

Higher Dimensional FRW String Cosmological Model with Bulk Viscous Quark and Strange Quark Matter in Lyra Manifold

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We investigate higher dimensional Friedmann-Robertson-Walker (FRW) universe for a cloud of string with perfect fluid attached with quark and strange quark matter in Lyra geometry. The Einstein field equation is solved using time dependent deceleration parameter. The average scale factor is taken as an analogous to gamma function $a(t) = e^{-t} t^{m-1}$, which always give a time-dependent deceleration parameter. It is further assumed that the displacement vector is also a function of time. The behavior of flat, open and closed models is presented and discussed under various scenarios.

1. Introduction

At very early stages of evolution of the Universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the Universe is broken spontaneously. This can give rise to topologically stable defects such as domain walls, strings and monopoles [1]. Recent observations of type Ia-supernovae showed that our universe is accelerating [2,3]. Scientists have proposed two possible approaches to clarify this acceleration of the Universe. One of these considerations is dark energy that is dominating the Universe and has a negative proportion between its own pressure and density [4]. The other consideration is alternative gravitational theories [5]. Alternative gravitational theories can be categorized as bi-metric theories, scalar-tensor theories, scalar theories and vector-tensor theories [6,7]. The most known theories are the Weyl theory [8], $f(R)$ theory [9], Brans-Dicke theory [10] and Lyra geometry [11]. Weyl's theory is a modification of the Riemannian manifold in order to combine electromagnetism and gravitation [12,13]. Lyra [11] using the Weyl geometry suggested a modification of the Riemannian geometry [14]. Sen [15] and Sen-Dunn [16] established a new scalar-tensor theory of the Einstein field equation based on Lyra manifold. Also, Halford [17] showed that in Lyra manifold the constant displacement field vector ϕ_i plays the role of cosmological constant Λ [18]. Recently,

many researchers have examined various models of the Universe with different energy-momentum components in Lyra's geometry. Aygun et al. [19] have searched the non-existence of a massive scalar field for the Marder universe in Lyra and Riemannian geometries. Singh [20] investigated various solutions of the Einstein field equations for cylindrically symmetric metric in Lyra's geometry. At the cosmological scale, the present universe is defined by a Friedmann-Robertson-Walker (FRW) universe. Also, FRW universes in which the metric arises from material source alone are satisfied in the framework of Mach's principle [21]. Singh et al. [22] have investigated FRW models in the presence of a bulk viscous fluid with deceleration parameter in the Lyra geometry.

In the early phase of evolution of the Universe it is believed that quark-gluon plasma existed when the temperature was $\sim 200\text{MeV}$. The two ways of formation of quark matter are the quark-hadrons phase transition and the conversion of neutron stars into strange ones at ultra high densities [23-25]. The equation of state based on the phenomenological bag model of quark matter where quark confinement is described by an energy term proportional to the volume is used to model the strange quark matter. In this model, quarks are considered as degenerate Fermi gas, which exist only in a region of space endowed with a vacuum energy density B . Moreover, in this model the quark matter is composed of light u , d quarks, massive s quarks and electrons. In our present study, we have considered a simplified version of this model with mass-less and non-interacting quarks.

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Yilmaz et al. [26] studied strange quark matter for the Robertson-Walker model in the context of the general theory of relativity. Yilmaz and Yavuz [27] obtained higher-dimensional Robertson-Walker cosmological models in the presence of quark-gluon plasma in the general theory of relativity. Khadekar et al. [28] confined their work to the quark matter attached to topological defects in general relativity. Mahanta et al. [29] investigated Bianchi type-III cosmological models with strange quark matter attached to string cloud in self-creation theory. Khadekar and Wanjari [30] studied the geometry of quark and strange quark matter in higher-dimensional general relativity. Katore and Shaikh [31] investigated cosmological models with strange quark matter attached to the string cloud in the general theory of relativity for axially symmetric space-time. Mahanta and Biswal [32] studied string cloud and domain walls with quark matter in Lyra geometry. Rao and Neelima [33] discussed axially symmetric space-time with strange quark matter attached to the string cloud in self-creation theory and general relativity. Rao and Sireesha [34] discussed axially symmetric space-time with strange quark matter attached to the string cloud in the Brans-Dicke theory of gravitation.

In string theory, the innumerable particle types are replaced by a single fundamental building block, called a ‘string’. These strings can be closed, like loops, or open, like a hair. As the string moves through time it traces out a tube or a sheet, according to whether it is closed or open. Furthermore, the string is free to vibrate, and different vibrational modes of the string represent the different particle types, since different modes are seen as different masses or spins.

Massive closed loops of string serve as seeds for the formation of large structures like galaxies and cluster of galaxies. While matter is accreted onto loops, they oscillate violently and lose their energy by gravitational radiation and therefore they shrink and disappear. If string exists, they can produce a number of characteristic observational effects detectable with existing astronomical instruments. Kaiser and Stebbins [35] pointed out that such strings would produce density fluctuations on very large scales and may be responsible for the formation of large scale structures. The gravitational effects of cosmic strings are responsible for creation of galaxies and clusters [36,37]. Letelier [38] investigated Bianchi type I and Kantowski-Sachs string cosmological models, which evolve from a pure massive string dominated era to a particle dominated era, with or

without remnant of strings. Reddy et al. [39-41], Venkateswarlu et al. [42-45], Satish [46], Rao et al. [47-49] and Pradhan et al. [50-52] are some of the authors who have studied various aspects of string cosmologies in theory of relativity as well as in alternate theories of gravitation.

In view of the importance of bulk viscosity in cosmological models with respect to the accelerated expansion phase of the Universe, in this work we construct bulk viscous string cloud cosmological model with strange quark matter attached to one dimensional cosmic string in Lyra geometry. In the presence of quark matter in the Universe, the total matter energy density is considered to consists of quark energy density ρ_q and the vacuum energy density B_c ,

$$\rho_m = \rho_q + B_c \tag{1}$$

and the total pressure is

$$p_m = p_q - B_c \tag{2}$$

Where, the quark pressure is expressed as

$$p_q = \frac{\rho_q}{3} \tag{3}$$

2. Metric and Field Equations

Einstein field equation based on Lyra’s geometry, which on normal gauge may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -T_{ij} \tag{4}$$

Where, ϕ_i is the displacement vector defined as

$$\phi_i = (0,0,0,\beta(t)) \tag{5}$$

Here, we consider the five dimensional FRW metric of the form

$$ds^2 = -dt^2 + R^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] + A^2(t) d\mu^2 \tag{6}$$

Where, $R(t)$ is the scale factor and $k = 0, -1$ or $+1$ is the curvature parameter for flat, open and closed universe, respectively. The fifth coordinate μ is also

assumed to be space like coordinate. The average scale factor is defined as

$$a(t) = (R^3 A)^{\frac{1}{3}} \tag{7}$$

The energy-momentum tensor for a cloud of massive strings with bulk viscosity can be written as

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij} - \lambda x_i x_j - \xi u_{;i}^i (u_i u_j - g_{ij}) \tag{8}$$

Here ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and ξ is the bulk coefficient of viscosity. The velocity u^i describes the five-velocity, which has components (1, 0, 0, 0, 0) for a cloud of particles and x^i represents the direction of string that satisfies

$$u^i u_i = -x^i x_i = 1 \quad \text{and} \quad u^i x_i = 0 \tag{9}$$

The total rest energy density is the sum of energy density due to particles ρ_p and tension density,

$$\rho = \rho_p + \lambda \tag{10}$$

We assume that the Universe consists of mass-less non-interacting quarks of all possible flavors, which contribute to the energy density due to particles $\rho_p = \rho_q + B_c$ and hence

$$\rho = \rho_q + \lambda + B_c \tag{11}$$

The direction of the strings is taken to be along X_4 -axis so that we have $X^i = (0, 0, 0, 0, 1/A)$. Now the field equations for the metric (Eqn. (4)) can be written as

$$2 \frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 2 \frac{\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} + \frac{k}{R^2} + \frac{3}{4} \beta^2 = \xi\theta - p \tag{12}$$

$$3 \frac{\ddot{R}}{R} + 3 \frac{\dot{R}^2}{R^2} + 3 \frac{k}{R^2} + \frac{3}{4} \beta^2 = \rho \tag{13}$$

$$3 \frac{\ddot{R}}{R^2} + 3 \frac{\dot{R}\dot{A}}{RA} + 3 \frac{k}{R^2} - \frac{3}{4} \beta^2 = \xi\theta - p + \lambda \tag{14}$$

Where, overhead ‘dot’ denotes ordinary differentiation with respect to t .

The energy conservation equation $T_{\mu;\nu}^\mu = 0$ leads to

$$\dot{\rho} + \left(3 \frac{\dot{R}}{R} + \frac{\dot{A}}{A}\right)(\rho + p) - \lambda \frac{\dot{A}}{A} - \xi(n+3)^2 \frac{\dot{R}^2}{R^2} = 0 \tag{15}$$

and

$$\left(R_\mu^\nu - \frac{1}{2} g_\mu^\nu R\right)_{; \nu} + \frac{3}{2} (\phi_\mu \phi^\nu)_{; \nu} - \frac{3}{4} (g_\mu^\nu \phi_k \phi^k)_{; \nu} = 0 \tag{16}$$

Eqn. (16) leads to

$$\frac{3}{2} \beta \dot{\beta} + \frac{3}{2} \beta^2 \left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right) = 0 \tag{17}$$

3. Solutions of Field Equations

To get a deterministic solution of field equations, additional conditions are required. One can introduce more conditions either by an assumption corresponding to some physical condition of an arbitrary mathematical assumption. Here, we take a time-dependent deceleration parameter. The reason being that the Universe was decelerating in the past and accelerating at the present time, that is, the deceleration parameter shows signature flip. So, in general, the deceleration parameter is not a constant but time variable. This motivates us to choose an average scale factor that yields a time-dependent deceleration parameter.

Generally, the utmost importance is given to average scale factor $a(t)$ that explains practically all cosmological phenomena of the Universe. Pradhan [50] used a time-dependent deceleration parameter taking the average scale factor as a function of time. Following this, we take an analogous approach to gamma function for the average scale factor as

$$a(t) = e^{-t} t^{m-1} \tag{18}$$

Where, m is non-zero constant. The above equation gives rise to a time dependent deceleration parameter.

Since the gamma function is capable of representing constant, increasing and decreasing rate of deterioration, we consider the average scale factor as in Eqn. (18) whose evolution with respect

to time is given in Fig. 1. For large m , the graph of average scale factor $a(t)$ looks like a open curve and becomes flat as the value of m decreases.

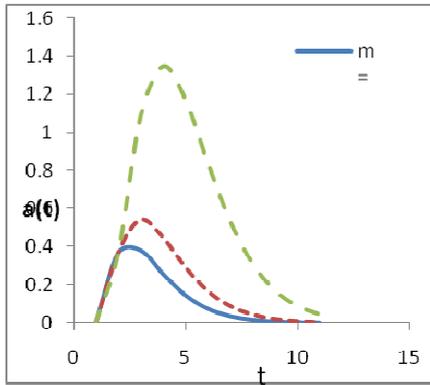


Fig.1(a): Evolution of the average scale factor versus time for $m=2.5,3,4$

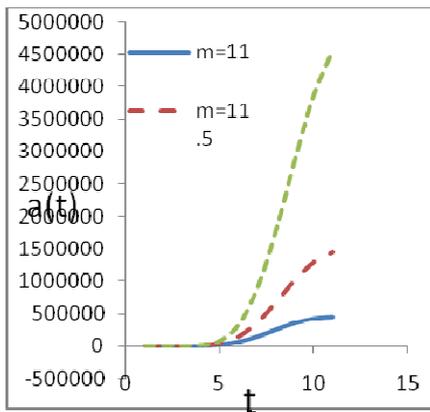


Fig.1(b): Evolution of the average scale factor versus time for $m=11,11.5,12$

Also a power law equation is considered due to the fact that there is still anisotropy for the flat and homogeneous universe and $\theta \propto \sigma_{ij}$ (shear tensor), so we use the following polynomial relation between metric coefficients

$$A=R^n \tag{19}$$

Where, n is an arbitrary constant.

Therefore, from Eqns. (7), (19) and (20), we obtain the scale factors explicitly as

$$R(t) = \left(e^{-t} t^{m-1} \right)^{\frac{3}{n+3}} \tag{20}$$

$$A(t) = \left(e^{-t} t^{m-1} \right)^{\frac{3n}{n+3}}$$

And, the displacement vector is given by

$$\beta(t) = c_1 e^{3t} t^{3(1-m)} \tag{21}$$

The variation of β with respect to time is given in Fig. 2.

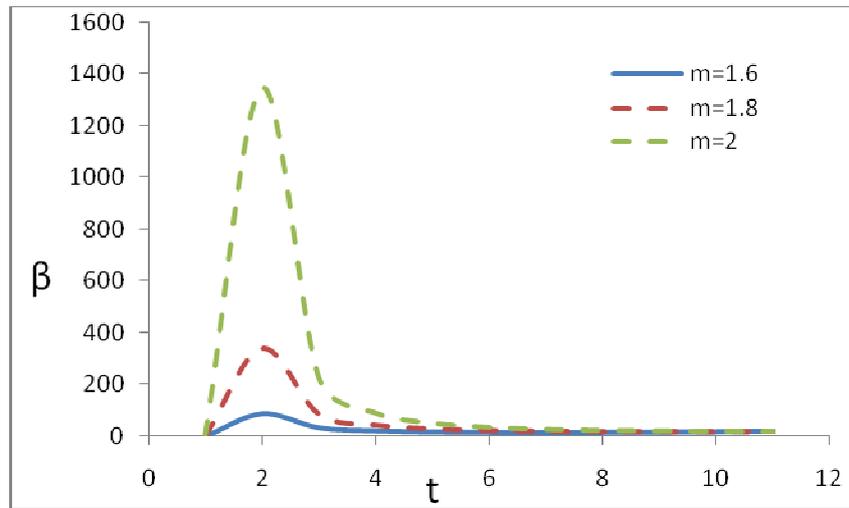


Fig.2: The evolution of displacement vector β for $c_l=1$

Thus, the metric in Eqn. (6) for FRW model takes the form

$$ds^2 = -dt^2 + \left[\left(e^{-t} t^{m-1} \right)^{\frac{6}{n+3}} \right] \left[\frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] + \left(e^{-t} t^{m-1} \right)^{\frac{6}{n+3}} d\mu^2 \tag{22}$$

From Eqns. (12)-(14), we obtained the string tension density as

$$\lambda = \frac{(3 - 2n - n^2)[9t^2 - 18(m-1)t] + 3(m-1)[(4n^2 + 6n - 10) + m(7 - 4n - 3n^2)]}{(n+3)^2 t^2} + \frac{2k}{\left(e^{-t} t^{m-1} \right)^{\frac{6}{(n+3)}}} \tag{23}$$

Fig. 3 is the plot of string tension density (λ) versus time (t). It is clear from the figure that λ is positive decreasing function of time converging to a small positive value at late times for closed ($k=1$). For flat model ($k=0$), the string tension density is almost coincident with zero and for open model ($k=-1$), λ is negative increasing function of time converging to a small negative value at late times.

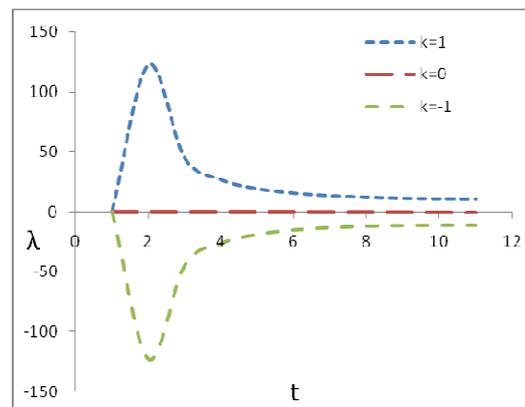


Fig.3: Evolution of the string tension density for $n=0.5, m=2$

From Eqn. (13), we carry out the energy density

$$\rho = \frac{3(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{(n+3)^2 t^2} - \frac{3c_1^2}{4} e^{6t} t^{6(1-m)} + \frac{3k}{(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} \tag{24}$$

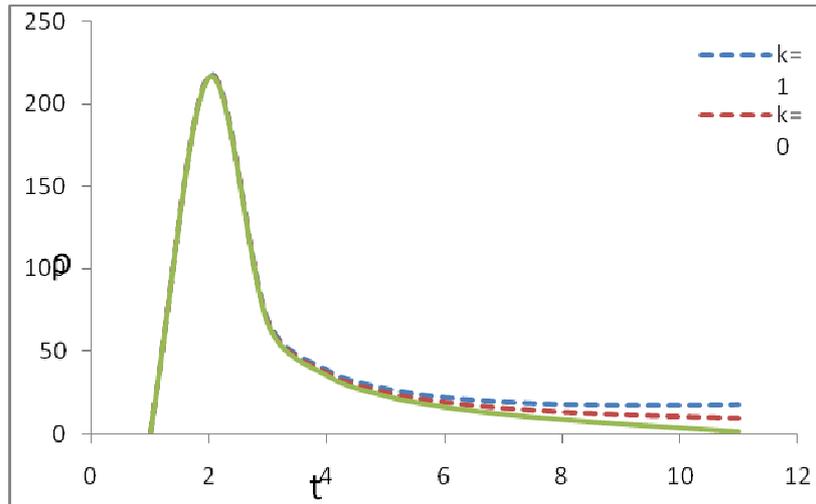


Fig.4: Evolution of the energy density for $n=3, m=0.25, c_1=.001$

Fig. 4 depicts the positive decreasing behavior of energy density (ρ) versus time (t). It clearly elaborates that ρ starts from a very large value and approaches to zero as anticipated. This is true for all closed, flat and open models.

The energy density due to particles obtained as

$$\begin{aligned} \rho_p = \rho - \lambda = & \frac{3(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{(n+3)^2 t^2} - \frac{3c_1^2}{4} e^{6t} t^{6(1-m)} \\ & \frac{(3-2n-n^2)[9t^2 - 18(m-1)t] + 3(m-1)[(4n^2 + 6n - 10) + m(7 - 4n - 3n^2)]}{(n+3)^2 t^2} \\ & + \frac{k}{(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} \end{aligned} \tag{25}$$

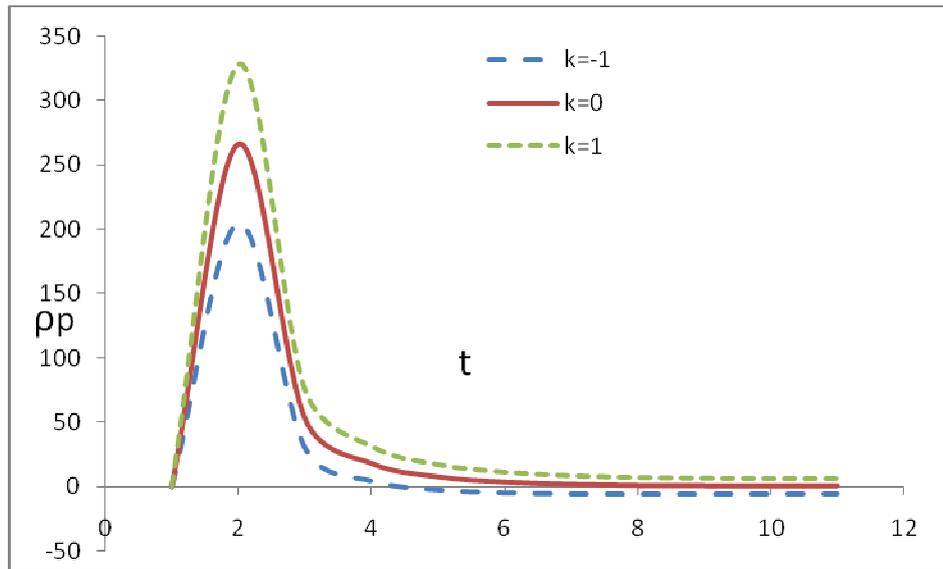


Fig.5: Plots of rest energy density ρ_p vs. time t for $c_1=0.001, m=5, n=0.5$

From Eqn. (25), it is observed that the rest energy density ρ_p is a decreasing function of time and $\rho_p > 0$ always for open model ($k = -1$). The rest energy density versus time has been plotted in Fig. 5. It is evident that the rest energy density remains positive in closed model ($k=1$). However, it increases more sharply with the cosmic time in the closed model. Whereas in the flat model ($k=0$), the rest energy density approaches to zero to negative values.

Using Eqn. (15), we get the pressure as

$$p = \frac{n\lambda}{(n+3)} - \frac{\dot{\rho}}{(n+3)\frac{\dot{R}}{R}} - \rho + \xi(n+3)\frac{\dot{R}}{R} \quad (26)$$

Where

$$\begin{aligned} \dot{\rho} = & \frac{54(1+n)(m-1)(t+1-m)}{t^3(n+3)^2} \\ & + 18k(e)^{\frac{6t}{(n+3)}} t^{\frac{(5-6m)}{(n+3)}} \frac{(t+1-m)}{(n+3)} \\ & - \frac{9c_1^2}{2} e^{6t} t^{(5-6m)} (t+1-m) \end{aligned}$$

$$\begin{aligned} P = & \left(\frac{n}{(n+3)} \right) \left\{ \frac{(3-2n-n^2)[9t^2 - 18(m-1)t] + 3(m-1)[4n^2 + 6n - 10] + m(7-4n-3n^2)}{(n+3)^2 t^2} + \frac{2k}{(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} \right\} \\ & - \left(\frac{t}{3(m-1)-3t} \right) \left\{ \frac{54(1+n)(m-1)(t+1-m)}{t^3(n+3)^2} + 18k(e)^{\frac{6t}{(n+3)}} t^{\frac{(5-6m)}{(n+3)}} \frac{(t+1-m)}{(n+3)} \right. \\ & \left. - \frac{9c_1^2}{2} e^{6t} t^{(5-6m)} (t+1-m) \right\} - \frac{3(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{(n+3)^2 t^2} + \frac{3c_1^2}{4} e^{6t} t^{6(1-m)} \\ & - \frac{3k}{(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} + \xi(n+3) \left(\frac{3(m-1)-3t}{(n+3)t} \right) \end{aligned} \quad (27)$$

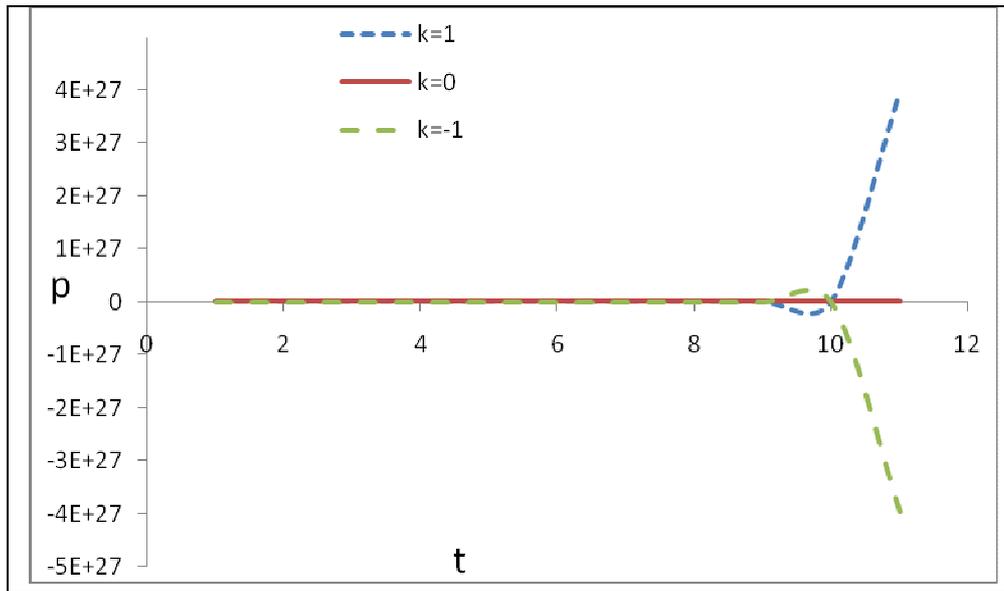


Fig.6: Plots of Pressure p vs. time t for $n=3, m=2, c_1=0.001$

Fig. 6 shows the variation of isotropic pressure versus time in all three cases for ($k=0, k=1,$ and $k=-1$). We now consider our new energy-momentum tensor of perfect fluid form. The energy-momentum tensor can be given as the energy-momentum tensor for quark matter, according to the result of the experiments of the Brookhaven laboratory [53]. By using a linear equation of state, we investigate strange quark matter for a higher dimensional FRW universe in the Lyra geometry as

$$p = \epsilon(\rho - \rho_0) \tag{28}$$

Where, ρ_0 represents the energy density at zero pressure [54]. If one considers $\epsilon = \frac{1}{3}$ and

$$B_c = \frac{3(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{4(n+3)^2 t^2} - \frac{3c_1^2}{16} e^{\delta t} t^{6(1-m)} + \frac{3k}{4(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} - \left(\frac{3n}{4(n+3)} \right) \left\{ \frac{(3-2n-n^2)[9t^2 - 18(m-1)t] + 3(m-1)[(4n^2 + 6n - 10) + m(7 - 4n - 3n^2)]}{(n+3)^2 t^2} + \frac{2k}{(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} \right\}$$

$\rho_0 = 4B_c$, in Eqn. (28), the equation of state for the bag model (SQB) is obtained by

$$p = \frac{\rho - 4B_c}{3} \tag{29}$$

Where, B_c is named bag constant [55]. On the other hand, if one considers $\epsilon = \omega$ and $\rho_0 = 0$, in Eqn. (28), we obtain the EOS for quark matter as follows

$$p = \omega\rho \tag{30}$$

From Eqn. (29) the equation of state for the bag model (SQB) is obtained by

$$\begin{aligned}
 & + \frac{3}{4} \left(\frac{t}{3(m-1)-3t} \right) \left\{ \frac{54(1+n)(m-1)(t+1-m)}{t^3(n+3)^2} + \frac{27}{2} k (e)^{\frac{6t}{(n+3)}} t^{\frac{(5-6m)}{(n+3)}} \frac{(t+1-m)}{(n+3)} \right. \\
 & \left. - \frac{27c_1^2}{4} e^{6t} t^{(5-6m)} (t+1-m) \right\} - \frac{9(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{4(n+3)^2 t^2} \\
 & + \frac{9c_1^2}{16} e^{6t} t^{6(1-m)} + \frac{9k}{4(e^{-t} t^{m-1})^{\frac{6}{(n+3)}}} - \frac{3}{4} \xi(n+3) \left(\frac{3(m-1)-3t}{(n+3)t} \right)
 \end{aligned} \tag{31}$$

Adding Eqn. (12) and Eqn. (14), we have the coefficient of bulk viscosity as

$$\begin{aligned}
 \xi = \frac{t}{3(m-1-t)} & \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+5)}{4(n+3)^2 t^2} + \right. \\
 & \left. + \frac{9(n^2 + n + 4)(m-1-t)^2}{(n+3)^2 t^2} + \frac{3}{2} c_1^2 e^{6t} t^{6(1-m)} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} + 2p - \lambda \right)
 \end{aligned} \tag{32}$$

Quark energy density

$$\rho_q = \rho - B_c = \frac{3(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{(n+3)^2 t^2} - \frac{3c_1^2}{4} e^{6t} t^{6(1-m)} - B_c, \tag{33}$$

Quark pressure

$$p_q = \frac{\rho_q}{3} = \frac{(1+n)[9t^2 - 18t(m-1) + 9(m-1)^2]}{(n+3)^2 t^2} - \frac{c_1^2}{4} e^{6t} t^{6(1-m)} - \frac{B_c}{3} \tag{34}$$

For the case $k = 0$, we get the almost equivalent expression for ρ and p . Also for the case $k = \pm 1$ we get the equivalent expression for quark energy density and quark pressure. It is also observed that in all the cases the total pressure is negative for some t due to B_c (bag constant or vacuum energy density). Because Milton [56,57] has pointed out that the normalized zero point energy of the confined fields in the MIT bag model is repulsive rather than attractive. So, vacuum energy density (bag constant) may have negative pressure. In this case quarks and gluons which are confined move freely. They are not moving collectively, i.e., as a perfect liquid.

Using Eqns. (12), (13), (19) and (20), we get exact solutions of the higher dimensional FRW universe for SQL and SQB in the Lyra geometry with the aid of Eqns. (28) and (29). Also, if we consider Eqn. (30), we obtain exact solutions for quark matter.

(i) Exact solutions of the higher dimensional FRW universe for SQL (strange quark linear) with the $p = \epsilon(\rho - \rho_0)$ condition.

Using Eqns. (12), (13) and also Eqn. (28), we get, respectively, the energy density, pressure and displacement field vector for strange quark matter in linear form as

$$\rho = \frac{1}{(1-\epsilon)} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} \right) - \frac{\epsilon \rho_0}{(1-\epsilon)} - \frac{3(m-1-t)\xi}{(1-\epsilon)t} \tag{35}$$

$$p = \frac{\epsilon}{(1-\epsilon)} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} \right) - \frac{\epsilon^2 \rho_0}{(1-\epsilon)} - \frac{3\epsilon(m-1-t)\xi}{(1-\epsilon)t} \tag{36}$$

$$\beta^2 = \frac{4k}{3(e^{-t} t^{m-1})^{\frac{6}{n+3}}} + \frac{2(m-1-t)\xi}{(1-\epsilon)t} - \frac{2}{3} \frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} - \frac{2}{3} (n^2 - 2n - 2) \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} - \frac{2}{3} \rho(1+\epsilon) + \rho_0 \frac{2}{3} \tag{37}$$

(ii) Exact solutions of the higher dimensional FRW universe for SQB with the $p = \frac{\rho - 4B_c}{3}$ condition.

displacement field vector for strange quark matter in bag model as

From (12), (13) and also Eqn. (29), we get, respectively, the energy density, pressure and

$$\rho = \frac{3}{2} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} \right) - 2B_c - \frac{9(m-1-t)\xi}{2(1-\epsilon)t} \tag{38}$$

$$p = \frac{1}{2} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} \right) - \frac{10B_c}{3} - \frac{9(m-1-t)\xi}{2(1-\epsilon)t} \tag{39}$$

$$\beta^2 = \frac{4k}{3(e^{-t} t^{m-1})^{\frac{6}{n+3}}} + \frac{2(m-1-t)\xi}{(1-\epsilon)t} - \frac{2}{3} \frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} - \frac{2}{3} (n^2 - 2n - 2) \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} - \frac{8}{9} (B_c - \rho) \tag{40}$$

(iii) Exact solutions of the higher dimensional FRW universe for quark matter with the $p = \omega\rho$ condition. We obtain the energy density, pressure

and displacement field vector, respectively, from Eqns. (12), (13) and also using Eqn. (30) as

$$\rho = \frac{1}{(1-\omega)} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{(e^{-t} t^{m-1})^{\frac{6}{n+3}}} \right) - \frac{3(m-1-t)\xi}{(1-\varepsilon)(1-\omega)t} \tag{41}$$

$$p = \frac{\omega}{(1-\omega)} \left(\frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} + \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} + \frac{4k}{e^{-t} t^{m-1} \frac{6}{n+3}} \right) - \frac{\omega}{(1-\omega)} \frac{3(m-1-t)\xi}{t} \tag{42}$$

$$\beta^2 = \frac{4k}{3(e^{-t} t^{m-1})^{\frac{6}{n+3}}} + \frac{2(m-1-t)\xi}{t} - \frac{2}{3} \frac{[9t^2 - 18t(m-1) + 3(m-1)(m-n-4)](n+2)}{4(n+3)^2 t^2} - \frac{2}{3} (n^2 - 2n - 2) \frac{(n+2)^2 9(m-1-t)^2}{(n+3)^2 t^2} - \frac{2}{3} (\omega+1)\rho \tag{43}$$

An important cosmological parameter is the dark energy equation of state $\omega = \frac{p}{\rho}$, which is exactly equal to -1 for a cosmological constant.

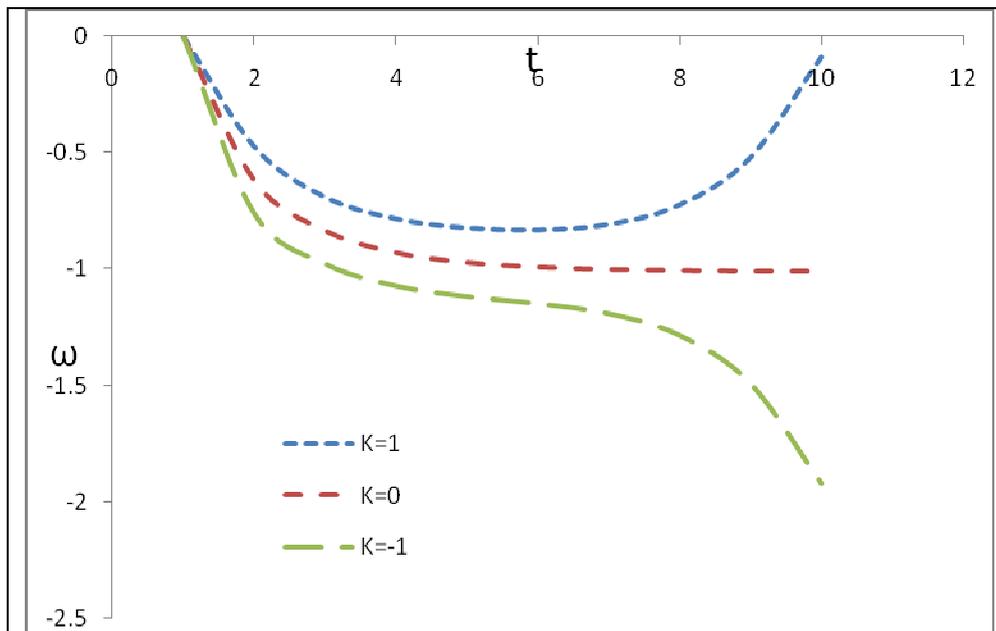


Fig.7: Evolution of EoS parameter $c_1=1, n=2, m=3$

From Fig. 7, the equation of state parameter takes on negative values in the range $-1 \leq \omega < -0.4$ in Lyra geometry. Therefore, dark energy density transits from vacuum era to quintessence era in Lyra manifold. At the early stage EoS parameter in Lyra geometry mimic vacuum era, which is mathematically equivalent to the cosmological

constant. This class of value of EoS parameter is called quintessence $\omega < \frac{-1}{3}$, which is a necessary condition to accelerate the Universe (Sahni et al. [58]). The higher dimensional flat FRW universe the EoS parameter of dark energy transits from

quintessence $\frac{-1}{3} > \omega > -1$ era toward vacuum era $\omega = -1$. From this figure we observe that the dark energy model for $c_1=1, n=2,$ and $m=3$ evolves from the matter dominated era to quintessence era and ultimately approaches to cosmological constant era. The future of cosmic acceleration studies depends partly on the facilities built to enable them, partially on the ingenuity of experimenters and theorists in controlling systematic errors and fully exploiting their data sets, and partly on the kindness of nature. The next generation of experiments could merely tighten the front around $\omega = -1$, ruling out many specific theories but leaving us no more enlightened than we are today about the origin of cosmic acceleration. However, barely a decade after the first supernova measurements of an accelerating universe, it seems unwise to bet that we have uncovered the last “surprise” in cosmology. Equally important, the powerful data sets required to study cosmic acceleration support a broad range of astronomical investigations. These observational efforts are natural next steps in a long-standing astronomical tradition: mapping the Universe with increasing precision over ever larger scales, from the solar system to the Galaxy to large scale structure and to the CMB.

4. Evolution of the Universe with Time Dependent Deceleration Parameter

The deceleration parameter q is defined as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} \tag{44}$$

Where, $a(t)$ represents the average scale factor, which gives information about the expansion of the Universe. Any change of state of the motion of the Universe from decelerated to accelerated phase should cause the deceleration parameter q to change its sign from positive to negative. The functional form of q in Eqn. (44) clearly indicates that it can change sign (say from positive to negative) as a function of time, only if \ddot{a} changes sign in the opposite manner (say from positive to negative). Let us choose a simple functional form for average scale factor $a(t)$ such that its double derivative \ddot{a} shows a signature flip at a certain instant of time t . For this purpose, let us take the following functional form of the scale factor given by Eqn. (18) as

$$a(t) = e^{-t} t^{m-1} \tag{45}$$

Using the above functional form of the average scale factor, the deceleration parameter comes out to be

$$q(t) = -\frac{[t^2 - 2(m-1)t + (m-1)(m-2)]}{[(m-1)-t]^2} \tag{46}$$

The Universe would exhibit decelerating expansion if $q > 0$, an expansion with constant rate if $q = 0$, accelerating power-law expansion if $-1 < q < 0$, exponential expansion (also known as de Sitter expansion) if $q = -1$ and super-exponential expansion if $q < -1$. However, with the average scale factor given by Eqn. (62), the Universe inevitably evolves into constant rate if $t = (m-1) - \sqrt{(m-1)}$. We observed that accelerating power-law expansion survive for $m > 14$ and exponential expansion (de Sitter expansion) obtained for $m = 1$. We get the super-exponential expansion for $m < 0$, which is one of the possible fates of the Universe according to the cosmological observations. Signature flip part would exhibit for higher value of m , that is, $1 < m < 14$. These models are shown in Figs. 8-12.

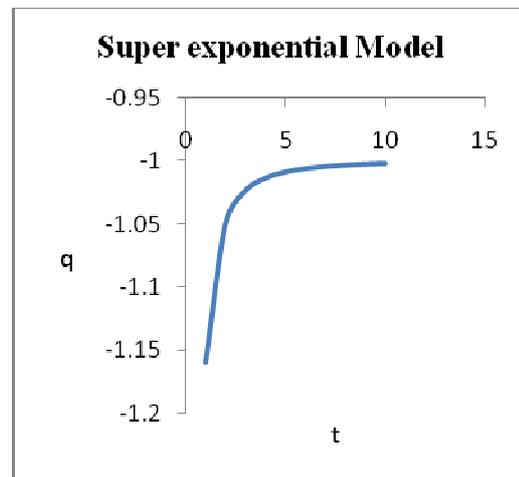


Fig.8: Evolution of DP for $m=-2$

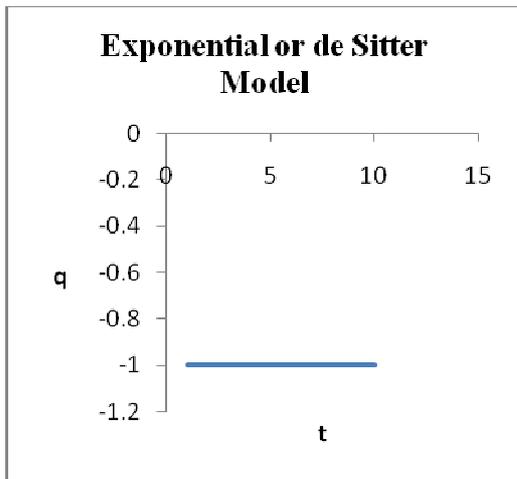


Fig.9: Evolution of DP for $m=1$

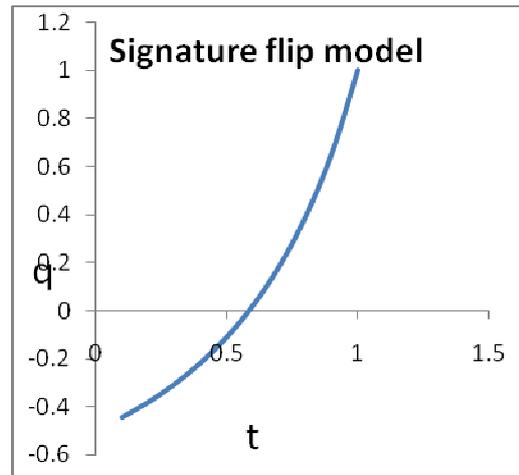


Fig.11: Evolution of DP for $m=3$

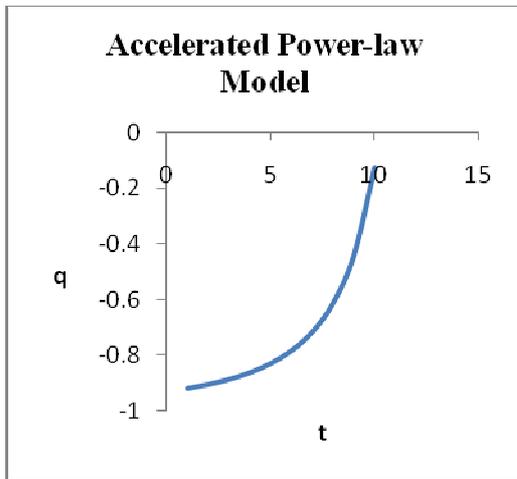


Fig.10: Evolution of DP for $m=15$

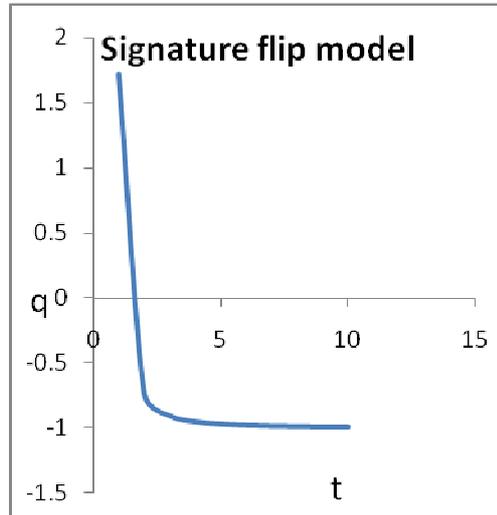


Fig.12: Evolution of DP for $m=1.55$

We can now express the scale factor as a function of deceleration parameter q as follows

$$a(q) = \left[(m-1) - \left(\left(\frac{1+q}{m-1} \right)^{\frac{-1}{2}} \right) \right]^{(m-1)} \exp \left[\left(\frac{1+q}{m-1} \right)^{\frac{-1}{2}} - (m-1) \right] \tag{47}$$

The cosmological red-shift z is related to the scale factor $a(t)$ by the relation

$$z = a^{-1} - 1 \tag{48}$$

or

$$z = -1 + \left[(m-1) - \left(\frac{1+q}{m-1} \right)^{\frac{-1}{2}} \right]^{(1-m)} \exp \left[(m-1) - \left(\frac{1+q}{m-1} \right)^{\frac{-1}{2}} \right] \quad (49)$$

From model dependent or independent analyses of the cosmological observations in the literature [59,60], the transition red-shift of the accelerating expansion is given by $0.3 < z < 0.8$. In particular, the kinematic approach to cosmological data analysis provides direct evidence to the present accelerating stage of the Universe, which does not depend on the validity of general relativity, as well as on the matter-energy content of the Universe.

One may observe that the accelerating expansion begins at $z \cong 0.5$ consistent with the observational data. We particularly would like to demonstrate that by choosing $m = 2.3$, $m=2.4$ and $m= 2.5$, we can obtain the value of red-shift as $z = 0.15$, $z=0.26$ and $z=0.38$, respectively, which fits quite well for spatially flat universe given by Li et al. [61] in a recent paper, where they analyzed the data from SNIa, BAO and CMB simultaneously.

5. Conclusions

In this paper, exact solutions for the generalized higher dimensional FRW universe are obtained in the presence of quark and strange quark matter attached to a perfect fluid in the Lyra geometry. From Eqns. (12)-(14), it is clearly seen that the cloud of string with perfect fluid energy-momentum tensor is survived for the higher dimensional FRW model. The extra dimension $A(t)$ is amenable for contraction if $n = 0$. The dominant energy conditions implies that $\rho > 0$ and $\rho^2 \geq \lambda^2$. These energy conditions do not restrict the sign of λ and accordingly the expressions given by Eqn. (25) satisfies all these conditions. Study of early stages of the evolution of cosmological models in the frame work of gravitation plays an important role. In the present work, we have investigated models in Lyra manifold with the hybrid expansion (product form of exponential and polynomial) model; we observed that the Universe expands with a fastest rate. The anisotropy of the expansion decreases monotonically as time increases.

In the present study we have chosen the scale factor $a(t)$ in such a way that the deceleration parameter q is based on it and q evolves as a function of time. Our choice is $a(t) = e^{-t} t^{m-1}$. The functional dependence between $a(t)$ and $q(t)$ have been analyzed in detailed. The law suggested gives

the opportunity to generalize many of these models with better consistency with the cosmological observations.

The matter content of the Universe is assumed to be a cloud of cosmic strings with particles attached to them. The particles are considered to consist of strange quark matters. Since the anisotropic cosmic fluid is having dissipative phenomena, we have incorporated bulk viscosity in the model. In most of the earlier investigations, the bulk viscosity is assumed to be a power function of the energy density or a power function of the scalar of expansion. In this work we have obtained models in higher dimensional FRW model without considering such functional forms for bulk viscosity. In the present work, we have restricted ourselves to a variable deceleration parameter as predicted from observations.

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