$E_{\text{com-hm}}=C+\delta b^2-\frac{\mu(a-A\delta)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu(B+b)+(l+1/2)^2}\right)^2} \tag{60}$$

The explicit diagonal elements [ $(\hat{H}_{nc-yh})_{11}$ , $(\hat{H}_{nc-yh})_{22}$ , $(\hat{H}_{nc-yh})_{33}$ ] and [ $(\hat{H}_{nc-hm})_{11}$ , $(\hat{H}_{nc-hm})_{22}$ , $(\hat{H}_{nc-hm})_{33}$ ] of two operators $\hat{H}_{nc-yh}$ and $(\hat{H}_{nc-hm})$ for (MIQYH) and (MIQHM) potentials in (*NC-3D: RSP*) can be deduced as

$$(\hat{H}_{nc-yh})_{11}=-\frac{\Delta}{2\mu}+\frac{(b-V_0)}{r^2}+\frac{(2V_0-a-b\delta)}{r}+(b-2V_0)\delta^2+$$
$$+k_+\left\{\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}+\left(\chi\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)BL_z$$
$$\text{if } j=l+\tfrac{1}{2}\Rightarrow \text{spin-up} \tag{61}$$

$$(\hat{H}_{nc-yh})_{22}=-\frac{\Delta}{2\mu}+\frac{(b-V_0)}{r^2}+\frac{(2V_0-a-b\delta)}{r}+(b-2V_0)\delta^2+$$
$$+k_-\left\{\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}+\left(\Theta\left(\frac{(b-V_0)}{r^4}+\frac{(2V_0-a-b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right)BL_z$$
$$\text{if } j=l-\tfrac{1}{2}\Rightarrow \text{spin-down} \tag{62}$$

$$(\hat{H}_{nc-yh})_{33}=-\frac{\Delta}{2\mu}+\frac{(b-V_0)}{r^2}+\frac{(2V_0-a-b\delta)}{r}+(b-2V_0)\delta^2 \tag{63}$$

And

$$(\hat{H}_{nc-hm})_{11}=-\frac{\Delta}{2\mu}+\frac{B+b}{r^2}-\frac{a-A+b\delta}{r}+(C+b\delta^2)+$$
$$+k_+\left\{\Theta\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}++B\left(\chi\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)L_z$$
$$\text{if } j=l-\tfrac{1}{2}\Rightarrow \text{spin-down} \tag{64}$$

$$(\hat{H}_{nc-hm})_{22}=-\frac{\Delta}{2\mu}+\frac{B+b}{r^2}-\frac{a-A+b\delta}{r}+(C+b\delta^2)+$$
$$+k_-\left\{\Theta\chi\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}+B\left(\chi\left(\frac{(B+b)}{r^4}-\frac{(a-A+b\delta)}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)L_z$$
$$\text{if } j=l-\tfrac{1}{2}\Rightarrow \text{spin-down} \tag{65}$$

$$(\hat{H}_{nc-hm})_{33}=-\frac{\Delta}{2\mu}+\frac{B+b}{r^2}-\frac{a-A+b\delta}{r}+(C+b\delta^2) \tag{66}$$

It is well known that the atomic quantum number $m$ can be takes ( $2l+1$ ) values and we have also two possible values for eigen-values $j=l\pm\frac{1}{2}$ , thus every state in usually three-dimensional space for (*MIQYH*) and (*MIQHM*) potentials will be replaced in (*NC-3D: RSP*) by $2(2l+1)$ sub-states and then the degenerated state can be take $2\sum_{i=0}^{n-1}(2l+1)\equiv 2n^2$ values. It is important to notice that our recent study can be extended to molecular case with spin $s\neq\frac{1}{2}$ , we replace the factors $k_\pm\equiv\frac{1}{2}\left\{\left(l\pm\frac{1}{2}\right)(l+\frac{1}{2}\pm 1)+l(l+1)-\frac{3}{4}\right\}$ by new factor $k(j,l,s)\equiv\frac{1}{2}\{j(j+1)+l(l+1)-s(s+1)\}$ with $|l-s|\leq j\leq|l+s|$ , which allow us to obtain the modifications to the energy levels $\left(E_{\text{nc-hy}}, E_{\text{nc-hm}}\right)$ for (*MIQYH*) and (*MIQHM*) potentials

$$E_{\text{nc-hy}} = (2V_0 - b)\delta^2 - \frac{\mu(2V_0 - a - b\delta)^2 / 2}{\left(n + \frac{1}{2} + \sqrt{2\mu(b - V_0) + (l + 1/2)^2}\right)^2} + \\ + \frac{|N_n|^2}{2} k(j,l,s)\left\{\Theta T_{yh} + \frac{\bar{\theta}}{2\mu}\bar{T}_{yh}\right\} + \frac{1}{2}|N_n|^2 B\left\{\chi T_{yh} + \frac{\bar{\sigma}}{2\mu}\bar{T}_{yh}\right\} m \tag{67}$$

And

$$E_{\text{nc-hm}} = C + \delta b^2 - \frac{\mu(a - A\delta)^2 / 2}{\left(n + \frac{1}{2} + \sqrt{2\mu(B + b) + (l + 1/2)^2}\right)^2} + \\ + \frac{|N_n|^2}{2} k(j,l,s)\left\{\Theta T_{hm} + \frac{\bar{\theta}}{2\mu}\bar{T}_{hm}\right\} + \frac{1}{2}|N_n|^2 B\left\{\chi T_{hm} + \frac{\bar{\sigma}}{2\mu}\bar{T}_{hm}\right\} m \tag{68}$$

And the corresponding non-commutative two Hamiltonian operators $\hat{H}_{nc-yh}$ and $\hat{H}_{nc-hm}$ can be fixed by the following results:

$$\hat{H}_{nc-yh} = -\frac{1}{2\mu}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial\varphi^2}\right) \\ + \frac{(b - V_0)}{r^2} + \frac{(2V_0 - a - b\delta)}{r} + (b - 2V_0)\delta^2 + \left\{\Theta\left(\frac{(2b - 2V_0)}{r^4} + \frac{(4V_0 - 2a - 2b\delta)}{2r^3}\right) + \frac{\bar{\theta}}{\mu}\right\}\vec{\mathbf{L}}\vec{\mathbf{S}} \\ + \left(\chi\left(\frac{(b - V_0)}{r^4} + \frac{(2V_0 - a - b\delta)}{2r^3}\right) + \frac{\bar{\sigma}}{2\mu}\right) BL_z \tag{69}$$

$$\hat{H}_{nc-hm} = -\frac{1}{2\mu}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial\varphi^2}\right) + \\ + \frac{B + b}{r^2} - \frac{a - A + b\delta}{r} + \left(C + b\delta^2\right) + \left\{\Theta\left(2\frac{(B + b)}{r^4} - \frac{(2a - 2A + 2b\delta)}{r^3}\right) + \frac{\bar{\theta}}{2\mu}\right\}\vec{\mathbf{L}}\vec{\mathbf{S}} \\ + B\left(\chi\left(\frac{(B + b)}{r^4} - \frac{(a - A + b\delta)}{2r^3}\right) + \frac{\bar{\sigma}}{2\mu}\right) L_z \tag{70}$$

It is important to notice that the appearance of the polarization states of a fermionic particle for (*MIQYH*) and (*MIQHM*) potentials indicates the validity of obtained results at very high energy, where the two relativistic equations Klein-Gordon and Dirac will be applied, which allow us to apply these results to various nano-particles at nano-scales. Finally, if we take the two limits $\left(\theta, \bar{\theta}\right) \to (0,0)$ simultaneously, we obtain all results of ordinary quantum mechanics.

## 5. Concluding Remarks

In this article, the new energy eigenvalues of Schrodinger equations for (*MIQYH*) and (*MIQHM*) potentials are successfully investigated by applying the generalization of Boopp's' shift method and standard perturbation theory in (*NC-3D: RSP*). We showed that the obtained degenerated spectrum depended on new discrete atomic quantum numbers ($m$, $j = l \pm \frac{1}{2}$ and $s_z = \pm\frac{1}{2}$) and the validity of obtained corrections can be prolonged to nano-particles at nano and Plank's scales. In addition, we recover the ordinary commutative spectrums when, we make the two simultaneously limits: $\left(\theta, \bar{\theta}\right) \to (0,0)$ for (*MIQYH*) and (*MIQHM*) potentials.

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