Higher Dimensional LRS Bianchi type - I Cosmological Model Universe Interacting with Perfect Fluid in Lyra Geometry

Kangujam Priyokumar Singh[1] and Mahbubur Rahman Mollah[2,*]
[1] *Department of Mathematical Sciences, Bodoland University, Kokrajhar.BTC, Assam-783370,*
[2] *Department of Mathematics,Commerce College, Kokrajhar.BTC, Assam-783370,*

Considering the LRS Bianchi type I metric we study here a perfect fluid cosmological model in the context of Layra's geometry by using constant deceleration parameter. Exact solutions of field equations are obtained with certain physical assumptions in different cases. The physical behavior of the model is also discussed in details.

1. Introduction

Einstein developed his general theory of relativity, where gravitation is described in terms of geometry. Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the Universe, because without the cosmological term his field equations admit only non-static cosmological models for non-zero energy density. Later, Weyl [1] proposed a more general theory in which electromagnetism is also described geometrically. He showed how one can introduce a vector field in the Riemannian space-time with an intrinsic geometrical significance. But this theory was based on non-integrability of length transfer so that it had some unsatisfactory features, and hence this theory which is known as Weyl's geometry still today did not gain general acceptance. After having these concepts, Lyra [2] suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structure-less manifold, which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. Halford [3,4] pointed out that in the normal general relativistic treatment the constant displacement vector field ϕ_i in Lyra's geometry plays the role of cosmological constant and the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits, as far as the classical solar system test are concerned (as in the Einstein's theory of relativity).

* mr.mollah123@gmail.com

In the past and recent years many prominent researchers like [5-27] have investigated and proposed different cosmological models and ideas of the Universe within the framework of Lyra's geometry and other theories of relativity in different context. But the main problem in Astrophysics is the discovery, about two decades ago, that our Universe expansion is accelerating, instead of slowing down as predicted by the Big Bang theory [28]. Observational evidence for accelerated expansion in the Universe has been growing during this period [29,30,31]. Independent confirmation using observations of high red shift supernovae [32-37] along with observations of cosmic microwave background radiation (CMB) [38-40] and large scale structure [41] have made this result more acceptable to the community. In fact, the recent observations of Type SNeIa supernova, CMB anisotropies the large scale galaxies structures of universe and Sachs Wolf effects have led to the idea that our universe undergoes accelerated expansion at the present epoch tending to a de-Sitter space-time as predicted by inflation theory [42-46].

Moreover, solutions of Einstein field equations in higher dimensional space times are believed to be of physical relevance possibly at extremely early times before the Universe underwent the compactification transitions. As a result, now the higher dimensional theory is receiving great attention in both cosmology and particle physics. Particle physicists and cosmologists predicted the existence of GUT (Grand Unified Theory). Using a suitable scalar field it was shown that the phase transitions on the early universe can give rise to such objects which are nothing but the topological knots in the vacuum expectation value of the scalar field and most of their energy is concentrated in a small region. As the necessity to study higher-dimensional space-time in this field aiming to unify gravity with other interactions the concept of extra dimension is relevant in cosmology. In particular, for the early stage of the Universe and theoretically the present four dimensional stage of the Universe might have been preceded by a multi-dimensional stage.

So, in this paper we discuss the higher dimensional cosmological models in Lyra geometry by considering locally rotationally symmetric (LRS) Bianchi Type-I metric with the use of deceleration parameter and certain physical assumption to find out the solutions compatible with the observational facts.

2. Field Equations and Their Solutions

Here we consider the five dimensional plane symmetric metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2(dy^2 + dz^2) + C^2 dm^2 \tag{1}$$

with the convention $x^1 = x,\ x^2 = y,\ x^3 = y,\ x^4 = m,\ x^5 = t$ where *A*, *B* and *C* are functions of time 't' only. Here the extra coordinate is taken to be time-like.

Einstein's field equations based on Lyra's Geometry as used by [47] and [48] is

$$R_{ij} - \tfrac{1}{2} g_{ij} R + \tfrac{3}{2}\phi_i\phi_j - \tfrac{3}{4} g_{ij}\phi^k\phi_k = -8\pi T_{ij} \tag{2}$$

Where, ϕ_i is the displacement vector given by

$$\phi_i = (0,0,0,0,\beta(t)) \tag{3}$$

and T_{ij} is the energy momentum tensor for the perfect fluid given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{4}$$

Where, ρ is the energy density, p is the pressure and u^i is the five velocity vector given by

$$u^i = (0,0,0,0,\tfrac{1}{A}) \tag{5}$$

Also let, $x^i = (\frac{1}{A},0,0,0,0)$ so that

$$g_{ij}u^i u^j = -1 = -x^i x_i\ ,\ and\ \ u^i x_i = 0 \tag{6}$$

In co-moving coordinate system, we have from Eqn. (4)

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p\ ;\quad T_5^5 = \rho$$

$$T_j^i = 0 \quad \text{for all}\ \ i \neq j \tag{7}$$

Using Eqns. (3)-(7), the surviving field equations of Eqn. (2) for the metric in Eqn. (1) are obtained as

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} + \frac{3}{4}\beta^2 = A^2 p \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = A^2 p \tag{9}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{3}{4}\beta^2 = A^2 p \tag{10}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{4}\beta^2 = -\rho A^2 \tag{11}$$

Now from Eqns. (9) and (10) we have

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_1}{B^2 C} \tag{12}$$

Where, $k_1 > 0$ is an integrating constant.

Since the field equations (8)-(11) are highly non-linear, so in order to obtain the exact solution of Eqns. (8)-(11), we use the following scale transformations as used by [49].

$$A = e^a\ ,\qquad B = e^b\ ,\qquad C = e^c$$
$$\text{and}\quad dt = AB^2 C dT \tag{13}$$

Using transformations of Eqn. (13) in Eqns. (8)-(11) we have

$$2b^{//} + c^{//} - 4a^/ b^/ - 2b^/ c^/ - 2c^/ a^/ - {b^/}^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\beta^2 e^{2(a+2b+c)} \tag{14}$$

$$a^{//} + b^{//} + c^{//} - 3a^/ b^/ - 2b^/ c^/ - 2c^/ a^/ - {a^/}^2 - {b^/}^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\beta^2 e^{2(a+2b+c)} \tag{15}$$

$$a^{//} + 2b^{//} - 4a^/ b^/ - 2b^/ c^/ - c^/ a^/ - {a^/}^2 - {b^/}^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\beta^2 e^{2(a+2b+c)} \tag{16}$$

$$2a^{/}b^{/} + 2b^{/}c^{/} + c^{/}a^{/} + b^{/^2} = -\rho e^{2(2a+2b+c)} + \frac{3}{4}\beta^2 e^{2(a+2b+c)} \tag{17}$$

Where, dashes denote derivative with respect to time 'T'

Solving Eqns. (14)-(16) we have

$$a = b = c \tag{18}$$

Therefore, from Eqn. (13) we have

$$A = B = C \tag{19}$$

By using Eqn. (19) in Eqns. (8)-(11) we have

$$3\frac{\ddot{A}}{A} + \frac{3}{4}\beta^2 = A^2 p \tag{20}$$

$$6\frac{\dot{A}^2}{A^2} - \frac{3}{4}\beta^2 = -A^2\rho \tag{21}$$

There are two independent equations involving four unknowns A, β, p and ρ. So, in order to get deterministic solutions of the above set of highly nonlinear Eqns. (20)-(21), we shall use the special law of variation of Hubble's parameter proposed by Bermann [13] that gives constant deceleration parameter as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{22}$$

Where, 'q' is a constant and

$$R = (A^2B^2C)^{\frac{1}{4}} \tag{23}$$

is the overall scale factor.

Here the constant 'q' is taken as negative, so the model is an accelerating model of the Universe.

Solving Eqn. (22) we have

$$R = (\alpha t + \gamma)^{\frac{1}{q+1}} \tag{24}$$

Where, $\alpha \neq 0$ and γ are constants and $q + 1 \neq 0$.

By using Eqn. (19) in Eqn.(24) we have

$$A = R^{\frac{4}{5}} \tag{25}$$

Therefore, from Eqns. (19), (24) and (25) we have

$$A = B = C = R^{\frac{4}{5}} = (\alpha t + \gamma)^{\frac{4}{5(q+1)}} \tag{26}$$

We shall now consider two cases described below.

Case-I: When β is a constant

From Eqns. (20) and (21) we have

$$p = \frac{3}{4}\frac{\beta^2}{(\alpha t + \gamma)^{\frac{8}{5(q+1)}}} - \frac{12\alpha^2(5q+1)}{25(q+1)^2(\alpha t + \gamma)^{\frac{8}{5(q+1)}+2}} \tag{27}$$

$$\rho = \frac{3}{4}\frac{\beta^2}{(\alpha t + \gamma)^{\frac{8}{5(q+1)}}} - \frac{96\alpha^2}{25(q+1)^2(\alpha t + \gamma)^{\frac{8}{5(q+1)}+2}} \tag{28}$$

Here, the integrating constant α and are to be chosen in such a way that ρ and p are non-negative.

Now, the metric in Eqn. (1) using Eqn. (28) can be written as

$$ds^2 = (\alpha t + \gamma)^{\frac{8}{5(q+1)}}\{dx^2 + dy^2 + dz^2 + dm^2 - dt^2\} \tag{29}$$

The above equation (29) together with Eqns. (27) and (28) will be the exact 5-D LRS Bianchi type-I perfect fluid cosmological model in Lyra Geometry when β is a constant.

Now, if we take $q = \beta = 0$ then we have from Eqns. (27) and (28) we have

$$p = -\frac{12\alpha^2}{25(\alpha t + \gamma)^{\frac{18}{5}}}$$
$$\rho = -\frac{96\alpha^2}{25(\alpha t + \gamma)^{\frac{18}{5}}}$$

Since both ρ and the p are negative so from the above two equations we have

$$\rho = 8p$$

Which satisfies the general equation of state: $p = \delta\rho$.

Case-II: When β is a function of *t*

There are two independent field equations, Eqns. (20) and (21), involving three unknowns ρ , p and β. So, in order to get deterministic solution we must have to assume a physical or mathematical condition amongst the unknowns. Here we consider the equation of state (i.e., physical condition) as

$$p = \delta\rho \tag{30}$$

Case II-(a): Dust (or, in coherent matter) distribution [δ=0, i.e., $p = 0$ and $\rho \neq 0$]
When $\delta = 0$, then from Eqn. (30) we have

$$p=0 \tag{31}$$

Putting $p = 0$ in Eqn. (20), we have

$$\frac{3}{4}\beta^2 = \frac{12\alpha^2(5q+1)}{25(q+1)^2(\alpha t+\gamma)^2} \tag{32}$$

Using Eqns. (32) in (21) we have

$$\rho = \frac{12\alpha^2(5q-7)}{25(q+1)^2(\alpha t+\gamma)^{\frac{8}{5(q+1)}+2}} \tag{33}$$

The above equation (29) together with equations (31)-(33) will constitute an exact 5-D LRS Bianchi type-I coherent matter distribution model universe in Lyra geometry.

Case II-(b): Stiff (or, Zel'dovich) fluid distribution [$\delta = 1$]
When δ=1 then from Eqn. (30) we have

$$p = \rho \tag{34}$$

When δ =1 i.e., when p=ρ then we can see that it is not possible to find out a physically meaningful solution for the field equations.

Therefore, when β is a function of time t then Bianchi type-I cosmological stiff fluid universe does not exist in this theory.

Case II-(c): Disordered distribution of Radiation (or, Radiation Universe) [$\delta = \frac{1}{3}$]
When $\delta = \frac{1}{3}$ then from Eqn. (30) we have

$$\rho = 3p \tag{35}$$

Using ρ = 3p in Eqns. (20) and (21) we have

$$p = \frac{6\alpha^2(5q-7)}{25(q+1)^2(\alpha t+\gamma)^{\frac{8}{5(q+1)}+2}} \tag{36}$$

$$\rho = \frac{18\alpha^2(5q-7)}{25(q+1)^2(\alpha t+\gamma)^{\frac{8}{5(q+1)}+2}} \tag{37}$$

Therefore, from Eqn. (20) (or (21)) we have

$$\frac{3}{4}\beta^2 = \frac{6\alpha^2(3q-1)}{5(q+1)^2(\alpha t+\gamma)^2} \tag{38}$$

Eqn. (29) together with Eqns. (35)-(38) will constitute an exact 5-D LRS Bianchi type-I radiating model universe in Lyra geometry.

Case II-(d): Matter distribution in inter-nebular space [$\delta = \frac{2}{3}$]
When, $\delta = \frac{2}{3}$, then from Eqn. (30) we have

$$\rho = \frac{3}{2}p \tag{39}$$

Using $\rho = \frac{3}{2}p$ we have from Eqns. (20) and (21)

$$p = \frac{24\alpha^2(5q-7)}{25(q+1)^2(\alpha t+\gamma)^{\frac{8}{5(q+1)}+2}} \tag{40}$$

$$\rho = \frac{36\alpha^2(5q-7)}{25(q+1)^2(\alpha t+\gamma)^{\frac{8}{5(q+1)}+2}} \tag{41}$$

Therefore from Eqn. (20) (or (21)) we have

$$\frac{3}{4}\beta^2 = \frac{12\alpha^2(15q-13)}{25(q+1)^2(\alpha t+\gamma)^2} \tag{42}$$

Eqn. (29) together with Eqns. (39)-(42) will constitute an exact 5-D LRS Bianchi type-I cosmological model universe in the matter distribution in inter-nebular space in Lyra geometry.

In all the cases, II-(a)-(d) the reality condition $\rho > 0$ is obtained as

$$q > \frac{7}{5} \tag{43}$$

Now in all the above four cases we see that the value of the deceleration parameter $q > 0$ i.e., our model is an accelerating one.

3. Physical and Geometrical Properties of the Solutions

Here, the spatial volume V and the average scale factor $R(t)$ for the Bianchi type-I plane symmetric metric (Eqn. (1)) defined by $V = R^4(t) = (-g)^{\frac{1}{2}} = A^2B^2C = A^5$ of the model are given by

$$V = (\alpha t + \gamma)^{\frac{4}{q+1}} \tag{44}$$

and

$$R(t) = (\alpha t + \gamma)^{\frac{1}{q+1}} \tag{45}$$

We observed that the volume V is increasing with the increase of time if $q+1 > 0$ i.e., if $q > -1$ and the volume V is decreasing with the increase of time and tend to zero as $t \to \infty$ if $q+1 < 0$ i.e., if $q < -1$. Also the scale factor R is increasing with the increase of time if $q+1 > 0$ i.e., if $q > -1$ and the scale factor R is decreasing with the increase of time and tend to zero as $t \to \infty$ if $q+1 < 0$ i.e., if $q < -1$

Also, the mean Hubble's parameter H is obtained as

$$H = \frac{\alpha}{(q+1)(\alpha t+\gamma)} \tag{46}$$

From the above Eqn. (46) it has been observed that in the initial stage, when $t = 0$ or $t= \frac{\alpha}{(q+1)\gamma}$. Again, the value of H decreases with the increase of time t and finally H becomes zero whenever $t \to \infty$. Also the Hubble's parameter H becomes infinite whenever $q = -1$ or $t = -\frac{\gamma}{\alpha}$.

The expansion factor Θ calculated for the flow vector u^i is given by

$$\Theta = u^i_{;i} = \frac{1}{A}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 4\frac{\dot{A}}{A^2} = \frac{16\alpha}{5(q+1)(\alpha t+\gamma)^{\frac{4}{5(q+1)}+1}} \tag{47}$$

The model has a singularity at $t = -\frac{\gamma}{\alpha}$ and the scalar expansion $\Theta \to 0$ as time $t \to \infty$ if $q > -\frac{1}{5}$.

The components of the shear scalar σ for the metric in Eqn. (1) are given by

$$\sigma^1_1 = \frac{1}{A}\left(\frac{\dot{A}}{A} - \frac{A\Theta}{4}\right)$$

$$\sigma^2_2 = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{A\Theta}{4}\right)$$

$$\sigma^3_3 = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{A\Theta}{4}\right)$$

$$\sigma^4_4 = \frac{1}{A}\left(\frac{\dot{C}}{C} - \frac{A\Theta}{4}\right)$$

$$\sigma^5_5 = 0$$

Therefore, the shear scalar σ for the metric in Eqn. (1) is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}[(\sigma^1_1)^2 + (\sigma^2_2)^2 + (\sigma^3_3)^2 + (\sigma^4_4)^2 + (\sigma^5_5)^2] = 0 \tag{48}$$

Since $\sigma^2 = 0$ so our model universe is shear free. Also since $\frac{\sigma}{\Theta} = 0$ for all values of 't, so our model universe is always an isotropic one.

4. Conclusion

In this work, we have considered a LRS Bianchi type I cosmological model universe interacting with perfect fluid in the context of Layra's geometry by using constant deceleration parameter. We have discussed different distributions like dust, stiff fluid, disordered distribution and Matter distribution in inter-nabular space and it is observed that our model universe is always an isotropic one.

Acknowledgements

The authors are very thankful to honorable referees for their valuable comments and suggestions for the improvement of this paper. We are also thankful to the UGC, India for funding under the sanction Order No. F. 5-332/2014-15/MRP/NERO/2386 to carry out this work successfully.

References

[1] H. Weyl, Sitzungsberichte Der Preussischen Academie Der Wissenschaften, Sitzungsberichte, zu Berlin (1918) p.465.

[2] G. Lyra, Mathematische Zeitschrift, **54**, No.1, 52 (1951); doi:10.1007/BF01175135.

[3] W. D. Halford, Australian J. Phys. **23**, 863 (1970).

[4] W. D. Halford, J. Math. Phys. **13**, 1699 (1972).

[5] M. S. Berman and F. M. Gomide, Gen. Rel. Grav. **20**, No.2 (1988).

[6] V. B. Johri and K. Desikan, Gen. Rel. Grav. **26**, No.12 (1994).

[7] A. Beesham, Gen. Rel. Grav. **25**, No.6 (1993).

[8] D. R. K. Reddy, M. V. S. Rao and G. K. Rao, Astrophys Space Sci. **306**, 171 (2006); DOI 10.1007/s10509-006-9210-0.

[9] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, Int. J. Theor. Phys. **46**, No.6, June (2007); DOI 10.1007/s10773-006-9283-0.

[10] D. R. K. Reddy, R. L. Naidu and K. S. Adhav, Astrophys Space Sci. **307**, 365 (2007); DOI 10.1007/s10509-006-9282-x.

[11] K. S. Adhav, A. S. Nimkar, M. R. Ugale and M. V. Dawande, Int. J. Theor. Phys. **47**, 634 (2008); DOI 10.1007/s10773-007-9487-y.

[12] V. U. M. Rao and G. S. D. Kumari, Int. J. Theor. Phys. **51**, 311 (2012); DOI 10.1007/s10773-011-0909-5.

[13] M. S. Bermann, Nuovo Cimento Series 11, **74B**, 1826, Italy (1983).

[14] T. Singh and K. Agarwal, Nuovo Cimento **108B**, No.5, 541 (1993).

[15] F. Rahaman, N. Begum, G. Bag and B. C. Bhui, Astrophys. Space Sci. **299**, 211 (2005).

[16] V. U. M. Rao, T. Vinutha and M. V. Santhi, Astrophys Space Sci. **314**, 213 (2008); DOI 10.1007/s10509-008-9757-z.

[17] K. S. Adhav, Eur. Phys. J. Plus **126**, 122 (2011); DOI 10.1140/epjp/i2011-11122-9.

[18] R. Zia and R. P. Singh, Romanian J. Phys. **57**, Nos.3-4, 761, Bucharest (2012).

[19] M. R. Mollah, K. P. Singh and K. M. Singh, Int. J. Astron. & Astrophys. **5**, 90 (2015), http://www.scirp.org/journal/ijaa http://dx.doi.org/10.4236/ijaa.2015.52012.

[20] F. Darabi, Astrophys. Space Sci. **343**, 499 (2013); DOI 10.1007/s10509-012-1250-z.

[21] U. K. Panigrahi and B. Nayak, Int. J. Mathematical Archive 5(5), 123 May (2014), ISSN 2229-5046, 2014.

[22] K. P. Singh, Prespacetime J. **6**, No.10, 984 (2015).

[23] A. Asgar and M. Ansary, J. Theor. and Applied Physics **8**, 219 (2014); DOI: 10.1007/s40094-014-0151-7.

[24] A. Asgar and M. Ansari, The African Review of Physics **9**, 145 (2014).

[25] P. K. Sahoo and B. Mishra, Turk. J. Phys. **39**, 43 (2015); DOI:10.3906/fiz-1403-5.

[26] K. S. Adhav, P. R. Agarwal and M. A. Purandare, The African Review of Physics, **10**, 65 (2015).

[27] V. U. M. Rao, V. Jayasudha and D. R. K. Reddy, Prespacetime Journal, **6**, Issue 9, 787, September (2015).

[28] J. Silk, *The Big Bang* (Freeman and Company, New York, 1989).

[29] J. P. Ostriker and P. J. Steinhardt, Nature **377**, 600 (1995).

[30] J. S. Bangla, T. Padmanabhan and J. V. Narlikar, Comments on Astrophys. **18**, 275 (1996).

[31] G. Efstathiou1, W. J. Sutherland and S. J. Maddox, Nature **348**, 705 (1990).

[32] P. M. Garnavich et al., Astrophys. J. **509**, 74 (1998).

[33] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999).

[34] J. L. Tonry et al., Astrophys. J. **594**, 1 (2003).

[35] B. J. Barris et al., Astrophys. J. **602**, 571 (2004).

[36] A. G. Riess et al., Astrophys. J. **607**, 665 (2004).

[37] P. Astier et al., astro-ph/0510447 (2005).

[38] A. Melchiorri et al. [Boomerang Collaboration], Astrophys. J. Suppl. **536**, L63 (2000).

[39] D. N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003).

[40] E. Komatsu et al., Astrophys. J. Suppl. **180**, 330 (2009).

[41] W. J. Percival et al., Astrophys. J. **657**, 645 (2007).

[42] A. G. Riess et al., Astron. J. **116**, 1009 (1998).

[43] B. R. Schmidt et al., Astrophys. J. **507**, 46 (1998).

[44] P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).

[45] M. Persic, P. Salucci and F. Stel, Mon. Not. R. Astron. Soc. **281**, 27 (1996).

[46] J. V. Cunha, Phys. Rev. D **79**, 047301 (2009).

[47] D. K. Sen, Z. Phys. **149**, 311 (1957).

[48] D.K. Sen and K. A. Dunn, J. Maths Phys. **12**, 578 (1971).

[49] D. R. K. Reddy and R. Venkateswarlu, Astrophys. Space Sci. **154**, 115 (1989).

Received: 30 October, 2015
Accepted: 23 March, 2016